External Influences on Spray Patterns (EPAS)

Report 12: Flow of salt particles on a spinning spreader disk
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Abstract
This work researches the salt spreading process with respect to the complex radial movement of salt particles on a spinning disk. Today, manufacturers that design new spreaders must rely merely on experience and testing since accurate prediction of the salt outflow is still out of reach. Mathematically, conventional flow models for salt motion are usually based on the assumption of continuous flows and single particle mechanics.

The objective of this study was to model the salt flow on a full-scale spreader disk. The idea was to introduce a novel algorithm able to predict the outflow by using a small number of parameters for that particular spreader design. The identification of these parameters was based on the examination of a large number of high-speed videos of salt motion at different rotation speeds. From these videos phenomena such as sliding, rolling and repositioning of the salt particles along their trajectory were enlightened. The key finding was the notion of the salt moving as sub-clusters. This sub-cluster flow was characterized by a steady salt inflow, a complex acceleration over the disk and an outflow that looks like salt-swarms. Each sub-cluster was seen to include many salt particles at the same time; however, their number and constellation were ever-changing. Geometrically, this appeared as changes in the shape of the sub-cluster. The characteristics of these salt constellations and their parameterization have been the focal point of this study.

It was found that the salt motion for a straight-bladed disk could be described mathematically as a sub-cluster motion. In particular, for a spreader with three radial straight blades it was found that only three parameters were required. From these three parameters, it was verified that for rotating speeds in the range: 80-215 (rpm) the outflow predicted by the novel model was comparable to measured outflows.

In the future, this novel sub-cluster approach may be integrated in the design of salt spreaders as a tool to predict the outflow in an early stage of the design process so testing expenses and development period can be diminished.

Keywords: sub-cluster flow, image recognition, density estimation, parametric fitting
1. Introduction
Salt is spread on the roads every year to combat the winter effects of ice and snow which make the roads dangerous for driving (Figure 1). Basically, salt works by lowering the freezing point of water. The most common system to spread the salt is the spinning disk. Its popularity is motivated by its low production costs, its large spreading width, its small size and its simple construction (Aphale et. al, 2003).

![Figure 1. Spreading salt on icy road (BREDAL A/S, 2014)](image1)

The process of the salt in the truck (Figure 2) starts in the tank where it is stored and thanks to the feeder mechanism it is supplied to the spreader disk where it is distributed into the air. Eventually, after being airborne it ends up jumping or sliding at the road surface.

![Figure 2. Process of spreading salt](image2)

The centrifugal spreader consists of a rotating disk with blades fixed to the surface. The salt is poured onto the spreader disk colliding with the rotating blades, accelerating outwards, before it eventually leaves the spreader.
There is a vast variety of spreader disk designs (Figures 3 and 4). Normally, they are distinguished according to whether they have a conical or flat disk, with radial or pitched, curved or straight blades. Furthermore, spreader disks are characterized by their number of blades, geometry of the blades, diameter of the disk, angle of the disk and the disk material.

The objective of this study was to describe the flow of rock salt particles on a full-scale spreader disk and to develop a parametric model based on a sub-cluster approach. The strategy is to introduce a small set of parameters that can characterize the physics of the complex flow on a spreader disk from a simple mathematical model.

1.1. State of the art
The performance of spinning spreader disks has been widely investigated (Patterson & Reece, 1962; Hao, Jianqun & Hong, 2013; Macías, 2014). Abundant are the papers which analyse spreading of fertilizer and less are the ones with salt as object of the investigation. Nevertheless
the physics behind the movement of both materials on the spreader disk is comparable. Therefore, research on salt spreading on roads as well as the distribution of fertilizers for agricultural purposes have been studied.

The objective of the studies has been quite varied. While many authors have developed analytical models of the particles on the disk or/and in the air before landing on the ground, others have focused on factors influencing the spreading pattern.

