Predictability of the Aggregate Danish Stock Market

A theoretical and out-of-sample empirical analysis
Abstract
Individual predictor variables have historically been popular in academia for forecasting stock markets returns. Researchers in finance have long tried to predict these stock returns, especially on the American market, but there is still no clear agreement if returns are even predictable, or to what extent.

Given the curiosity in predictability, this thesis pursues finding empirical evidence using different theoretically suggested forecasting methodologies for the predictability of the Danish stock market, using data from 1973 to 2014. More specifically it will emphasise recent methodologies to overcome the drawback of only using individual predictive regressions.

On a theoretical basis two different frameworks for assessing predictability will be presented; the constant expected return model versus the time-varying return model, and the hypothetical relationship between 10 predictor variables and the equity premium will be inspected. Besides utilizing bivariate regression forecast, the paper will also feature the following multivariate forecasting approaches; combination forecast, sum-of-parts, diffusion index and complete subset regression.

A thorough analysis of the aforementioned relationships will be carried out on the Danish equity market, and assessed in regards to the statistical and economic performance. Specifically, the paper will produce three investment criteria which a successful and reliable method should accomplish: 1) achieve positive utility gains, 2) derive no dominating fraction of predictive power from a single period and 3) have a consistent and positive upward trend in predictive power. This should sort out the sub-optimal methods, and produce models which are ideally useful on a future ex-ante basis.

An out-of-sample forecasting period of 1990-2014 will be separated into business cycles, and economically motivated restrictions from Campbell and Thompson (2008) will be imposed.

The forecasting models, representing the time-varying expected return model, will be evaluated relative to the historical average forecast, representing the constant expected return model. A hypothesis test using the MSFE-adjusted statistic proposed by Clark and West (2007) will be conducted, and rejecting the null would imply time-varying expected returns could exist and, more specifically, that stock return predictability would be possible to a certain extent.

The bivariate forecasts proved to have poor predictive power relative to the historical average. The German short-term interest rate fulfilled, as the only individual predictor variable, all the investment criteria, significantly explaining 2% more than the benchmark in the overall period, while the mean-
variance investor would be willing to pay a yearly certainty equivalent return of 2.51% for having this forecast accessible.

The combination forecasts and complete subset regression both fulfilled all the investment criteria, but the best complete subset regression \((k = 5)\) enjoyed the highest predictive power of 2.21% with a certainty equivalent return of 2.38%, consequently representing the best forecasting method in the thesis.

The origin of predictability was primarily found in recession periods for both approaches, which is similar to other academic literature, and the models have notably improved over the last seven years. The findings generally displayed superiority of the multivariate approach over the simple bivariate method, as these enjoyed higher relative predictive power, while reducing exposure to potential biases.

On the basis of time-varying risk premium from fluctuating risk aversion, the results suggest predictability of the Danish stock market.
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Chapter 1: Introduction

Efforts to predict stock returns have a long-standing tradition within finance, especially on the American market, but there is still no clear consensus if returns are predictable. Historically individual predictor variables have been popular in the academic literature, but the seminal paper from Goyal and Welch (2008) dramatically changed how academia view stock return predictability. Recent articles have tried to rebuttal the conclusions put forth by Goyal and Welch (2008) by introducing more complicated forecasting methodologies, such as those in Rapach et al. (2013) and Elliot et al. (2013). However, even the best model would only be able to explain a small fraction of stock returns, since it contains a sizeable unpredictable component, which also rationalizes the fact why there is no consensus on the predictability of stock returns. Establishing predictability would have several beneficiaries, such as academics and practitioners within the industry. In corporate finance, predictability would improve the ability to assess cost of capital calculations in project and firm valuations, and researchers would be able to produce more realistic asset pricing models to explain financial data. This thesis will therefore try to investigate return predictability in the Danish stock market, focusing on using some of the more recently proposed methodologies to overcome the limitation of only using bivariate regression.

1.1 Problem Statement

Given the interest in predictability, this thesis seeks to find empirical evidence into the predictability of the aggregated Danish stock market from 1990 to 2014 by using different theoretically suggested forecasting methodologies.

To facilitate this discussion, the paper will introduce two different frameworks; constant expected return, which represents no predictability, versus time-varying return, which represents predictability. Specifically it will examine the theoretical relationship between 10 predictor variables and the equity premium, and in addition assess their ability to predict stock returns.

An in-depth empirical analysis of this relationship will be conducted on the Danish equity market using bivariate and multivariate forecasting practices, and the findings will be evaluated according to suggested investment criteria, which emphasises both statistical and economic performance. To further elucidate the results, the out-of-sample (OOS) forecasting period of 1990-2014 will be divided into business cycles to find underlying patterns in predictability relative to a historical average forecast, which represents the constant expected return model. Ultimately outperforming the benchmark could imply that time-varying returns exist in the market, and stock return predictability would be plausible to some extent. Theoretically motivated restrictions from Campbell and Thompson (2008) will also be imposed to accommodate potential economic shortcomings of the forecasts.
1.2 Delimitation
As mentioned, the thesis will be limited to only evaluate predictability on the aggregate Danish stock market, which is motivated by the limited amount of empirical research on the matter. Predictability will only be evaluated at a 1-month horizon, since this provides a more substantial amount of data over a relatively short timespan, as well as encouraging more active portfolio management. This paper seeks to utilize conventional predictor variables suggested by financial literature, however due to the limited Danish data available some have been excluded, e.g. the book-to-market ratio. Furthermore, it is not the aim to establish to what extent abnormal profits could have been achieved through the forecasting period, and hence not within the scope of the thesis to test the efficient market hypothesis. Likewise transaction cost will be ignored. More extensive econometric tests to validate assumptions and results have also been excluded. Predictive power will only be evaluated out-of-sample, because this simulates a more realistic environment for a real-time investor. To assure symmetry, the in-sample period 1974:01-1989:12 and the out-of-sample period 1990:1-2014:12 covers both expansions and recessions in the economy. The sample start was chosen as earlier data was unavailable. Finally, the paper simply assumes mean-variance properties for the Danish investors, thus excluding alternative and more complex utility metrics.

1.3 Structure
This dissertation is decomposed into three parts: theory (Ch. 2 & 3), methodology (Ch. 4) and application (Ch. 5). Chapter two accounts for predictability of the stock market, assuming constant expected returns or time varying expected returns.

Chapter three examines the theoretical causal link between the chosen and potential predictor variables against the equity premium.

Chapter four introduces bivariate and multivariate forecasting approaches, alongside economically motivated restrictions. Moreover, this section also provides a framework for critically evaluating stock return predictability and how bias in earlier studies could influence the findings.

Chapter five contains the empirical applications of the prescribed procedures on the Danish stock market, including critique of the proposed model.

Chapter six finally concludes on the findings presented in the dissertation, and makes final remarks upon the possible predictability in the Danish market.
Chapter 2 – Theory of Predictability

In order to apply the proper framework when interpreting the results produced in this paper, it is relevant to contemplate what degree of predictability one should expect from stocks, with respect to the underlying assumptions supporting the conclusions. This section will thus introduce and discuss constant-expected returns and time-varying expected returns, and lastly conclude on the most realistic model.

2.1 Constant Expected Returns

The expected return of a stock is defined notionally by Fama (1970) as:

\[ E(p_{j,t+1}|\Phi_t) = \left[1 + E(r_{j,t+1}|\Phi_t)\right]p_{jt}. \]  

(2.1)

This expression implies that tomorrow’s expected stock price, \( p_{j,t+1} \), equals the current stock price, \( p_{jt} \), multiplied with the expectation of tomorrows expected stock return, \( r_{j,t+1} \), given the information set, \( \Phi \). The traditional way of interpreting stock price movements is by assuming constant expected returns. This model implies stocks are growing at a constant rate, and investors will consider holding a stock as long as it has positive constant return. Theoretically this is expressed as:

\[ E(r_{j,t+1}) = R, \quad R > 0. \]  

(2.2)

Utilizing the implicit nature of this assumption, combined with the theories of the random walk of stocks (Burton, 1973), the development of the stock price follows a random walk with a positive drift:

\[ E(p_{j,t+1}|\Phi_t) = \left[1 + (R|\Phi_t)\right]p_{jt}, \]  

(2.3)

simply expressed as:

\[ E(p_{j,t+1}) = (1 + R)p_{jt}, \]  

(2.4)

hence the price of the stock would be modelled as:

\[ p_{j,t+1} = \alpha + p_{jt} + \epsilon_t, \]  

(2.5)

with \( \epsilon_t \) being the unpredictable shock, alpha equals the constant positive drift factor, and \( p_{jt} \) the current price of the stock. The intuition behind the equation is that tomorrows stock price of asset j, depends on the current price of the stock, plus some constant positive return factor, which is the expected constant return. This is what Fama (1970) defines as a submartingale process, implying that on average it is not possible to make abnormal profits, mathematically shown as:
\[ x_{j,t+1} = p_{j,t+1} - E(p_{j,t+1} | \Phi_t), \] (2.6)

then

\[ E(x_{j,t+1} | \Phi_t) = 0. \] (2.7)

Hence, by definition, the ability to make abnormal profits expressed as the series \( x_{jt} \), follows a stochastic process, which on average always yields 0.

To gain deeper insight into the formation of a stock price, the realized one-period return on stock \( j \) is defined as:

\[ \eta_{j,t+1} = \frac{(p_{j,t+1} + d_{j,t+1}) - p_{j,t}}{p_{j,t}}, \] (2.8)

the equation can be rewritten and solved for \( p_{j,t} \). The term includes unknown variables, thus expectations are taken to both sides

\[ p_{j,t} = \frac{E_t(p_{j,t+1} + d_{j,t+1})}{E_t(1 + \eta_{j,t+1})}, \] (2.9)

however, since it is assumed that \( \eta_{j,t+1} \) is in fact a constant, the equation can be rewritten:

\[ p_{j,t} = \frac{E_t(p_{j,t+1} + d_{j,t+1})}{(1 + R)}. \] (2.10)

In order to compute the multi-period return formula, the law of iterated expectations is utilized:

\[ E(x|\Phi_t) = E(E(x|\Phi_{t+1})|\Phi_t). \] (2.11)

The law implies, that it is not possible to get a better estimate of \( x \), utilizing information set \( \Phi_{t+1} \), when the value of the forecast of \( x \) is already contingent on information set \( \Phi_t \). The next period return will be:

\[ p_{j,t+1} = \frac{E_{t+1}(p_{j,t+2} + d_{j,t+2})}{(1 + R)}, \] (2.12)

which is substituted into the original model:

\[ p_{j,t} = \frac{E_t\left(\frac{E_{t+1}(p_{j,t+2} + d_{j,t+2})}{1+R} + d_{j,t+1}\right)}{(1+R)} = \frac{E_t(d_{j,t+1})}{(1+R)} + \frac{E_t(E_{t+1}(p_{j,t+2})}{(1+R^2)} + \frac{E_t(E_{t+1}(d_{j,t+2})}{(1+R^2)}. \] (2.14)
Collecting the terms gives:

\[ p_{j,t} = E_t \left[ \sum_{i=1}^{K} \left( \frac{1}{1+R} \right)^i d_{j,t+i} \right] + E_t \left[ \left( \frac{1}{1+R} \right)^K p_{j,t+K} \right], \quad (2.15) \]

and it is assumed that:

\[ \lim_{K \to \infty} E_t \left[ \left( \frac{1}{1+R} \right)^K p_{j,t+K} \right] = 0, \quad (2.16) \]

which finally yields

\[ p_{j,t} = E_t \left[ \sum_{i=1}^{\infty} \left( \frac{1}{1+R} \right)^i d_{j,t+i} \right]. \quad (2.17) \]

Consequently this gives rise to the intuitive implication that the stock price of asset \( j \) today, is contingent on the value of future expectations dividends, discounted by the expected constant rate of return, which infers that the value of a stock today is the markets expectations to future dividends. Moreover, as above, it is assumed that the dividend grows by a constant rate and the dividends for period \( t+1 \) is known, meaning expected dividends can be expressed as:

\[ E_t(D_{t,j}^i) = (1 + G) E_t(D_{t+1+j}) = (1 + G)^i D_{t+1}, \quad (2.18) \]

this expression combined with the previous expression finally yields Gordon’s Growth Model (Gordon, 1962):

\[ p_t = \frac{E_t[D_{t+1}]}{R - G} = \frac{(1 + G)D_t}{R - G}, \quad (2.19) \]

which suggests that today’s stock price is positively dependent on today’s paid dividends, and negatively on the constant rate of return. However, as it was established that the expected return of stocks is constant with unpredictable variations, it can be deduced that the stock markets must be unpredictable. This motivates the conclusion, that under the assumption of constant expected returns, predicting stock movements should be impossible.

Principal financial theory dictates that investors are paid a risk premium for taking on systematic risk in their portfolio, meaning that a clear correlation between risk and return exists, violating the previous assumption of constant expected return. This suggests that expected returns, in addition to risk, should vary along the business cycle, forcing a need for a new way to model expected returns.
2.2 Time-Varying Expected Returns

On the basis of section 2.1 it makes sense to introduce the concept of time-varying expected returns. This assumption benefits from the idea that expected returns on the market should be in coherence with the business cycle. Moreover, since a time series of prices are to be investigated, it will be beneficial to involve log returns, due to their time additive nature. However, the introduction of log returns complicates the relationship between price and expected return since it is no longer linear; \( \log(a + b) \neq \log(a) + \log(b) \). Thus the log linear approximation as put forth by Campbell and Shiller (1988a) has to be introduced.

Under this theoretical framework, the continuously compounded log returns will be utilized between time periods, hence the return for an investor holding an asset from time \( t \) until \( t+1 \) is modelled by the non-linear relationship:

\[
    r_{t+1} = \log \left( \frac{p_{t+1} + d_{t+1}}{p_t} \right),
\]

where \( p_t \) is the current stock price, and \( p_{t+1} \) and \( d_{t+1} \) is the price and dividend in time period \( t+1 \), respectively. It is evident that this relationship is non-linear due to the sum of \( \log(p_{t+1} + d_{t+1}) \). To approximate this linear relationship a first order Taylor expansion is utilized. This process takes the derivatives of the function values from a single “centre” point. In this context, the centre point will be the dividend price ratio mean, which is assumed to follow a stationary stochastic process. The relationship is derived as:

\[
    r_{t+1} \cong \rho p_{t+1} + (1 - \rho)d_{t+1} - p_t + k,
\]

with \( p_t = \log(p_t) \) being the price at the end of \( t \), \( d_t = \log(D_t) \), which is the sum of dividends paid in the period \( t \), \( \rho = \frac{1}{1 + e^d - p} \), and \( k = -\log(\rho) - (1 - \rho)\log\left(\frac{1}{\rho - 1}\right) \), is a constant. The log-linear approximation has some important implications for the model. If the assumption of mean reversion is violated, it is subject to an approximation error. In earlier academic literature, it was found that the linear approximation was accurate (Campbell and Shiller, 1988; Cochrane, 1992; Engsted et al., 2010).

Moreover, the multi-period return from this function would be estimated as:

\[
    r_t = \sum_{j=0}^{i-1} \rho^j * r_{1,t+j},
\]

By imposing the terminal condition of no price bubbles in the equity markets i.e. \( \lim_{j \to \infty} \rho^j p_{t+j} = 0 \), and by taking conditional expectations, one can utilize above equations and solve forward for the price like in formula (2.15):
\[ p_t \cong \frac{k}{1 - \rho} + E_t \sum_{j=0}^{\infty} p^j [(1 - \rho) d_{t+1+j} - r_{t+1+j}] \tag{2.23} \]

This relation is imperative to understand time-varying returns. It states that if prices today are high, one must expect future returns to be lower and/or future dividends to be higher. Equivalently, if prices today are low, it is expected that future returns will be high and/or expected future dividends low. When returns are time-varying, Campbell and Cochrane (1999) conclude that stock prices do not follow a random walk like the constant expected return framework. Moreover, Campbell and Cochrane (1999) investigates to what extent the risk aversion, like expected returns, fluctuates over the business cycle, which is done by introducing the concept of habit formation. Habit formation is an economic theory, which links the investor’s utility levels to the business cycle, and it is a function of both what investor consumes today and in previous periods. It is defined as:

\[ U_t = \sum_{t=0}^{\infty} \delta^t \frac{(C_t - X_t)^{1-\gamma} - 1}{1-\gamma}, \tag{2.24} \]

let \( \delta \) be the subjective time discount factor, \( X_t \) the level of habit consumption, \( C_t \) aggregate consumption and \( \gamma \) the relative risk aversion. The surplus consumption ratio is defined as

\[ S_t = \frac{C_t - X_t}{C_t}, \tag{2.25} \]

or put in words, the fraction of consumption that is not classified as habit consumption. This implies that if \( S_t = 0 \), then \( X_t = C_t \) which indicates a “bad” state with no consumption above the habit level. Let \( X_t \) be constant, then the curvature of the utility function can be examined by taking the second derivatives of the function, and hence the relative risk aversion \( \eta_t \), is now given by:

\[ \eta_t = -\frac{C_t u_{cc}(C_t,X_t)}{u_c(C_t,X_t)} = \frac{\gamma}{S_t} \tag{2.26} \]

Where \( u_c \) and \( u_{cc} \) are the first and second derivatives of utility with respect to consumption, \( \gamma \) is now the curvature parameter of the utility function. Ultimately, as surplus consumption falls, i.e. approaches the habit level, risk aversion increases. This means that in recessions where the surplus consumption is low, investors demand higher utility for absorbing risk, hence demanding higher risk premiums, thus fluctuating risk aversion has a countercyclical effect.

Since the risk and return for the aggregate equity index is allowed to fluctuate contingent on the fundamentals of the economy, it is on the basis of time-varying risk premium possible to theoretically
suggest stock market predictability, contrarily to the constant expected return framework. However, this will not necessarily be in violation with the Efficient Market Hypothesis (EMH). Since an efficient market is defined as a market where abnormal profits are impossible to obtain for the average investor, predictability does not explicitly signal inefficiencies. In conventional financial theory, abnormal returns can only be present when there is no extra associated risk to the return. In such a situation, the inefficiencies would, theoretically, be instantly arbitraged away by investors - conditional on the equality of their information sets. Time-varying risk premiums implicitly signals changes in risk aversion, which makes it possible to make inference about future expected returns. For example when risk is high, investors require correspondingly higher compensation for participating in equity markets investments, which results in declining market prices and higher risk premiums. Hence when a model predicts higher risk premium it could be due to higher risk aversion. Therefore predictions made by the models, if successful, cannot be interpreted as a “free lunch” and expected predictability is not in breach of EMH.

With the established expectations to predictability under the time-varying returns assumption, chapter 3 will discuss and justify individual predictor variables ability to theoretically predict returns in the aggregate equity market.

Chapter 3: Return Predictability using Predictor Variables

As mentioned in the previous section, there is theoretical evidence to suggest that time-varying risk-premia exists in the market, which gives rise to the possibility of stock return predictability. This section will look at stock return predictability using selected individual predictor variables, account for previous literature on the subject, and explain how these implications could be justified from a purely theoretical perspective.

As mentioned in the introduction, academics within finance have sought to uncover stock return predictability for a long period of time, typically with bivariate regression for independent variables on the equity premium. There is still no clear consensus suggesting if and why returns are predictable when using individual variables, however among others, Fama and French (1989) and Cochrane (2007), rationalized the fact that variables which correlate with business-cycle fluctuations, i.e., variables that are able to predict the state of the economy, can be used to predict stock returns. Variables which are profoundly covered in academia comprise; price-dividend ratio (Fama and French, 1988; Cochrane, 2008, Price-earnings ratio (Campbell and Shiller, 1988). Output gap is examined by Cooper and Priestley (2007, 2009). The realized variance for predicting equity premium
was introduced by Merton (1980) and also applied by Goyal and Welch (2008). Rapach et al. (2012) investigate the predictive power of the lagged US return index on countries outside of the US. Moreover, Fama and Schwert (1977) and Campbell and Vuolteenaho (2004) shows return forecastability using inflation, and lastly Hjalmarsson (2010) shows that stock returns are predictable out-of-sample using short-term nominal interest rate and term spread, especially the latter displays significant predictive power.

Furthermore, Fama and French (1988) shows that predictive power is generally stronger over a longer period of time, which is due to the fact that the variance of individual variables grows quicker relative to the variance of the residuals once the time horizon is increased.

Even though many of these studies finds significant forecastability, the validity of these findings are severely questioned by Goyal and Welch (2008), and for the period after 1975 they conclude that "no model had superior performance OOS and a few had acceptable performance IS." (Goyal and Welch, 2008, pp. 1504). These findings were somewhat controversial, and challenged the same year by Campbell and Thompson (2008) who imposed economically justifiable restrictions that showed an opposite conclusion to the Goyal and Welch study. Rapach and Zhou (2013) also found significant OOS $R^2$ when imposing the Campbell and Thompson restrictions on the same data as Goyal and Welch (2008).

The ambiguous conclusions proposed by leading scientists surely stress the complex nature of assessing the forecastability of stock returns. It can seem confusing which findings are “most correct”, as return predictability can be rather unreliable, however their results could have been influenced by several biases, some of which will be discussed in section 4.3.

### 3.1 Dividend-Price Ratio and Dividend Yield

This section will provide a more detailed explanation of the dividend-price ratio and dividend yield, and their potential ability to predict stock returns.

To explain the relationship between the dividend price ratio and expected returns, one can use a dynamic version of the Gordon’s Growth model, by rearranging equation (2.10), called the dividend-ratio model (Campbell and Shiller, 1988). The log dividend-price ratio can be approximated as the discounted value of all future returns and future dividend growth rates, by using a linear approximation to log expected returns.

\[
    d_t - p_t \cong E_t \sum_{j=0}^{\infty} p^j(r_{t+j} - \Delta d_{t+j}) - \frac{k}{1 - p},
\]

where $d_t - p_t$ is the log dividend-price ratio, $p$ and $k$ are constants, $r_{t+j}$ is the future expected return, $\Delta d_{t+j}$ is the future expected growth rate of dividends.
Formula (3.1) states that if the dividend price ratio is high, investors expect higher future returns and/or lower growth in dividends. A positive correlation exists between the dividend-price ratio and the expected returns, since a higher future discount rate leads - ceteris-paribus - to future dividends being discounted at a higher rate, ultimately leading to a lower stock price today.

It was mentioned earlier that stationarity is a main assumption of the approximation for the Gordon’s Growth model. However, both prices and/or dividends can be non-stationary as long as the cointegrated ratio is stationary. Since D/P ratio can be seen as an indicator of the fundamental value relative to the price of the stock, it can show whether or not a stock is under- or overpriced. If the stock is overpriced, future dividends will be high in order to compensate for this, and vice versa. The market value of the stock should therefore return to its fundamental value, thereby showing mean-reversion.