**Analytical models**

The motion of a single spherical particle on a flat spreader disk with radial straight blades was studied by (Patterson & Reece, 1962). They distinguished between rolling and sliding motion of the particles. The analytical models developed assumed a near-centre feed and neglected bouncing of particles against the blades. A comparison between these models and experimental data using steel balls from bearings showed reasonable agreement with the radial and total velocities of the particles leaving the disk as well as on the angle between where particles hit the road and where they leave the disk. A model for an off-centre feed using the same disk configuration but taking into account the existence of impacts between the particle and the blade was also developed (Inns & Reece, 1962). This model satisfied the movements of the steel balls but performed poorly when studying irregular shaped particles. Models for a flat disk with straight and curved blades and for concave disks were developed by (Cunningham, 1963). A couple of years later, (Cunningham & Chao, 1967) proved the consistency of the models by getting experimental data of the angle at which various fertilisers left the disk.

Abundant is the research focused on developing models for the motion of the off-spinner particle through the air after being released from the spreader disk. Keeping this in mind, (Reints & Yoerger, 1967) made a simulation of the trajectory of a particle through the air. Moreover, the trajectory for an off-spinner particle considering a non-horizontal release was described (Mennel & Reece, 1963). They studied both spherical and irregularly-shape particles analytically but they only got experimental data using steel balls from bearings. They claimed good agreement between model and experiments. A couple of years later, the off-spinner expressions for particle trajectories through the air were simplified (Pitt, Farmer & Walker, 1982). They obtained an error less than 7% between their solution and the model of (Mennel & Reece, 1963).

The models described above have been the basis for several simulation studies. (Griffis, Ritter & Matthews, 1983) based their simulations of the trajectory of the particle on the disk upon the model of (Patterson & Reece, 1962). They made a comparison between their predicted spreading patterns and experimental data using two types of fertilizers and did not find good agreement. They blamed particle interactions and irregularly-shaped particles of the discrepancies found. Other authors also attributed the abundant discrepancies between their predicted spreading pattern and their experiments to particle interactions (Olieslagers, Ramon & De Baerdemaeker, 1996). They gathered data from a spreader manufacturer and compared it to simulations developed for the same fertilizer and spreader. In order to improve the results they tried to introduce several parameters in the simulation with little success until they took into account the importance of the point on the disk where the particles drop down. When using a set of parameters representing the drop area of particles the simulation was remarkably
improved. This ‘drop area’ parameter was also included by other authors in their simulations to satisfactorily predict spreader distribution patterns for a conical disk equipped with pitched blades (Dintwa et al., 2004b; Dintwa et al., 2004a). The model was derived as a generalisation of an earlier model which could only characterize particle motion on a flat disk with radial blades. These authors determined statistically a distribution pattern by treating the collective flow as a superposition of a large number of particles motion varying their initial positions on the disk. (Takai, 2013b) developed a set of equations to simulate the motion of salt particles through the air. He considered micro and macro systems and took into account size differences in salt particles (Figure 5). Another analytical solution of a particle motion on a concave disk spreader with pitched blades was presented by (Villette et al., 2005). The new form for the differential equation proposed is in agreement with the model elaborated by (Patterson & Reece, 1962) and (Cunningham, 1963). However, it shows some discrepancies with the model stated by (Dintwa et al., 2004b).

![Figure 5. Simulation of particle trajectories in Z-X plane for 1.2, 2.2 and 4.8 mm salt particles into calm air after being ejected horizontally with an initial velocity of 10 m/s (Takai, 2013b)](image)

**Factors influencing spreading pattern**

A multitude factors have been identified as influencing the spreading pattern from a spinner disk dedicated to distribute fertilizer or salt. These can be divided in three main groups: the environment where the spreading takes placed, the machine used and the material spread’s characteristics. A small number of authors have studied the effect of factors from the three groups in the same report, such as (Olieslagers, Ramon & De Baerdemaeker, 1996).

**Environment**

For just one of these groups, (Strøm, 2012a; Strøm, 2012c; Strøm, 2012b) focused their effort on the characterization of environmental factors, specifically air movement. Therefore, he studied the effect of cross wind and drafts created behind the truck. He also discussed some methods to maintain desirable shape on the resulting distribution pattern.