Cochrane (2008) examines the logic behind the predictability of stock returns found from the price-dividend ratio. Cochrane states that if dividend growth and returns are constant and thus unforecastable, then price/dividend ratio should also be constant, which it is clearly not. This is also confirming the assumptions about expected returns following a time-varying nature, as mentioned in section 2.2. Put differently, in order for the assumption regarding price-dividend ratio being stationary, either dividend growth or expected returns must be forecastable in order to bring the ratio back to its mean-reverting value following a shock. It is important to stress, that it is the variation in the price-dividend ratio that will be explained by either dividend growth or expected returns.

To test which of the two components of the dividend-price ratio is forecastable, Cochrane argues that if returns are not forecastable, then the change in dividend growth must be forecastable in order to explain the variation in the dividend-price ratio. He sets up a null test where returns are not forecastable, and the variation in the price-dividend ratio is explained by dividend growth, which will test if either dividend growth or return growth is forecastable. The result of the test shows, that the lack of forecastability in dividend growth is strong evidence that returns are in fact forecastable. The dividend-price ratio should therefore be a good indicator of return forecastability.

Fama and French (1988) found significant $R^2_{OOS}$ between 1967-1986 from the dividend-price ratio ranging from 1 to 5% $R^2$ for monthly and quarterly returns, however the predictive power increased significantly with a longer return horizon. Goyal and Welch (2008) came to the opposite conclusion, since they had access to a longer sample period, and stated that all individual predictive variables consistently underperformed the historical average benchmark. Ang and Bekaert (2003) also found weak OOS predictability for the dividend yield with a more meticulous structural model. Rapach and
Zhou (2013), however, found significant $R^2_{OOS}$ when imposing certain restrictions on the same analysis as Goyal and Welch (2008). In summary, one should expect predictability from the dividend-price or dividend-yield from a theoretical perspective, however the empirical findings make it hard to evaluate whether or not the ratios have been successful in predicting returns or not.

3.2 Price-Earnings Ratio

Another variable often used as a predictor variable for stock return forecasting, is the price-earnings ratio (P/E), as mentioned in the beginning of the section. The argument for using price-earnings ratio, is that it is a good indicator of the business-cycle fluctuations (Campbell and Shiller, 2008). Like the D/P ratio, P/E will show mean-reversion, since it is also an indicator of fundamental value relative to price, thus reversing over- or undervalued market prices to the fundamental value. In addition, Campbell and Shiller (2008) argue that yearly earnings are highly volatile, and thus in order to reduce the noise they smooth out earnings using a moving average of both 10 years and 30 years.

To explain the relationship between P/E and stock returns, the dynamic Gordon’s growth model will be used once again. In order to substitute earnings with dividends in the formula, a strict relationship between earnings and dividends will be assumed. Furthermore it will be assumed that firms pay a constant rate, $c_t$, of earnings, $E_t$, as dividends, $D_t$:

$$D_t = c_t \times E_t.$$  \hspace{1cm} (3.2)

Under this assumption, earnings can be substituted into the dynamic Gordon’s growth model, and thus be used to explain the price/earnings ratio from expected growth in earnings and expected returns:

$$p_t - e_t \approx E_t \sum_{j=0}^{\infty} p^j (-r_{t+j} + \Delta e_{t+j}) + \frac{k}{1-p},$$  \hspace{1cm} (3.3)

where $p_t - e_t$ is the log price-earnings ratio and $\Delta e_{t+j}$ is the future expected growth rate of earnings.

Formula (3.3) states that if the price-earnings ratio is high, investors expect lower future returns and/or higher growth in earnings. A negative correlation exists between the price-earnings ratio and the expected returns. This formula is again prone to an approximation error, but the approximation will be assumed to be precise, as was argued for the dividend-price ratio. According to Cochrane (2008), the variation of the price-earnings ratio will need to be explained by either the growth in earnings or in expected returns, similar to the price-dividend ratio. Cochrane tested this relationship, and found absence of
predictability in earnings, enforcing the conclusion that returns must be forecastable. The P/E ratio should therefore theoretically also be a good predictor of return predictability.

3.3 Realized Stock Variance
Another classical variable for predicting the expected future equity premium is the realized variance in stock prices, which is usually measured as the sum of squared daily returns (Guo, 2006; Goyal and Welch, 2008).

It was mentioned earlier that investors demand the risk-return relationship to hold, which has its theoretical foundation in the CAPM model and the Security Market Line. An increasing realized variance will thus be a signal of an increase in the systematic risk of holding stocks, making investors demand a proportional higher expected return, creating a positive correlation between risk and return in the SML.

The SML implies the reward-to-risk ratio to be positively linear, however, as mentioned in section 2.2, fluctuating risk aversion can be attributed to business cycle fluctuations, which means that the reward-to-risk ratio might not be linear. A high variance will thus create correspondingly higher risk premiums in the short run, but due to the fluctuating business cycles effect on risk aversion, the relationship might not be clear-cut in the longer run.

3.4 SP500 Lagged Index
One variable that has an international perspective, but seldom appear in the literature, is the lagged US return index prediction on non-US countries markets, as proposed by Rapach et al. (2012). They found that lagged US returns had considerable predictive power for non-US returns in industrialized countries. A possible explanation for this, put forth by Rapach et al. (2012), is the information friction between the US as the leading market, and outside countries which are less in focus by investors. Therefore information on macroeconomic fundamentals in the US is only gradually captured across non-US countries. They also specified an empirical news-diffusion model to point out the importance of information friction. The result was that return shocks in the US market was fully integrated into non-US markets with a lag, which supported their argument of information frictions being a possible explanation for the predictive power. They also admitted that other alternative explanations could exist, but none of these will be discussed.

3.5 Risk-Free Rate and Short-Term Rate (Germany)
The appealing motivation for including interest rate variables is their capability to serve as a proxy for the discount factors, see formula 2.23, and thereby also their ability to gauge future risk, which
naturally implies that higher rates signals lower stock returns. Moreover, as business cycle conditions vary, central banks are dedicated to influence interest rates to accommodate cyclical changes in the economy. Usually central banks conduct open market operations, where they interact on the secondary debt markets by purchasing (selling) bonds, and as a result decrease (increase) interest rates. They can also change the prevailing federal fund rate, also called key policy rate, which is the rate banks borrows money from the central bank. If this rate is lowered it will push short rates e.g. Copenhagen Inter Bank Offered Rate (CIBOR) down, which is the rate at which Danish banks borrows from each other. This would result in lowering floating bond rates, since they are usually benchmarked against the CIBOR. Additionally, when inflation is too low, central banks frequently pursue an expansionary monetary policy, hence either purchasing bonds or lowering the key policy rate, thereby coercing down interest rates. This infers that economic activity should surge in the face of lower rates, and as a result consumers will have higher disposable income. This entails firms having higher profits and increase in value due to the higher value of their discounted cash flows (Geske and Roll, 1983). Consequently interest rates close connection with business cycles is a main argument for including it as a predictor variable.

This thesis applies the Danish short-term rates, as well a German short-term rate. The German rates are interesting to contemplate, since Germany is Denmark’s dominant trade partner (Kureer, 2012), thus implying the dependability of Danish exports on the German economy. Furthermore, Denmark has made several obligations to the international community regarding a fixed currency regime. Starting with intentions of reducing variations in interest rates in the ERM agreement of 1979, later pegging the Krone to the D-mark in 1983, and finally pegging the Krone to the Euro in 1999\(^1\).

### 3.6 Term-Spread

The term-spread can be defined as the difference between short-term and long-term rates on otherwise identical bonds (Goyal and Welch, 2008). The term-spread will influence the aggregate stock returns in a similar fashion as above mentioned interest rates. As the monetary policy usually has a relatively slower influence on the long-term rates due to the longer maturity (Hull, 2008), changes in the monetary policy affects the terms spread. This implies, that as short rates increase (decrease), the spread will decrease (increase), and rationally entail lower (higher) stock market returns (Fama and French, 1989). This thesis will apply a German term-spread, due to the lack of data for a Danish long-term interest rate, which is justifiable according to the previous argument of the close connection between the Danish- and German economy.

---

3.7 Inflation

The Consumer Price Index (CPI) will be used as a proxy for inflation. The consumer price index measures the average percentage increase in the price of a representative basket of goods, usually consumed by an urban consumer. Theoretically, inflation should be able to forecast stock market changes due to its close link with interest rates. One of the core aims for central banks is to secure stable and modest inflation in economies\(^2\), thereby the ability to influence interest rates serve as remedies to control inflation, and the mechanics as mentioned in 3.5 still applies.

3.8 Output Gap

A more recently introduced variable used in stock return forecasting is the output gap, which is a prime indicator of business-cycles fluctuations (Cooper and Priestley, 2007, 2009). The output gap is defined as a macroeconomic measure of the difference between an economy’s actual output and potential output, often measured by GDP or total production index. Potential output refers to the output produced at full capacity for the economy, and needs to be estimated quantitatively. A negative output gap signals that the economy is producing at a lower rate than it potentially could, and a positive gap signals that the economy is over-producing. The positive output gap could happen if demand is extremely high, thus forcing suppliers to produce in excessive amounts above their efficient capacity, whereas the negative gap could happen when e.g. suppliers have excess capacity due to weak demand. Neither situations are desirable, hence the most optimal situation would happen at a output gap of zero. There are several other macroeconomic explanations for changes in the output gap, however they will not be discussed any further in this paper.

One popular method of estimating the potential output is by using a linear and quadratic trend, which only uses past data (Clarida, Gali and Gertler, 2000; Fuhrer and Rudebusch, 2004; Cooper and Priestley, 2009). The formula used to measure the output gap is:

\[
y_t = a_0 + a_1 * t + a_2 * t^2 + \epsilon_t, \tag{3.4}
\]

where \(y_t\) is the log of total industrial production index, \(t\) is a time trend, \(a_1, a_2\) are the linear and quadratic trend coefficients respectively, and \(\epsilon_t\) is an error term which measures the output gap.

Cooper and Priestley (2009) found statistically and economically significant \(R^2\) of 2% at the one-month horizon, which increases to 5% at quarterly horizon and 11% at one-year horizon. They also perform a Monte-Carlo experiment to evaluate the bias in the Newey-West t-statistics arising at near-unit root for the predictor variables. There is no significant bias in the size properties at the one-month horizon, but it deteriorates at a longer horizon. The predictive ability of the output gap is still

\(^2\) Ibid
significant when using the empirically derived t-statistics compared to actual t-statistics, despite the bias in the t-statistics.

3.9 Other Predictor Variables

Researchers have applied other predictive variables in the academic forecasting literature, which are not utilized in this dissertation. Some of the most significant will briefly be examined in this section. Lettau and Ludvigson (2001) utilize the macroeconomic variable consumption-wealth or cay, as a predictor of both excess and real stock returns on short and intermediate time horizons. They find that the deviations in cay successfully predict fluctuations in stock returns with adjusted R² ranging from 9% to 16% depending on the forecasting horizon. Furthermore, they argue that cay is the single best predictor, dominating the classical variables which this paper features e.g., the dividend yield, dividend-price and price-earnings ratio. Conversely, Cooper and Priestley (2007, 2009) argues that cay could suffer from being influenced by stock mispricing, and correspondingly nest a fad since the computation of the variable involves market prices levels, therefore arguing that output gap would theoretically be a better predictor.

Ponteff & Schall (1998) examines the book-to-market ratio’s ability to predict future returns on the Dow Jones Industrial Index (DJIA), and finds significant portions of returns explained, hence motivating the conclusion that B/M ratio has information that indicates magnitudes of future cash flows. The conclusion put forth further emphasises that the B/M ratio has had the highest return predictability for small firms, and also before 1960’s.