**Machine features**

Many researchers have dedicated their works to analyse the relation between machine features, such as salt mass flow or rotation speed of the disk, and spreading patterns. A custom-made prototype spreader for calibrating the model for changes in several spreader settings was used by (Dintwa et al., 2004a). They were able to quantify the influence of spreader adjustments.
such as mass flow, disk width, rotation speed, blade pitch radius or orifice radial dimensions on the spreading pattern. The drop area of the particle was found to have big influence on the spreading pattern (Pitt, Farmer & Walker, 1982; Grift & Kweon, 2006) and that it is possible to get highly uniform shapes of the fertilizer spreading pattern by changing the drop area. (Fulton et al., 2001), using the ASAE 341.2 standardized collection tray method, developed an investigation on the flow rate of fertilizer. They observed a desirable bell shape spreading pattern when using a low application rate, a mediocre M-shape at medium values and an undesirable W-shape at high application rates. Experiments with changes in mass flow and rotational speed have also been carried out (Villette et al., 2012). Results showed that the standard deviation of the distribution of fertilizer increases with the mass flow and decreases with the rotational speed.

Material
The last group is composed of authors who focused their investigations on identifying which fertilizer characteristics change spreading patterns. For this purpose, (Hofstee & Huisman, 1990) investigated the influence of some factors on particle motion on and off the disk. That included fertilizer characteristics such as particle size distribution, coefficient of friction, coefficient of restitution, particle strength and aerodynamic resistance. The effect on the spreading pattern from the particle size distribution has been studied by (Yule, 2011). This study included an analysis of 1700 tray samples which revealed the effect small particles have on the spreading pattern. As the proportion of small particles increases it was observed that the peak value around the centre line also increased. Nevertheless, the author claims that if the proportion remains below 15%, of particles smaller than 0.4 mm the coefficient of variation of spread is not disturbed more than a 5%.

1.2. Sub-cluster behavior
As an alternative to the analytical methods discussed in the state of the art section, instead of using single particle approaches, this research was focused in studying the behaviour of a sub-cluster of salt particles on a spinning spreader disk. It was observed that the flow of particles could be described as a sub-cluster that contained a varying number of particles. The sub-cluster starts growing from when the first particle drops down on the disk. Then more particles follow and when the blade reaches the first particle on the disk it starts collecting them at its front. More and more particles are collected and at a certain stage, particles located at the tip of the blade start leaving the spreader. More particles follow and finally all particles have left the spreader. However, since the particles leave the spreader over only a portion of a full turn, the outflow is characterized by a full sub-cluster of particles that forms a swarm.

The sub-cluster flow of particles is separated into three phases: inflow, underway and outflow as it can be seen in the following figure.
Flow of salt particles on a spinning spreader disk

Figure 6. Characteristic phases of the sub-cluster flow: inflow, underway and outflow

Figure 6 illustrates how the salt is dropped onto the spinning disk (inflow), caught up by the blade (underway) and finally cast out as particle swarms (outflow). During the inflow, particles build up on the rotating disk in front of the blade. Underway (a), when the blade collects the particles a non-stationary irregular sub-cluster is formed. At a following stage (b) the particles start leaving the disk and finally, after all particles have left the disk, their motion in the air can be regarded as a particle swarm\(^1\). Even if the inflow is constant, during one turn the sub-cluster flow transforms from a continuous inflow to a discontinuous outflow.

2. Method

2.1. Spreader disk features

The spreader disk used for the experiments was kindly provided by the company Bredal AS, Denmark. The disk was flat and had a diameter of 0.5 m (Figure 7). It was originally equipped

\(^1\)If the spreader is designed with three blades then three swarms are generated per turn. Also note that when the spreader disk is rotating with a high number of revolutions per minute the salt swarms cannot be recognized by the human eye; however, when the spreading is recorded by a high speed camera, the salt swarms become clearly visible.
with three curved blades but one of them was replaced by a straight blade since in this first approach the purpose was to study a simple geometry. In general conventional blades on the industrial market have non-straight geometries. However, if the kinematics of the salt can be modelled for the chosen simple geometry then probably the modelling of more complex geometries with other characteristic parameters would also be possible.

The blade was 0.281 m long and 0.05 m high (Figure 8).

2.2. Notation

In general, notation for discretization is taken from (Brøchner, 2013) and supplemented with notations for space and time in accordance with ISO 80000-3:2006(E). Notation for mechanics is in accordance with ISO 80000-4:2006(E) and is supplemented with special terms relevant for the description of spinning salt spreaders.

Figure 9 illustrates the global Cartesian frame (left) with origin at road level at the centre of the rear end of the spreading vehicle and with its vertical Z axis upwards. Furthermore it defines the local Cylindrical frame (right) with local origin typically 0.4 m above ground. The local
system is oriented with a vertical z-axis of rotation (green) and its radial axis is directed outwards (blue) and perpendicular to z. This means, that the global Z axis and the local z-axis are parallel. The disk is defined to have a positive angular speed when the disk is spinning counter clockwise around the local z-axis in accordance with the conventional right-hand rule.

The global stationary origin \( x_{xyz,0} \) is defined at road surface in Cartesian coordinates, i.e.:

\[
x_{xyz,0} = [x_0, y_0, z_0]^T
\]  

The kinematics of any point \( P \) on the disk is defined in a local Cylindrical frame with position, velocity and acceleration vectors given by the well-known formulas:

\[
\mathbf{r}_{\rho\phi z} = \begin{bmatrix} r, \rho, \phi, z \end{bmatrix}^T
\]

\[
\mathbf{v}_{\rho\phi z} = \begin{bmatrix} v, \rho \phi, \phi \end{bmatrix}^T
\]

\[
\mathbf{a}_{\rho\phi z} = \begin{bmatrix} a, \rho \phi^2, \phi \end{bmatrix}^T
\]

**Steady Rotation**

For conventional spreader disks in operation the angular velocity \( \dot{\phi} \) is usually constant. A constant angular velocity \( \omega \) also means that the angular acceleration \( \ddot{\phi} \) can be assumed to be zero, i.e.:

\[
\dot{\phi} = \text{Const} = \omega \rightarrow \ddot{\phi} = \frac{d\phi}{dt} = 0
\]

Therefore, for a spinning spreader disk the velocity description in the \( \rho\phi \)-plane can be completely defined by motion variables \( \{ \rho, \dot{\rho}, \dot{\phi} \} \) that originates from the radial direction only (obviously including the z-direction as well), i.e. the angular variables are absent.

\[
\mathbf{v}_{\rho\phi} = \begin{bmatrix} v, \rho \omega, \dot{z} \end{bmatrix}^T
\]

**2.3. Data acquisition from high speed videos**

In order to observe the kinematics of the flow of rock salt on the spreader disk, videos were recorded using a modular and compact high speed camera. The camera was placed focusing on the disk with some angle of inclination. This configuration was adopted to obtain data about the salt along the \( \rho\phi \)-plane and the \( \rho z \)-plane. Throughout filming, the tractor was fixed (Figure 9) and the only movement came from the salt spreader unit, attached to the tractor.

Limitations in the lighting equipment available forced the shoots to be taken outdoors in daytime. Therefore, the sun has been the main and unique light source. A different setting was
used for practically every shooting made because the changing nature of the weather. Besides, a mirror was used to concentrate sun light in the right position on the straight blade avoiding shades in critical places. In short, the lighting condition in every filming session highly influenced the quality of the images obtained.

In Figure 11, it is shown one example of layout used to take the shoots outdoors and the equipment needed.

![Figure 11. Film set](image)

The measures of the mass flow, required for characterizing the salt flow, were obtained indoors in order to avoid, as far as possible, imprecisions due to environmental factors. All equipment needed was a box, a timer and a balance. Further explanation is provided in section 2.5.