Lamont (1998) advocates the aggregate dividend pay-out ratio as a significant predictor of future returns. It is argued, that due to the correlation between earnings and the business cycle, and current dividends serving as a proxy for future expected dividends, this variable contains information about future returns not captured by other variables.
Chapter 4 – Forecasting Methodology

Having established a theoretical foundation for all predictor variables, it will be necessary to setup the forecasting framework before conducting the actual equity premium forecast. This section will start by introducing the different forecasting methods used in connection with the time-series data, how to evaluate the performance of the methods, and lastly what kind of biases could arise when working with the data.

4.1 Time Series Data

This thesis focuses exclusively on time-series data and excludes cross-sectional elements, as only the Danish equity premium is examined. Using time-series in connection with OLS regression could have some econometric limitations, however, these will not be discussed in this paper.

4.1.1 Linear Regression

The conventional way of analyzing equity premium predictability is by standard linear regression (OLS), which can be expressed as:

\[ r_{t+1} = \alpha_t + \beta_t \times x_{i,t} + \epsilon_{i,t+1}, \]  

(4.1)

where \( r_{t+1} \) is the log excess return from time \( t \) to \( t+1 \), measured as the gross log return minus the log risk-free rate, \( x_{i,t} \) is a predictor variable such as the P/E ratio, and \( \epsilon_{i,t+1} \) is a zero-mean error term.

Formula (4.1) will be estimated over the in-sample period, and will be used to predict out-of-sample forecasts of \( r_{t+1} \), using similar framework as Campbell and Thompson (2008), Goyal and Welch (2008) and Neely et al. (2012). This is expressed as

\[ \hat{r}_{i,t+1} = \hat{\alpha}_{i,t} + \hat{\beta}_{i,t} \times x_{i,t}, \]  

(4.2)

where \( \hat{\alpha}_{i,t} \) and \( \hat{\beta}_{i,t} \) are the ordinary least squares estimates of \( \alpha_t \) and \( \beta_t \) from formula 4.1, respectively, from regressing the lead equity premium, \( \{ r_{s} \}_{s=2}^{t} \) on a constant and the lagged individual predictors, \( \{ x_{i,s} \}_{s=1}^{t-1} \). A more complex alternative to individual linear regression is multivariate regression, referred to as the ‘kitchen sink’, which regresses several predictor variables on a single dependent variable. This method has proved to be severely inferior to the more simple methods (Goyal and Welch, 2008; Rapach et al., 2013), due to the expected in-sample overfitting, and will therefore be excluded from the analysis.
The main focus of this paper will be on the out-of-sample equity premium forecast, and in order to evaluate this forecast it will be necessary to divide the complete sample T, into an in-sample period, \( v_1 \), and an out-of-sample period, \( v_2 \), so that \( T = v_1 + v_2 \).

\( \hat{a}_{i,t} \) and \( \hat{\beta}_{i,t} \) are estimated using an expanding (recursive) window, which implies, that for each time period the formula utilizes all the information available at t, to re-estimate the OLS parameters. Therefore the OOS equity premium forecast based on the individual predictors \( x_{i,t} \) is regressed 
\[
\{\hat{r}_{i,t+1}\}_{t=v_1}^{T-1}.
\]

To counter structural breaks the equity premium could also be estimated using rolling window estimation, however, Rapach et al. (2013) state that a rolling window will generally not be an optimal estimation technique when breaks are present. For evaluation purposes, researchers have implemented a recursive historical average benchmark forecast of the equity risk premium (Goyal and Welch, 2008; Neely et al., 2012). The historical average forecast can be expressed as

\[
\bar{r}_{t+1} = \left( \frac{1}{t} \right) \sum_{s=1}^{t} r_s,
\]

which is equivalent to the constant expected return model as discussed in section 2.1. Relating back to chapter 2, one might say that the objective of benchmarking to the historical average is to prove that the returns from the proposed forecasts beat the constant-expected returns, and thus time-varying returns could exist in the market (Cooper and Priestley, 2009).

This benchmark dramatically changes the positive findings in various empirical literature, and Goyal and Welch (2008) show, as mentioned earlier, that predictive regression fails to outperform the historical average benchmark OOS. Campbell and Thompson (2008) counter this finding however; by imposing weak restrictions on sign coefficients and return forecasts, and conclude that many predictor variables actually beat the historical average. These restrictions will be introduced in the next section.

### 4.1.2 Economically Motivated Model Restrictions

As mentioned in previous section, in order to overcome some of the shortcomings in the bivariate regression framework, Campbell and Thompson (2008), introduces two economically motivated sign restrictions on \( \hat{r}_{i,t+1} \) and \( \hat{\beta}_{i,t} \), henceforth CT-restrictions. These restrictions should improve the predictability of bivariate regression should make results more stable.

The first restriction builds on Campbell and Thompson's (2008) theory; that a short IS estimation sample will yield "perverse" results for the individual predictor variables in certain periods with high fluctuation, such as the 1930’s. In order to evaluate the correct sign, the regressions are estimated over the full sample, and thus will determine the theoretically correct slope coefficient of the bivariate regression. Therefore, whenever the slope coefficient \( \hat{\beta}_{i,t} \) has an unexpected sign, the restriction will
set $\beta_{i,t} = 0$ in the forecast, which is equivalent to the historical average forecast (constant expected return model). A more specific version of this restriction relates to SVAR. According to financial theory $\beta_{SVAR}$ should always be non-negative, thus implying the restriction of $\beta_{SVAR} > 0$.

The next restriction also relies on basic financial theory. As mentioned in section 2.1, investors are paid a risk premium for absorbing systematic risk, which implies the equity risk premium should always be positive. Accordingly, whenever the equity risk premium $\hat{r}_{i,t+1} < 0$, the restriction will set $\hat{r}_{i,t+1} = 0$.

The findings of Campbell and Thompson (2008) are highly interesting, as the restrictions never weaken their results, and almost always improve their OOS predictability. The restrictions even make most predictor variables significant, despite the earlier findings from unrestricted variables by Goyal and Welch (2008). Campbell and Thompson (2008) also introduce other types of restrictions, but these will not be discussed.

### 4.1.3 Sum-of-Parts Forecast

Another forecasting practice is the sum-of-parts (SOP), as proposed by Ferreira and Santa-Clara (2011). The name originates from the principle of splitting the total stock return into three components; the dividend-price ratio, the growth rate of the price-earnings ratio and growth rate of earnings, and summing them up. To make this point clear, the total market return of an asset is split into two components:

$$R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t} = CG_{t+1} + DY_{t+1},$$

where $P_t$ is the stock price, $D_t$ is the dividends, $DY_{t+1} = \frac{D_{t+1}}{P_t}$ is the dividend yield and $CG = \frac{P_{t+1}}{P_t}$ is the gross capital gain. These are the two central elements of the total return of a stock. Furthermore, multiplying and dividing each term in $CG_{t+1}$ by earnings gives:

$$1 + CG_{t+1} = \frac{P_{t+1}/E_{t+1}}{P_t/E_t} \cdot \frac{E_{t+1}}{E_t} = \frac{M_{t+1}}{M_t} \cdot \frac{E_{t+1}}{E_t} = (1 + GM_{t+1})(1 + GE_{t+1}),$$

where $E_{t+1}$ is the earnings in t+1, $M$ is subscript for the price-earnings multiple, $GM_{t+1}$ and $GE_{t+1}$ is the growth rate of price earnings ratio and earnings growth from period t through t+1, respectively. Likewise it is also possible to decompose the dividend yield by multiplying and dividing by $P_{t+1}$:

$$DY_{t+1} = \frac{D_{t+1}}{P_{t+1}} \cdot \frac{P_{t+1}}{P_t} = DP_{t+1}(1 + GM_{t+1})(1 + GE_{t+1}),$$
where $DP_{t+1}$ is the dividend price ratio. When the two terms have been decomposed it is possible to substitute their new expressions into (4.4):

$$R_{t+1} = (1 + GM_{t+1})(1 + GE_{t+1}) + DP_{t+1}(1 + GM_{t+1})(1 + GE_{t+1})$$

$$= (1 + GM_{t+1})(1 + GE_{t+1})(1 + DP_{t+1}),$$

(4.7)
to make them additive the nature of logarithms is exploited:

$$r_{t+1} = gm_{t+1} + ge_{t+1} + dp_{t+1},$$

(4.8)

where lower case letters denote logs of the variable.

From (4.8) it is apparent to observe that total returns can be decomposed into the three elements, as mentioned earlier; dividend-price ratio, growth of price-earnings ratio and growth of earnings. By denoting them in logs, it is possible to establish equivalence between the sum of these variables and the total return. Consequently, this forecasting method is, in principle, relying on three individual forecasts, one for each component, and accordingly utilizing them in a regression framework. However, since earnings are nearly unpredictable and only contain a low frequency forecastable component (van Binsbergen and Koijen, 2010), a 15-year moving average will be employed. Additionally, to account for the high variance of earnings a 12-month moving sum of earnings will be utilized. Moreover, the most simple version of SOP forecast is employed; in this setting $gm_{t+1} = 0$ and $dp_{t+1} = dp_{t},$ which is in line with the variables following a random walk and embedding a high degree of persistency (Ferreira and Santa-Clara, 2011). Essentially, the SOP equity premium forecast is formally expressed as:

$$\hat{r}_{t+1}^{SOP} = \bar{g}e_t^{15} + dp_t - rf_{t+1},$$

(4.9)

where $rf_{t+1}$ is the log risk free rate known at the end of period $t.$ The computed forecast will be a function of (4.9), which has the variable $\bar{g}e_t^{15} - rf_{t+1}$ as intercept and the slope of $x_{i,t} = dp_t.$ It is clear that this model has a significant economic motivation behind it. Additionally, since the parameters have been theoretically deduced, there are no (OLS) parameters to estimate, which decrease deficiency from estimation error, and hopefully results in a stable forecast series and a higher $R^2_{OOS}.$ However, the choice of estimators is not uninformed, since a prior knowledge of the high degree of persistency in earnings growth and dividend-price ratio exists, which constitutes a central assumption for performing this forecast. This could deteriorate the applicability of the forecast, as the investor would not have known this information at the beginning of the forecasting period. By testing the robustness of their results, Ferreira and Santa-Clara (2011) are able to defuse this critique by utilizing different windows sizes for the earnings growth moving. Specifically they estimate an out of
sample first-order autoregression for the dividend price ratio, thus simulating the priors of an investor from the start of the forecasting period, and find no significant change in $R^2_{od}$.  

4.1.4. Combination Forecast

One major drawback of the bivariate forecasts is the possible structural instability/uncertainty embedded in the predictor variables, which means their predictive power can vary over time, thus creating highly improbable and volatile forecast, leading to poor performance (Goyal and Welch, 2008). Combination forecasts, as used by Rapach et al. (2010), sets out to provide a more stable forecast compared to the bivariate regression. This forecast method is able to accommodate these shortcomings by taking several individual predictors’ forecasts into account. The intuition behind this method is that each predictor variable should, ideally, be a source of distinct information available to the investor, thereby implicitly producing forecasts with relatively low inter-correlations. This should provide more nuanced inputs to the single combination forecast, and at the same time reduce redundant “noise” embedded in the individual forecasts. Therefore the risk of structural instability arising from solely relying on one predictor is diversified away, much like an investor can diversify away idiosyncratic risk exposure in his/her portfolio by considering more stocks (Rapach et al., 2010). Moreover, if the correlations between the forecasts are considered weak, the pooled forecast should estimate a forecasted time series with a more smoothed variance. This is similar to the historical average predictions, however combination forecast will still incorporate relevant information from other economic variables, which is neglected by the historical average forecast. As argued by Rapach et al. (2010); it is the combination forecasts ability to reduce forecast variance coupled with low bias, which makes it outperform bivariate regression.  