### 2.4. Characterization of sub-cluster kinematics

In the sub-cluster model presented in Figure 12, shown in a local frame of reference in Cylindrical coordinates \((\rho \varphi z)\), the salt particles collected by one blade of a spinning spreader disk are assumed to form a non-stationary space time constellation represented by a time dependent sub-cluster \(\Omega_y(t)\).
Flow of salt particles on a spinning spreader disk

In the previously described model, the sub-cluster $\Omega_\Psi$ is discretized into small sub-domains $\Omega$ that are characterized by having a homogenous mass flow inside their finite volumes $V_\Omega$. Each volume is assumed to be represented by its length $\Delta \rho$ and transversal area $A_\Omega$. Assuming that the volume of each subdomain goes towards zero the internal mass-distribution is substituted by the total sub-domain mass located at the instantaneous subdomain centre of mass.

2.5. Considerations taken to obtain radial velocity data

As it was suggested in section 2.4., radial velocity and transversal area of salt of each sub-domain are related. This relation can be specified through the following expression:

$$\dot{m} = \frac{dm}{dt} = \frac{\rho_\Omega V_\Omega}{dt} = \frac{\rho_\Omega A_\Omega \Delta \rho}{dt} = \rho_\Omega A_\Omega \dot{\rho} \quad \Rightarrow \quad \dot{\rho} = \frac{\dot{m}}{\rho_\Omega A_\Omega} \quad (7)$$
where, a sub-cluster of particles has been discretized underway into finite sub-domains \( \Omega \) with length \( \Delta \rho \). Therefore, to obtain the value of the radial velocity \( \dot{\rho} \), the value of the mass flow \( \dot{m} \) is needed. Also necessary are the area \( A_\Omega \) and the empiric density \( \rho_\Omega \) of each sub-domain along the radial axis considered (Figure 13).

![Area along the blade](image1)

![Density distribution](image2)

**Figure 13. Identification of area and density sections**

**First step: frame selection**

The spreader disk has been recorded spreading salt at five different rotation speeds. Each of these videos includes the system movement for two seconds. During this time, the disk turns several times (the exact number depends on the rotation speed). The search of the radial scale parameters is based on the study of one particular frame of these videos for each disk velocity ergo it is important to select the appropriate image in every case.

Figure 14 shows a frame selected from the high speed video recording and in parallel it is highlighted the position of the blade corresponding to that frame. The adequacy of the image resides in the situation of the blade at the moment taken. The straight blade has just left the inflow zone, when only a few percentage of the salt dropped has already left the disk and most of the salt has already been collected by the blade.
Flow of salt particles on a spinning spreader disk

Figure 14. Distribution of salt along the blade in a particular frame

Second step: obtaining the mass flow
Assuming a constant flow, the mass of salt which reaches the disk per unit time, was obtained using a box, a balance and a timer. With the salt spreader unit casting salt, the box collected the salt cast out. About 15 seconds later, the spreading stopped and measurements were taken of the weight of salt in the box and the exact time needed for spreading that amount of salt.

Before including the obtained value of mass flow (0.97 kg/s) in the calculations of the radial velocity, some further considerations had to be taken into account. In Figure 15 is illustrated the sub-cluster flow of particles since the moment they are dropped down (a) until the moment the straight blade reached the position of the frame studied (b).

Inflow. Particles are dropped down in front of the blade.

Underway. Particles are collected by the blade.

There is a percentage of salt not gathered by the blade at the instant the frame is taken. Some particles have already left the blade and some others have not yet been collected. This fact is considered in calculations by applying a reduction, in the form of a percentage, to the mass flow.
Flow of salt particles on a spinning spreader disk

previously measured (0.97 kg/s). Each of the five rotation speed corresponds to a percentage based on observations of the fraction of salt collected and not by the blade in the frame.