Forecast made at time $t$ for $r_{t+1}$ is some weight of $N$ individual forecasts based on (4.2):  

$$\hat{r}_{t+1}^{CF} = \sum_{i=1}^{N} \omega_{i,t} \hat{r}_{i,t+1}$$  

(4.10)

where $\{\omega_{i,t}\}$ is the weight assigned to each individual forecast. In general, two methods for choosing weights for the forecast exist. One is utilizing simple averages, medians or trimmed averages etc. The other method calculates the weights on the basis of each predictor’s statistical performance from a predetermined hold-out period. However, Rapach et al. (2010) shows that the latter category often yields weights approximate to the simple average. Moreover, Timmerman (2006) concludes that a simple combining method often dominates complex methods. Consequently, this paper will solely rely on a simple average weighting scheme, hence $\omega_{i,t} = 1/N$ for $i=1,\ldots,N$.  


### 4.1.5 Diffusion Index

Diffusion index, or principal component analysis (PCA), is another method for forecasting a single dependent variable using many predictor variables. Specifically it tracks the covariation from several different return predictors, and reduces all this information into a few estimated latent factors, thereby reducing the “noise” in the data. It is argued by Ludvigson and Ng (2007), that this method removes the dependence of a few number of individual exogenous predictors in the estimation, and instead includes a large number of economic variables, which are more likely to capture the whole picture, i.e. the unobservable or omitted information of the financial market.

The potential predictors can be expressed in the latent factor model structure as:

\[
X_{it} = \lambda_i' f_t + \epsilon_{it}, \quad i = 1, \ldots, N, \quad (4.11)
\]

where \(X_{it}\) denotes the N-vector of potential predictors, \(f_t\) is a q-vector of unobserved factors, \(\lambda_i\) is a q-vector of factor loadings, and \(\epsilon_{it}\) is the N x 1 idiosyncratic error term.

It is assumed for the strict factor model, that the latent factors and the idiosyncratic errors are contemporaneously and serially uncorrelated, whereas the approximate factor model relaxes these assumptions by a certain degree (Stock and Watson, 2002a).

Regardless of the strict or approximate factor model, the unobserved factors can be estimated from the principal component and is expressed in regards to the dependent variable \(r_{t+1}\) as:

\[
r_{t+1} = \alpha_{DI} + \beta_{DI}' f_t + \epsilon_t, \quad (4.12)
\]

where \(\alpha_{DI}\) is the OLS intercept and \(\beta_{DI}'\) is the q-vector of slope coefficient estimates, from regressing \(\{r_s\}_{s=2}^{t}\) on a constant and \(\{f_{s,t}\}_{s=1}^{t-1}\).

The equity premium forecast \(\hat{r}_{t+1}^{DI}\), from using diffusion index approach formula 4.12 will yield the following formula:

\[
\hat{r}_{t+1}^{DI} = \hat{\alpha}_{DI,t} + \hat{\beta}_{DI,t}' \hat{f}_{t,t}, \quad (4.13)
\]

where \(\hat{\alpha}_{DI,t}\) is the OLS intercept and \(\hat{\beta}_{DI,t}'\) is the q-vector of slope coefficient estimates, from regressing \(\{r_s\}_{s=2}^{t}\) on a constant and \(\{f_{s,t}\}_{s=1}^{t-1}\).

An important feature for the diffusion index is the selection of q, which is the number of principal components. Rapach and Zhou (2013) advise to keep q relatively small, which will help avoid an
over-parameterized forecasting model. Thus they only use the first principal component extracted from 14 economic variables.

A shared advantage of combination forecast and diffusion index is the ability to extract information from a large base of data, which will additionally provide some robustness against structural instability (Stock and Watson, 2002b). Accordingly, this might also increase the risk of parameter instability from estimation error, thereby creating a trade-off between parameter instability and structural instability. A clear statistical advantage for using the diffusion index over combination forecast is the fact that it tracks the most “relevant” variation between the variables, i.e. the co-movements, whereas the combination forecast uses an equal-weight for all variables.

Contrarily, the disadvantage of using the diffusion index from a forecasting perspective is that the estimated latent factors are modelled to explain the co-variation among the individual predictors themselves, thus not taking the explicit relationship between the predictor variables and the equity premium into account. Some research has been carried out to overcome this drawback, for example Kelly and Pruitt (2012) develops a three-pass regression filter, which only estimates the most relevant factors for forecasting the dependent variable, however this analysis will only rely on the conventional PCA.

4.1.6. Complete Subset Regression Forecast

Elliot et al. (2013) introduces complete subset regressions to equity premium forecasting as an alternative to former mentioned forecasts. The aim of introducing the subset regression into equity forecasting, is to recognize the complex dimensionality of the underlying models by including combinations of regressions across all dimensions. The method implies, that for a given set of potential predictors it creates forecasts for all possible linear regression models, while keeping the number of predictors fixed. Thereby from K potential predictor variables, there will be K unique bivariate regressions and \( n_{k,K} = \frac{K!}{((K-k)!k!)} \) different K variate models for \( k \leq K \), hence in total \( 2^K \) possible “short” regression models to explore. Additionally, the simple combination forecast (4.10) can be seen as a special case of a complete subset regression when \( k = 1 \).

Specifically the short regression forecasted return is modelled as:

\[
r_{2:t+1} = \alpha + (X_{1t} S) \beta + \epsilon_{2:t+1},
\]

where \( r_{2:t+1} \) is the forecasted equity premium, \( X = (x_0, x_1, ..., x_{t-1})' \) stores x observations into a complete \( T \times K \) regressor matrix and \( \epsilon_{2:t+1} \) is a vector of error terms. Furthermore, \( S \) is a \( K \times K \) selector matrix with zeros everywhere, except on the diagonal elements where 1 or 0 indicates
whether to include the predictor or not, e.g. when \( k=K \) then \( S=I_K \), meaning all predictors are included. \( \alpha \) and \( \beta \) each represents a \( K \times 1 \) vector with \( S \) determined OLS parameter values.

The subset regression forecast will, like combination forecast, follow a simple mean weighting scheme of each short regression forecast, thus the expression that gathers all short forecasts into the final forecast is expressed:

\[
\hat{r}_{t+1}^{CS} = \frac{1}{n_{K,K}} \sum_{j=1}^{n_{K,K}} (\hat{\alpha}_j + x_t' S_j \hat{\beta}_j) \quad s.t. \ tr(S_j) = k,
\]

(4.15)

where \( tr(S_j) \) is the trace operator which sums the diagonal elements, i.e. \( tr(S_j) = s_{11} + s_{22}, \ldots, \Sigma_{i=1}^{n} s_{ii} \). This paper will produce all \( K \) different forecast. A classical problem when forecasting financial variables is parameter estimation error, which typically arises when trying to estimate too many parameters with too little data. Conventionally, the statistician would conduct shrinkage on the estimates to fix this problem. But complete subset regression can be seen as a complex form of shrinkage, which helps reduce the effect from parameter estimation error, as concluded by Elliot et al. (2013). Moreover, as pointed out by Elliot et al. (2013), because more dimensions exist to construct the forecast, this method can further be utilized to trade-off the bias and the variance of the forecasts.

The variance of the forecasted return can be modelled as:

\[
var(\hat{r}_{t+1}^{CS}) = var(\hat{\alpha} + x_t S \hat{\beta}) = [t' t + x_t S(X'_{1:T}X_{1:T})^{-1}(x_t S)' ] \hat{\sigma}_e.
\]

(4.16)

From (4.16) it is clear, that the dimensionality of \( S \) is positively dependant on the forecast variance, but also increasing for \( \hat{\sigma}_e \). However, to what extent the variance is improving or reducing forecast performance depends on how correlated the variance is with the outcome.

### 4.2 Test Methodology

This section will introduce a two-dimensional evaluation scheme based on interpreting the statistical significance and the economic implication of the proposed forecast methodology. Fama and French (1989), Campbell and Cochrane (1999), and Cochrane (2007) argue that investors require a lower equity risk premium at the end of expansions, due to an increase in income and consumption, but also require a higher equity premium during recessions, where the opposite scenario applies (also see section 2.2). Therefore, it will also be interesting to evaluate the forecasts performance in various business cycles. This argument further emphasizes the point made in chapter 2, i.e., that stock returns follow business-cycle fluctuations. In addition, this countercyclical pattern speaks in favour of time-varying returns, and interestingly enough, Henkel et al. (2011), Rapach et al. (2010) and Rapach and Zhou (2013) find that stock return predictability is stronger during recessions vis-à-vis expansions.
The forecast evaluation will therefore be evaluated separately over the OOS overall period, expansions and recessions.

4.2.1 Definition of the Cycles

There are several ways to define the various business cycles when performing the forecast evaluation. US scientific articles conventionally utilize the NBER to define turning points in the economy (Rapach et al., 2012; Goyal and Welch, 2008), however, no known equivalent measure exists for the Danish economy. Therefore, the Barbeque (BBQ) measure by Harding and Pagan (2002) will be implemented in order to define the actual business cycles in Denmark. The BBQ measure is an extended version of the Bry and Boschan (1971) measure, which is an algorithm that performs three tasks regarding an aggregate economic activity proxy, such as the log GDP or the log industrial production index. The first task involves determining the peaks and troughs. A peak occurs at time $t$, when $y_{t+1}, \ldots, y_{t+k} < y_t > y_{t-k}, \ldots, y_{t-k+1}$, $k = 1, \ldots, K$, and a trough happens in the opposite scenario. $K$ will be defined by the user, which will be $K = 5$ for monthly data, $K = 2$ for quarterly, and finally $K = 1$ for yearly data.

The next step involves the definition of the periods between the troughs and peaks, which will be the expansions, and the periods between the peaks and troughs, which will be the recessions. The last step implements censoring rules to evaluate the durations, amplitude, asymmetric behaviour and cumulative movement of the phases within expansions and recessions. Furthermore it imposes several other restrictions upon the phases, such as a 2-quarter minimum rule for expansions and contractions. These restrictions will not be elaborated in further detail in this paper, but can be seen in Harding and Pagan (2002).

4.2.2 Statistical Evaluation

Once the various forecasts have been conducted, it will be necessary to introduce a goodness-of-fit test to evaluate their predictive power over a certain time horizon. The conventional statistical metric for measuring the predictive power of a forecasting model is the in-sample $R^2_{IS}$, which measures how much variation in the dependent variable can be explained by the variation in the individual predictive variable over the entire sample. However, due to the nature of equity premium forecasting, it is not sufficient to evaluate the in-sample predictive power, but to evaluate the OOS predictive power relative to the historical average benchmark (Goyal and Welch, 2008; Rapach et al., 2013).

A prominent measure for assessing the OOS predictive power of forecasting models is the mean squared forecasting error (MSFE) metric. The MSFE metric measures the expected squared deviation between the forecasted value and the true value. Following the notation in 4.1.1, the complete sample
T is divided into an in-sample period, \( v_1 \), and an out-of-sample period, \( v_2 \), so that \( T = v_1 + v_2 \). Recursive equity premium forecasts are calculated using formula 4.2 from section 4.1 over the last OOS observations. This yields the following MSFE formula for the forecast model:

\[
MSFE_i = \left( \frac{1}{v_2} \right) \sum_{s=1}^{v_2} (r_{v_1+s} - \hat{r}_{i,v_1+s})^2,
\]

(4.17)

where \( \hat{r}_{i,v_1+s} \) represents the OOS equity premium forecast based on the methods mentioned in chapter 3. The MSFE formula 4.17 is also applied to the constant expected return model, see formula 4.3, which yields the following formula:

\[
MSFE_0 = \left( \frac{1}{v_2} \right) \sum_{s=1}^{v_2} (r_{v_1+s} - \bar{r}_{v_1+s})^2.
\]

(4.18)

The performance of each model is evaluated relative to the historical average, using the Campbell and Thompson (2008) OOS \( R^2 \) formula:

\[
R^2_{OOS} = 1 - \frac{MSFE_i}{MSFE_0}.
\]

(4.19)

This \( R^2_{OOS} \) effectively measures the variation explained by the forecasting model (time-varying return model) relative to the historical average benchmark forecast (constant expected return model), for the OOS period. When \( R^2_{OOS} > 0 \), the proposed model is more accurate than the historical average model. Notice that \( R^2_{OOS} \) is expected to be small, however, even small \( R^2_{OOS} \) can mean important economic gains for a mean-variance investor (Campbell and Thompson, 2008), which will be elaborated in section 4.2.3.

In addition to evaluating the predictive power, it will be relevant to perform a statistical test to evaluate whether or not the results are significant. This formal hypothesis test can be formulated as the null

\[
H_0: R^2_{OOS} \leq 0,
\]

(4.20)

against the alternative

\[
H_A: R^2_{OOS} > 0.
\]

(4.21)

The null hypothesis tests if the predictive power of the historical average is significantly better than the proposed forecasting model, which would mean there is no predictability present, against the alternative that predictability is present. It is important to notice, that if the null is accepted, this could
be evidence that the equity premium follows the constant expected return model and not the time-varying return model, hence indicating no predictability.

There are several approaches for testing this hypothesis, one of them being the Diebold and Mariano (1995) and West (1996) (DMW) statistic. It was emphasized by Clark and McCracken (2001) and McCracken (2007) however, that the DMW statistic has a non-standard asymptotic distribution when comparing nested models. Since the predictive regression formula is equivalent to the historical average benchmark whenever $\beta_i = 0$, this implies that the statistic is indeed comparing nested models.

McCracken (2007) analyzed the underlying implication of using the DMW statistic for nested models with a non-standard asymptotic distribution. This showed that the conventional critical values can be seriously undersized, which in some cases would lead to a type 1 error. Clark and West (2007) then introduced a modified version of the DMW statistic to counter this, the MSFE-adjusted, which compares nested model forecasts with a well-approximated normal asymptotic distribution. This statistic looks up new critical values after each application, and thus overcome the limitation of having undersized critical values. The MSFE-adjusted statistic can be calculated after defining

$$d_{t\mid v_1+s} = (r_{v_1+s} - \bar{r}_{v_1+s})^2 - \left[ (r_{v_1+s} - \hat{r}_{v_1+s})^2 - (\bar{r}_{v_1+s} - \hat{r}_{v_1+s})^2 \right], \quad (4.22)$$

and regressing $\{d_{t\mid v_1+s}\}_{t=v_1}^{T-1}$ on a constant. The t-statistic of this constant can then be calculated and used to evaluate the hypothesis test. The errors are reported using Newey-West standard errors, which corrects for autocorrelation and heteroscedasticity using lags of s-1, where s is the horizon.

Goyal and Welch (2003, 2008) suggest using the Cumulative Deviations in the Squared Forecasting Errors (CDSFE) for a graphical assessment of the performance of the regressions, which is simply an extension of formula (4.22):

$$CDSFE_{i\mid \tau} = \sum_{s=1}^{\tau} (r_{v_1+s} - \bar{r}_{v_1+s})^2 - \sum_{s=1}^{\tau} (\bar{r}_{v_1+s} - \hat{r}_{v_1+s})^2, \quad for \quad \tau = 1, ..., v_2. \quad (4.23)$$

Whenever CDSFE increases, it implies that the proposed model forecast has a superior predictive power relative to the historical average forecast, and whenever it decreases, the benchmark performs better. Therefore the slope of this coefficient will be used in order to evaluate the overall performance of the proposed models.
4.2.3. Economic Significance

It is important to evaluate the statistical performance of the proposed models; nevertheless it is also important to assess the economic significance for the investor. As mentioned in 4.2.1, Campbell and Thompson (2008) shows that even small predictability gains can be economically meaningful. The analysis will be facilitated using utility metrics to quantify and elucidate the value of the forecast to economic agents, employing the same approach as illustrated in Campbell and Thomsen (2008). To do this, mean variance properties are assumed for the investor, and the risk aversion will be quantified by the constant relative risk aversion coefficient, $\gamma$. Together with the forecasted return and variance, this determines what fraction of resources shall be allocated between the market portfolio (equity index) and the risk-free return. The solution to the maximization problem yields an expression for calculating the allocation weights in time $t$, which maximizes utility:

$$\alpha_{i,t} = \left(\frac{1}{\gamma}\right) \left(\frac{\hat{r}_{t+1}}{\hat{\sigma}_{t+1}^2}\right),$$

(4.24)

where $\hat{\sigma}_{t+1}^2$ is the forecasted variance of the equity premium, and, for simplicity $\gamma = 5$ (Rapach and Zhou, 2013). To allow for time-varying variance, $\hat{\sigma}_{t+1}^2$ is modelled using the sample variance computed from a 5-year rolling window (Campbell and Thompson, 2008). The value of $\alpha_{i,t}$ is restrained to be non-negative and maximum 150%, which ensures that the model will not take too unrealistic positions in the market, thus also implying the utility gain of the constrained models will only diverge in situations where beta has an unexpected sign. The portfolio return for each time period will also need to be estimated. Moreover, simple discrete returns will be used instead of logs, as it will be beneficial to utilize the cross-sectional nature of simple returns. This means that the sum of the two asset weights multiplied with asset returns remains the portfolio return. The average utility produced from using different forecasting methods, indexed by $i$, can be estimated as:

$$\bar{U}(\hat{r}_i) = \mu_i - \frac{1}{2} \gamma \hat{\sigma}_i^2,$$

(4.25)

where $\mu_i$ and $\hat{\sigma}_i^2$ is the portfolio mean and variance, respectively, and is calculated on the basis of the inputs to (4.23). To retain consistency, the reported results will be set in perspective to the historical average benchmark forecast.

Therefore the allocation of equity for an investor relying solely on the historical average forecast will be:

$$\alpha_{0,t} = \left(\frac{1}{\gamma}\right) \left(\frac{\hat{r}_{t+1}}{\hat{\sigma}_{t+1}^2}\right),$$

(4.26)
where the estimate of $\hat{\sigma}_{t+1}^2$ is similar to the one in (4.24). Over the forecasting period the investor will realize average utility of:

$$U(\bar{r}_0) = \hat{\mu}_0 - \frac{1}{2} \gamma \hat{\sigma}_0^2,$$

(4.27)

where $\hat{\mu}_0$ and $\hat{\sigma}_0^2$ is the sample mean and variance, respectively, calculated on the basis of $\bar{r}_{t+1}$ and $\hat{\sigma}_{t+1}^2$ in the forecasting period.

Having established two expressions for calculating the average utility gain over the forecasting horizon, the average utility gain for utilizing the $i$ indexed forecasting models is notated as $\delta$, and satisfies the criteria:

$$U(\hat{r}_i - \delta) - U(\bar{r}_0) = 0.$$

(4.28)

This implies, that the reported utility gains will be the average yearly differences between the two sums in the forecasting period. These values therefore represent the incremental yearly average gains from utilizing the proposed forecasting methods, relative to the historical average forecasts in asset allocation. As pointed out by Leitch and Tanner (1991), $\delta$ can be interpreted as the economic value of the forecasts, or the percentage fee an economic agent would be willing to pay a portfolio manager to provide the forecasts, i.e., the Certainty Equivalent Return.

As the mean-variance investor is assumed to have a constant relative risk aversion coefficient over time, in contrast to formula (2.26), the variation in expected returns arising from time-varying risk aversion will not be taken into account in the utility gains. The mean-variance investor will thus not be representative of agents’ rational response to time-varying investment opportunities, which arguably accounts for the possibility of return predictability.

### 4.2.4 Investment criteria

Following the framework of Goyal and Welch (2008), which proposes 4 investment criteria in order for a model to be considered a “good” predictor, this paper sets up 3 similar criteria, which any model should fulfil in order to be a satisfactory forecasting model. These investment criteria will also include utility gains, which were not covered by Goyal and Welch (2008), and thereby make it applicable in both empirical matters, but also practical for a mean-variance investor.

The three different investment criteria are:

1. Positive utility gains
2. No significant outliers in the CDSFE measure, which would be the main reason that the model outperforms the historical average
3. A consistent and positive upward trend in the CDSFE measure
The first criterion is practical, as an investor would like to know the economic value of using the proposed model forecast, and would obviously never utilize models indicating negative utility gains. The second criterion could potentially ruin the predictive power of some predictors, as certain periods might be responsible for a model’s entire predictive power. This is what Goyal and Welch (2008) found, as the oil crisis of 1973-75 gave rise to most of the explanatory power for many models. The third criterion might be the most conservative restriction, as it states that the proposed model should consistently outperform the historical average over the entire OOS period, but it is crucial to make realistic inferences regarding future ex-ante performance.

4.3 Bias in Earlier Studies
This section will look at some of the biases which has caused problems in earlier academic studies for the return predictability using individual variables. This is especially an important implication for the unreliable and mixed results found in the academic literature, and this section seeks to shed light on what might have caused these variations.

4.3.1 Parameter Instability and Model Uncertainty
Pesaran and Timmermann (1995) emphasised the importance of parameter instability and model uncertainty when performing return forecasting. Parameter instability refers to the fact that the “best” model can change over time. Model uncertainty means, that the forecaster is unable to identify the best model specification, and at the same time know the corresponding parameter values. Parameter instability and model uncertainty are, like stock returns, subject to business-cycle fluctuations. Lettau and Van Nieuwerburgh (2008) discovered in their effort to counter structural changes in the economy, an important implication for their regime-switching model. Apparently the model successfully determined structural breaks, but was unable to estimate the true parameters quickly enough to produce superior forecast relative to the historical average. Due to this correlation between stock return forecasting and business-cycle fluctuations, this problem emphasised the importance of parameter instability and model uncertainty when producing real-time regression forecasts.

4.3.2 Structural Changes in the Explanatory Variables
Structural changes in any of the explanatory variables might influence the ability to predict stock returns. Such a structural change could be firms repurchasing stocks instead of paying out dividends, which would mean the dividend-price ratio would be a poor predictor of future returns. As mentioned earlier, there is a correlation between the return horizon and the predictive ability, however, by increasing the time horizon, the risk of having potential structural changes increases, which could lead to biases in the data. Viceira (1997) finds no evidence of structural changes between 1926 and 1995.
regarding the relationship between dividend-price ratio and the future expected returns. If this bias should arise, it could be corrected by adjusting the dividends with respect to the buy-backs in dividends.

4.3.3 Data-Snooping
Another important bias in empirical statistical research is data-snooping, which refers to the act of using the same dataset for repeated analysis, usually with the goal of finding interesting or significant results. More specifically, it has been pointed out by Pesaran and Timmermann (1995), among others, that the finance profession has searched for models to predict equity premium for the same single U.S. or OECD dataset. Lo and MacKinlay (1990) point out that since stock market prices might be the most studied economic variable, the statistical tests of asset pricing models might be especially biased. This means; the more published articles utilizing the same dataset, the higher chance of finding non-theoretically justifiable predictability (Lo and MacKinlay, 1990).