**Third step: calculation of the area of each sub-domain in the chosen frame**

Situating the high speed camera level with the disk, as in Figure 16a, it is appreciated a triangular shape sub-cluster when the salt is collected by the blade. The same geometry is present in different sizes along the blade (Figure 16b) and is caused by the movement of the spreader disk gathering the salt towards the blade and the gravity.

![Image](image)

**Figure 16. a) Frame from high speed camera level with the disk. b) Characterization of the area of salt along the blade c) Area of a sub-domain**

The areas of such triangular sub-domains are obtained by defining the piecewise linear lines (red) along the blade (Figure 17). To this end, first the coordinates of the points highlighted are determined and then the equations of the lines joining them are defined.

The equation of a straight line with gradient \( m \) and intercept \( c \) on the \( z \rho \)-plane is defined as: \( z = m_{z \rho} \rho + c_{z \rho} \) where, as it is shown in Figure 17, \( z \) provides the height of every sub-domain along the \( \rho \) axis.

With the aim of finding the base of the triangle, the red lines joining the points 4-5, 5-6 and 6-7 are characterized, in the \( \rho \varphi \)-plane, as: \( \varphi = m_{\rho \varphi} \rho + c_{\rho \varphi} \)
Fourth step: estimation of the empiric density of each sub-domain of the sub-cluster in the specific frame

A coefficient of compaction $C_c$ is defined in order to normalize relative to solid salt.

$$C_c = \frac{\rho_{\Omega}}{\rho_c}$$  \hspace{1cm} (8)

Where $C_c$ goes from 0 (not salt at all) till 1 (solid fraction).

The solid salt density $\rho_c$ is considered to be 2165 kg/m$^3$ (Weast & Astle, 1981-1982).

Therefore, the coefficients of compaction for each one of the sections must be defined.

The variations in the salt density along the blade from a single frame are revealed changing the level of contrast and brightness of the frame. An approximated subdivision into sections with different concentration of salt that are fitted by piecewise linear boundaries is illustrated in Figure 18.

Last step: obtaining results

Once the values of mass flow, density and area have been obtained for every radial position along the blade, it is time to apply the expression (7) obtained at the beginning of this chapter. These results are collected in section 3.2.

3. Results

3.1. Flow measurements and considerations

Data of the mass flow was obtained according to section 2.5. Three different combinations of weight and time were taken and the mass flow calculated for each one (Table 1).

<table>
<thead>
<tr>
<th>Table 1. Mass flow measurements</th>
</tr>
</thead>
<tbody>
<tr>
<td>Salt mass (kg)</td>
</tr>
<tr>
<td>----------------</td>
</tr>
<tr>
<td>Data collection 1</td>
</tr>
<tr>
<td>Data collection 2</td>
</tr>
<tr>
<td>Data collection 3</td>
</tr>
</tbody>
</table>

The average of these three measurements: $\dot{m} = 0.97 \text{ kg/s}$

In order to indicate how representative this average is, a measure of dispersion consisting in subtracting the lowest mass flow data measured to the highest and divide that by the average, can be presented: $\%D = 1.44\%$
Taking into account the limitations in the equipment, a value under 5% is acceptable (Villasuso, 2003). Thus, the average is considered as representative.

### 3.2. Radial scaling function

A MATLAB® algorithm has been developed for the determination of values for the radial velocity of each sub-domain along the blade at a certain time.

The interesting positions along the blade are presented in Figure 19.

Moreover, an example of results from this novel algorithm is shown in Figure 20 in a graphic way with the radial position \( \rho \) as abscissa and the radial velocity \( \dot{\rho} \) as ordinate. The graph clearly indicates an exponential tendency in the data obtained.

![Figure 19. Positions along the blade](image)

![Figure 20. Results for the radial velocity](image)

At the particular moment selected for the frame from the high speed camera, the model for each disk rotation speed is obtained by fitting the data to a mathematical function. The intention is to capture important patterns in the evolution of the radial velocity while filtering out noise.
The fitting was carried out using the MATLAB® Curve fitting tool. This tool has determined the mathematical function which best fits to a series of radial velocity data points for each radial position.