One way to avoid snooping bias could be to include a different market than the US Market, such as Cooper and Priestley (2009), Rapach and Zhou (2012). This will not only provide comparable out-of-sample results of the in-sample predictability from the U.S. returns, but also reduce the risk of data-snooping. Another potential way to reduce, but not eliminate, the data-snooping bias could be the model framework introduced by Lettau and Van Nieuwerburgh (2008). They investigate structural change on the basis of mean shifts in the independent variable, rather than on the basis of the forecasting regression. It is not evident how large data-snooping bias is, however Neely et al. (2012) uses an econometrical framework to assess the significance of the bias on equity premium, and find that predictability of the equity premium does not arise from data snooping alone.

4.3.4 Cherry-Picking
Cherry-picking has been discussed in various academic literature, and refers to the process of intentionally or unintentionally using an incomplete or incorrect methodology to obtain or confirm desired results. This could be researchers profound desire to find predictability in stock returns by choosing a specific and/or biased sample start, data frequency or specific method to improve the results. Goyal and Welch (2008) found cherry-picking to be an uneasy task however, which would reduce the importance of this bias.
Chapter 5 – Empirical Application

The focal aim of this paper is to account for the empirical predictability in the aggregate Danish equity market. This section will turn to the empirical application of the methodologies introduced in chapter 4, by firstly describing the data. Secondly it will evaluate and partially conclude on the bivariate and multivariate forecasts. Finally the section will discuss a model critique for further improvements of the empirical application.

5.1 Data

The Danish equity premium will be measured as the monthly log-return of the end-month Datastream (DS) calculated Total Return DS-Denmark Index, which constitutes 50 large and listed firms in Denmark, minus the monthly Danish risk-free rate. The index rebalances its constituents quarterly and will at all times represent minimum 75-80% of the total equity market. The data spans from 1973:01-2014:12. However, the first year is used to compute the variables using e.g. 12-month moving sums, therefore the initial in-sample estimation period, \( v_1 \) covers 1974:01 - 1989:12 (192 observations), and the forecasting period \( v_2 \) spans from 1990:1 - 2014:12 (300 observations).

10 predictor variables are computed and described:

<table>
<thead>
<tr>
<th>Name of the Variable</th>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log dividend-price ratio</td>
<td>Log(DP)</td>
<td>Dividends are calculated as the difference in return between the DS Denmark Total Return Index and the DS Denmark Price Index. The log of a 12-month moving sum of dividends is subtracted from the log DS Denmark Price Index.</td>
</tr>
<tr>
<td>Log dividend yield</td>
<td>Log(DY)</td>
<td>The dividend yield follows similar approach as Log(DP), however, subtracting the lagged log DS Denmark Price Index.</td>
</tr>
<tr>
<td>Log price-earnings ratio</td>
<td>Log(PE)</td>
<td>Computed as the log of 12-month moving average of earnings, deducted from the log DS Denmark Price Index.</td>
</tr>
<tr>
<td>Realized Stock variance</td>
<td>SVAR</td>
<td>Realized stock variance is computed as a 12-month moving window, trailing the realized volatility in the DS Total Return Index.</td>
</tr>
</tbody>
</table>

---

<table>
<thead>
<tr>
<th>Lagged SP500 index</th>
<th>SP500</th>
<th>The lagged DS calculated SP500 Total Return Index.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk-free rate</td>
<td>Rf</td>
<td>Interest rate on a Danish 1-year government bond. Data obtained from the Danish national bank.</td>
</tr>
<tr>
<td>Short-term interest rate - Germany</td>
<td>STR</td>
<td>Interest rate on a 3-month German government bond obtained from OECD.Stats Databank. Average arithmetic rates relating to all days in the month.</td>
</tr>
<tr>
<td>Term spread - Germany</td>
<td>TS</td>
<td>Difference between the long- (10-year) and short German government rate (3-month), which are obtained from OECD.Stats Databank. Average arithmetic rates relating to all days in the month.</td>
</tr>
<tr>
<td>Inflation</td>
<td>INFL</td>
<td>DS calculated CPI in Denmark (all items included). However, to accommodate for the late release of CPI estimates, the variable is modelled as: $x_{INFL,t-1}$ in (3, 1).</td>
</tr>
<tr>
<td>Output gap</td>
<td>OG</td>
<td>The output gap variable is estimated on the basis of the Danish Industrial Production Index gathered from Datastream and is computed as specified in section 3.8.</td>
</tr>
</tbody>
</table>

### 5.2 Monthly Equity Premium Predictability

The next section will introduce the actual monthly equity premium predictability using the framework set up in chapter 4, and evaluate it more closely to discover underlying patterns in the data.

The data period of 1973:01-2014:12 covers Denmark entering EF in 1973, the oil crisis’ of 1973 and 1979, the implementation of the Single Market in Europe from the mid 1980’s to 1993, the growth and expansion of the 1980s and 1990s following the modernisation of several industries and technology improvements, and lastly the Global Financial Crisis of 2007-08. The selection criteria regarding IS and OOS will always be somewhat biased and arbitrary, however, both the in-sample period 1974:01-1989:12 and the out-of-sample period 1990:1-2014:12 covers expansions and recessions in the economy.

### 5.2.1 Monthly Equity Premium Predictability using Individual Predictor Variables

This section turns to the individual predictor variables used in the bivariate regression, and more specifically to analyse their OOS performance.
Table 5.1 shows the monthly out-of-sample equity premium forecast for the Danish Market based on individual predictive regressions relative to the historical average for the period 1990:1-2014:12.\footnote{MATLAB has been used to compute the OOS statistics. The program is a modified version of the code by Rapach and Zhou (2013), which is available on David Rapach’s web page. See appendix A for the full code.} Group A reports the unrestricted predictive regressions, and group B reports the predictive regressions with the economically motivated restrictions imposed by Campbell and Thompson (2008). The results from the table are also divided into an overall model, an expansion period and a recession period, using the pre-defined business cycles calculated using the BBQ Measure (Harding and Pagan, 2002).

It can be seen in the second column for Group A for $R_{OOS}^2$, that 4 out of 10 predictors outperformed the historical average benchmark. The third column shows that 3 (SP500, STR, TS) of the 4 variables with positive $R_{OOS}^2$, are significant at the 0.05 level with Clark and West (2007) p-values, leaving only INFL insignificant. This could be the first indication that bivariate regression might not be the most optimal way to forecast the equity premium.

When the CT restrictions are imposed in Group B, the findings are dramatically changed as 8 predictors have positive $R_{OOS}^2$, in addition SP500, STR, TS are still significant at the 0.05 level. These economically motivated restrictions changes the findings from the unrestricted model, which is in accordance with the conclusion of Campbell and Thompson (2008), and thereby a rebuttal to the unrestricted findings in Goyal and Welch (2008).

Looking at the 5th column in Group A for expansions, 4 variables have positive $R_{OOS}^2$, and STR and TS are significant at the 0.05 level. Likewise the findings are changed in group B, where 5 out of 10 have positive $R_{OOS}^2$, and 3 are significant at the 0.10 level, however, only STR and SP500 are significant at the 0.05 level.

The last section in Group A shows positive $R_{OOS}^2$ for 6 predictors, where SP500 is significant at the 0.05 level and INFL at the 0.10 level. It is noteworthy that the $R_{OOS}^2$ for 6 variables changes signs from expansion to recession, and the 2 significant variables from the expansion (STR, TS) are not the same significant ones in recession (SP500, INFL).

Imposing the CT restrictions in Group B the findings changes once again, as 9 predictors have positive $R_{OOS}^2$; 5 are significant at the 0.10 level and only SP500 is significant at the 0.05 level.

To complement the statistical evaluation made in the following analysis, the utility gains reported in table 5.1 will be utilized to illustrate the economic value of the forecast.

To facilitate the bivariate predictor regression evaluation more closely, figure 5.2 and 5.3 will be introduced. Figure 5.2 shows the cumulative differences in squared forecast errors for the monthly out-of-sample equity premium forecast for the Danish Market based on individual predictive
regressions relative to the historical average. Recessions are depicted with grey vertical bars in the time series.

Figure 5.3 shows the monthly out-of-sample equity premium forecast for the Danish Market based on multiple predictor variables for different forecasting methods relative to the historical average. Recessions are depicted with grey vertical bars in the time series.

**Dividend-price ratio and dividend yield (Log(DP) and Log(DY)):**

As mentioned in section 2.3.1, Cochrane (2008) argues that the dividend-price ratio should be able to predict stock returns from a theoretical viewpoint. This is not evident in table 5.1, which shows that log(DP) and log(DY) are both highly insignificant. These findings are only somewhat changed when imposing the CT-restrictions, since both variables are still insignificant, but with a slight improvement in predictive power.

Figure 5.2 shows the period from 1992 to 1993 and 1994-1995 increased the performance of the unrestricted DP, as the slope increased above zero and thus outperformed the restricted forecast and historical average for these periods. The same scenario applies to DY, but in a much less dramatic fashion. Figure 5.3 provides an explanation since the unrestricted DP and DY forecasted a negative equity premium in these periods, which were actually present in the market in figure 5.1, whereas the CT restrictions sat the forecasts to zero, and thus weakened the predictive power relative to the unrestricted forecast.

In figure 5.2 for the period after 1995 up until 1998, the predictive power of the unrestricted forecasts decline significantly. It is apparent that the period 1995-1998 is where DP and DY deviate the most from the historical average. Combining this with the decline in predictive power, the two variables are obviously not quick enough to adjust to a positive trend in the equity premium after having been negative, which is why the variables are poor predictors. Obviously this means the restricted forecasts are relatively better than the unrestricted in this period, since they lie closer to the actual equity premium in figure 5.1, when the unrestricted forecasts are still negative. After 1998 and for the rest of the period in figure 5.2, the unrestricted remains below zero, only having small fluctuations in the last recession. The restricted forecast ends slightly above zero, but not enough to make it significant. The small fluctuations for this remaining period can be explained from figure 5.3, which shows that both predictors follow the historical average relatively well.

The period of 1995-1998 thus seems to be the significant contributor to the weak predictive power of the unrestricted DP and DY, which only improves slightly upon imposing CT restrictions.

The negative utility gains substantiate the inability of the variables to serve as a superior source of information compared to the historical average and thus would not benefit the investor in any profitable way.
Figure 5.1 Development in the Equity Premium in the Danish Market, 1990:1-2014:12.