An approximating function of the form: 

$$\hat{\rho} = a_i e^{b_i \rho} + c_i$$  \hspace{1cm} (9)

is used for the fitting. The three fitting parameters $a$, $b$ & $c$ can be determined from a standard curve fitting tool. Where, $\hat{\rho}$ is called the “radial scaling function” see (Macías, 2014), $\rho$ is the radial position, the parameter $a$ is called the function´s ordinate-intercept, $b$ is the constant rate and $c$ represents the vertical translation of the data in an exponential function with base $e$.

Figure 21 shows an example of radial velocity data, in black, fit with the exponential function mentioned (9), in blue. As already pointed out, the black dots of figures 20 and 21 are the calculated values for the radial velocity at every point using the expression (7).

The same process has been carried out for the 5 levels of rotation speed.
Flow of salt particles on a spinning spreader disk

Figure 22 shows the graphical representation of the fit functions along the radial position at a characteristic instant of time for the different levels of rotation speed.

Table 2. Collection of fit function parameters and calculation of radial velocities at the edge of the disk

<table>
<thead>
<tr>
<th>i</th>
<th>Disk rotation speed (rpm)</th>
<th>Coefficients of the exponential function</th>
<th>Radial velocity of the sub-domain leaving the disk (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>a</td>
<td>b</td>
</tr>
<tr>
<td>1</td>
<td>80</td>
<td>2.757·10^{-3}</td>
<td>27.94</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
<td>9.625·10^{-4}</td>
<td>31.45</td>
</tr>
<tr>
<td>3</td>
<td>160</td>
<td>6.820·10^{-4}</td>
<td>35.15</td>
</tr>
<tr>
<td>4</td>
<td>190</td>
<td>2.783·10^{-4}</td>
<td>38.25</td>
</tr>
<tr>
<td>5</td>
<td>215</td>
<td>1.503·10^{-4}</td>
<td>40.51</td>
</tr>
</tbody>
</table>

Table 2 shows the fitting parameters determined by using MATLAB® Curve fitting tool and the value of the radial velocity of the salt particles at the tip of the blade for every disk rotation speed.

After fitting data with the exponential function (9), the goodness of fit is evaluated. The first step is a visual examination of the fitted curve displayed what allows visualization of the entire data set at an instant. Beyond this, the MATLAB® Curve fitting tool also provides numerical methods to determine goodness of fit. Among others, the toolbox provides for goodness of fit statistics\(^2\): R-Square. In general good agreement is achieved as seen in Table 3.

Table 3. Summary of the goodness of fit

<table>
<thead>
<tr>
<th>Disk rotation speed (rpm)</th>
<th>Goodness of fit R-square</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
<td>0.9993</td>
</tr>
<tr>
<td>100</td>
<td>0.9968</td>
</tr>
<tr>
<td>160</td>
<td>0.9988</td>
</tr>
<tr>
<td>190</td>
<td>0.9885</td>
</tr>
<tr>
<td>215</td>
<td>0.9987</td>
</tr>
</tbody>
</table>

\( R – square \in [0.9885 – 0.9993] \)

Values close to 1 mean the model success in explaining the variation of the data.

\(^2\) For more details see: [http://www.mathworks.se/help/curvefit/evaluating-goodness-of-fit.html](http://www.mathworks.se/help/curvefit/evaluating-goodness-of-fit.html)
4. Discussion

In this first study, a series of assumptions and approximations were made.

The novel kinematic shape functions intend to incorporate the non-stationary flow complexities due to sliding, rolling and repositioning of the particles inside the salt sub-cluster. This means that the approach includes, at least in a mean value approach, the effects from impulsive forces and moments. This stochastic approach seems to be closer to reality than the analytical models discussed in the state of the art.

The parameterization of the radial scaling function, which is the key focus of this work, requires the identification of different density sections and their coefficients of compaction along the blade at a characteristic instant of time. This task, in this first approach, was developed by simple observation.

It is likely, that a partly automated and improved experimental setup would be more efficient and would increase the quality of the high speed videos in respect to contrast and focus and the number of angles for recording. This would facilitate the recognition of different density sections.

Limitations were also found in the equipment and data processing. The high speed videos were filmed focusing on a spreader disk, originally equipped with three curved blades. One of these blades was exchanged by the straight blade focus of this study. The differences in geometry and weight between the two types of blades installed on the disk caused vibrations in the system while the disk was spinning. For future research is highly recommended to use identical blades.

The geometry of the spreader unit itself hinders the filming of the spreading process from some interesting angles, such as top view. As this study is only focused on the kinematics of the sub-cluster on the disk, a future experimental setup might be designed to isolate the spreader disk from the unit and thereby enable filming with the high speed camera from any angle.

The high speed camera needs plenty of light to get good images. Limitations in the lighting equipment available forced us to record the movies outside, which unfortunately means that the wind also became a source of noise in the data collection.
The radial velocity data obtained from the high speed videos were fitted using the exponential function (9). The fitting was carried out using the MATLAB® Curve fitting tool. Figures 23 and 24 graph the values of the three parameters of the fit functions and the respective radial velocity of the sub-domains leaving the disk. Even though there is no relation established yet between these parameters obtained, the graphs show a clear tendency and existing relation between their values for the different rotation speeds. While the parameter a decreases with the rotation speed, the rest of the parameters increase as the rotation speed goes up. Moreover, the values obtained for the R-square [0.9885 – 0.9993] mean that the fit functions success in explaining the variation of the data. Overall, these findings support the hypothesis that this novel model for parameterization of radial kinematics can fit real world observations through a shape function with only three parameters. A next step could be to study if the model is able to run, as good as for straight blades, with more complex geometries.

Also this piloting study shows that further research in this area could accumulate a significant knowledge bank of experimental data based on high speed videos for various blade designs. Furthermore, this knowledge bank could be the basis for the development of dedicated simulation software that could predict the outflow for new designs at an early stage in the design process. Such an advanced prediction could cut down development time for the manufacturers and optimize the amount of salt spread. Moreover these predictions may be linked to others parts of the spreading process (Figure 2).

Although, the data of radial velocities of rock salt for the disk and blade configurations used in this study is not found in other research, the kinematic results can be compared to other works. Disk dimensions and blade geometries are not the same in comparable studies; however, the range of radial velocities obtained at the tip of the blade is in agreement with several authors (Takai, 2013a; Van Liederkerke et. al, 2009; Van Liederkerke et. al, 2006).
5. Conclusion

This work intended to shed some light on the spreading process with respect to the rotation speed and the radial movement of the salt particles on the disk. Focus was on the movement of the salt on the spreader disk and along the blades. The observation of high speed videos of such salt movements provided essential visual information on the flow complexities due to sliding, rolling and repositioning of the salt particles along their trajectory. Besides, it was observed that the flow of particles could be described as a sub-cluster that contained a varying number of particles.

Based on a large number of videos, a mathematical model was developed that defines the motion of salt particles by a novel sub-cluster approach. It differs from conventional models where the behaviour is usually based on a single particle. The sub-cluster approach is still at its first stage. However, it may lay the foundations for the development of a design tool that speeds up the development time and the optimization of components in future spreader designs.

The key finding of this study is that a novel model for the radial kinematics, based on the development of a radial scaling function with only three parameters, is able to fit real world observations. The model is able to capture important patterns in the evolution of the radial velocity and successfully explains its variation.

This piloting study is based on a simplified design with straight blades. In general, conventional blades on the industrial market have non-straight geometries. However, if the kinematics of the salt could be modelled for the simple geometry chosen then probably the modelling of more complex geometries with other characteristic parameters would also be possible. These promising results encourage extending studies on stochastic sub-cluster kinematics for spreader disks in close collaboration with manufacturers.

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References


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