Red lines show the monthly equity premium for the Danish Market with the horizontal line depicting the average over the entire period (left). The index development is shown for the DS-Total Market index versus the DS-Total Price Index (right). Recessions out-of-sample are illustrated with grey vertical bars in the time series.
### Table 5.1 Monthly Out-of-Sample Equity Premium Forecast Results in the Danish Market Based on Individual Predictor Variables, 1990:1-2014:12

<table>
<thead>
<tr>
<th>Individual Predictor Variable</th>
<th>Overall</th>
<th>Expansion</th>
<th>Recession</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$R_{OS}^2$ (p-value)</td>
<td>$\Delta$ (ann %)</td>
<td>$R_{OS}^2$ (p-value)</td>
</tr>
<tr>
<td>Log(DP)</td>
<td>-0.15</td>
<td>0.56</td>
<td>-0.16</td>
</tr>
<tr>
<td>Log(DY)</td>
<td>-0.23</td>
<td>0.82</td>
<td>-0.28</td>
</tr>
<tr>
<td>Log(PE)</td>
<td>-0.30</td>
<td>0.36</td>
<td>-0.53</td>
</tr>
<tr>
<td>SVAR</td>
<td>-1.08</td>
<td>0.48</td>
<td>1.04</td>
</tr>
<tr>
<td>SP500</td>
<td>1.47</td>
<td>0.01</td>
<td>1.41</td>
</tr>
<tr>
<td>Rf</td>
<td>-0.02</td>
<td>0.34</td>
<td>-0.31</td>
</tr>
<tr>
<td>TS</td>
<td>2.00</td>
<td>0.00</td>
<td>2.51</td>
</tr>
<tr>
<td>TS</td>
<td>0.69</td>
<td>0.02</td>
<td>1.00</td>
</tr>
<tr>
<td>TS</td>
<td>0.20</td>
<td>0.17</td>
<td>0.06</td>
</tr>
<tr>
<td>OG</td>
<td>-1.63</td>
<td>0.39</td>
<td>0.10</td>
</tr>
</tbody>
</table>

**Group A: Unrestricted predictive regression forecasts**

**Group B: Campbell-Thompson restricted predictive regression forecasts**

The first column measures the $R_{OS}^2$, defined as the mean squared forecast error (MSFE) reduction from using the individual predictive regression relative to the historical average benchmark forecast. Second column reports the p-values from conducting a hypothesis test using the MSFE-adjusted statistic from Clark and West (2007), where the null tests if the historical average benchmark outperforms the proposed regression in terms of MSFE, against the alternative that the proposed regression outperforms the historical average. The last column reports the percentage increase in utility gain for a mean-variance investor, by using the individual variable regression relative to the historical average. All three columns are divided into group A and B, which shows the unrestricted forecasts and the Campbell and Thompson (2008) restricted forecasts, respectively, and also evaluated in different business cycles according to the BBQ measure proposed by Harding and Pagan (2002).
Figure 5.2 Cumulative Differences in Squared Forecast Errors, monthly Out-of-Sample Equity Premium Forecast in the Danish Market Based on Individual Predictor Variables, 1990:1-2014:12

Black lines depict the cumulative differences in squared forecast errors for the unrestricted bivariate forecast based on the chosen variable in the heading, relative to the historical average, whereas the red lines shows the forecast relative to the historical average with the Campbell and Thompson (2008) restrictions imposed. Recessions are depicted with grey vertical bars in the time series, and are based on the BBQ measure proposed by Harding and Pagan (2002).
Figure 5.3 Monthly Out-of-Sample Equity Premium Forecast (in percent) in the Danish Market Based on Individual Predictor Variables, 1990:1-2014:12

Black lines depict the equity premium forecast (in percent) for the unrestricted bivariate forecast based on the chosen variable in the heading, relative to the historical average, whereas the red lines shows the forecast relative to the historical average with the Campbell and Thompson (2008) restrictions imposed. Recessions are depicted with grey vertical bars in the time series, and are based on the BBQ measure proposed by Harding and Pagan (2002).
Price-earnings ratio (Log(PE)):
Referring to section 2.3.2, Cochrane (2008) found absence of predictability in earnings from the price-earnings ratio, similar to the price-dividend ratio, as evidence that returns must be forecastable. Yet again the empirical evidence shows that this conclusion does not support the actual Danish monthly data, as the P/E ratio is insignificant in all periods. The findings are faintly changed when imposing restrictions, as the predictive power increases in the overall model and recession, but decreases in expansion. To analyse the underlying reason further, figure 5.2 and figure 5.3 will be assessed once again.

In figure 5.2 the unrestricted P/E ratio outperforms the historical average in the period 1990-1996 with one exception in 1994. One interesting finding in figure 5.2 is the steep positive slope in the unrestricted forecast between 1992-1993, the same period where the unrestricted DP ratio increased. Comparing to all the other models in figure 5.2, the P/E ratio is actually the best predictor variable for this short period, but the predictive power quickly disappears. The reason behind this can be seen in figure 5.3, where the P/E ratio predicts the absolute lowest equity premium forecast of all predictors in the years 1992-1996, approximately negative 1%, which is considerably lower than any other variable. The P/E forecasts do fit the actual picture of figure 5.1 for the 1992-1993 period, but is obviously not able to react to the sudden positive change in the market afterwards.

Relating to the rest of the period in figure 5.3, the P/E ratio keeps forecasting low equity premiums which obviously do not fit the actual picture, as the unrestricted forecast predictive power gradually declines in figure 5.2 with only small fluctuations. This finding is equivalent to the finding of the DP and DY ratios, as the regression cannot estimate the right parameters quickly enough for the model to react to changes in the economy, see section 4.3.1. Obviously the restricted forecast will capture the change towards positive equity premiums instantaneously, which is why it outperforms the unrestricted model for the period after 1996.

Overall the P/E ratio, whether restricted or not, seems to be a poor predictor variable relative to the historical average, which is also mirrored in the negative utility gain across all sub periods.

Realized stock variance (SVAR):
Table 5.1 shows that unrestricted SVAR is insignificant in all periods. This is changed when imposing the CT-restrictions, as the variable is significant at the 0.10 level with $R_{OOS}^2$ of 0.19% during recessions.

In Figure 5.2 SVAR can be divided into three periods; 1) 1990-1997:3, 2) 1998:4-2008:3 and 3) 2008:4 – 2014:4. The trend for period 1) shows the unrestricted SVAR consistently underperforms the historical average. Imposing the CT-restrictions has an immense impact on the SVAR variable, which is due to the restriction imposed on SVAR in section 4.1.2. Theory suggested $\beta_{SVAR} > 0$, even though
the estimated slope is always negative and thus unexpected in the data. This implies, that restricted SVAR always follows the historical average, however, the non-negativity restriction on the equity premium forecast still applies. Arguably, the name ‘restricted SVAR’ is misleading, as it never uses any information from the SVAR variable, consequently the restricted SVAR is actually more of a restricted historical average model. The only time when the historical average forecasts negative premium can be seen in figure 5.3, which happens in period 1). Consequently the rest of the period it has a slope of zero.

Period 2) is characterized by a positive CDSFE for both the unrestricted and CT-restricted model, meaning they both outperform the historical average.

From September 2008 to April 2009 the predictive power of SVAR increases radically. The upward going trend reaches its climax in April 2009, where SVAR beats the historical average by a remarkable 1.4%. Looking at figure 5.3, one can notice that SVAR predicts a highly negative value for this period, which obviously captures the financial crisis where stock market prices plummeted. SVAR thus seems to be decent at predicting the financial crisis, which would turn out to be the significant contributor to the predictive power of SVAR. The highly significant performance ends quickly the same year, as the predictive power falls dramatically to almost -1% in September 2009.

The actual stock prices turned positive in July 2009, whereas SVAR kept predicting highly negative values until mid-2010, which explains the significant and sudden decrease in predictive power.

The unrestricted SVAR seems rather inconsistent for the three chosen sub-periods relative to the historical average, and has a significant outlier in the financial crisis. Turning to the economic value of the forecast, there is an interesting and somewhat paradox observation. The unrestricted variable is highly insignificant, however, it has a positive utility gains in all periods, but the underlying reason cannot be seen in any of the graphs. As mentioned in section 4.2.3 in formula 4.24, there are several variables, which affect the weight invested in the risky asset. This ratio has obviously benefitted the SVAR forecast in both expansion and recession making the economic performance good, even though the statistical performance is poor.

The American Standard & Poor’s 500 lagged index (SP500):
The lagged S&P500 index has a $R^2_{OOS}$ of 1.47% at the 0.01 significance level for the overall model. Furthermore, it has notable performance during recession periods, where it explains 4.08% more than the benchmark, at the 0.00 significance level. However, during expansions it is inferior to the unconditional mean model, which indicates the overall performance is driven by recessions. Imposing CT restrictions moderately changes the prior analysis, as the predictor now entails significant positive performance across all periods. The restricted $R^2_{OOS}$ in the overall period is 1.32% at a p-value of 0.00 and during expansions (recessions) it is positive at $R^2_{OOS}$ of 0.57% (2.50%) at a 0.07 significance level.
(0.00). Remarkably, during expansions the CT restrictions transform negative explanatory power of -0.19% into 0.57% significantly. This is also evident by looking at figure 5.2, as the CT restrained line actually has a positive drift throughout the period.

Although the model has negative performance during expansions, investors would still acquire, on average, small positive utility gain of 0.29%, which is in deep contrast to the 5.16% increase in utility gains realized by the investor during recessions. Figure 5.2 depicts that the unconstrained model can accurately account for the drastic decline in equity prices in 2007 and for the following rebound. Leaving this period out of the sample could lead to this unconstrained variable being insignificant, consequently using this ex-ante for forecasting could be associated with uncertainty regarding the variable not catching the same “lucky strike” as before.

**Risk-free rate (Rf):**
As shown in table 5.1 the unrestricted forecasts is marginally inferior to the benchmark forecast by having negative and insignificant $R_{OOS}^2$. Moreover it only obtains small benefits by imposing the CT-restrictions, with general OOS performance increasing from -0.02% to 0.19%, at a 0.21 p-level. As seen on figure 5.3 the unrestricted predictor primarily forecasts negative equity premia until 1997. However, after the recession of 1992 the equity market in figure 5.1 has a bullish trend, which neither the historical average nor the unrestricted forecast detects. Compared to the CT counterpart, the unrestricted model actually benefits from predicting negative returns in the beginning of this small recession period. After the recession the unrestricted is too slow to adjust the growing trend in the market, whereas the CT model in this period forecasts equity premiums of zero – hence accumulating less errors than the unrestricted model. Moreover, this predictor also has three distinctive dips in the CSFE relative to the benchmark in 1998, 2000 and 2008. What they all have in common is that the model fails to explain the sudden drastic declines in the index, thus amassing errors compared to the historical average, since it only predicts modest or slightly negative equity premiums. This is further substantiated by the $R_{OOS}^2$ performance of -0.56% during recessions vs. 0.31% in expansions. It is also clear that the utility gains are roughly in line with the aforementioned statistical results, but the utility measure is especially negative in recession periods, where the investor would have had a utility loss of -1.57%.

**Short-term interest rate – Germany (STR):**
From looking at table 5.1 the STR predictor is the sole best bivariate model, with the highest overall $R_{OOS}^2$ performance of 2% above the benchmark, and significant at 0.00 in the overall period. In expansions (recessions) the model explains 2.75% (0.81%), at a 0.00 p-level (0.18). Looking at figure 5.2 further enforces this claim, as both the unrestricted and restricted model mainly display a positive
drift throughout the entire period, which naturally leads to a high $R^2_{OOS}$, and the model especially gains momentum in the interim period between the fourth and the fifth recession period. However, the overall model actually worsens when imposing CT-restrictions, with a marginal decrease in the explanatory power in expansions, though the recessions achieves an $R^2_{OOS}$ of 1.25% at a 0.12 p-level. Nonetheless, it is relatively weak at explaining sudden downfalls in equity prices, thus exercising the same fall in explanatory power during the 1998, 2000 and 2008. The investor faces a certainty equivalent return of 2.51% in the overall period, which is mostly driven by the extreme performance during expansions where the investor would have achieved 2.79% in average utility gain.

**Term Spread - Germany (TS):**

The term-spread seems to have strong performance relative to historical average, as it predicts the equity premium 0.69% better, and clearly rejects the null at a 2%-level. Moreover, along with the STR variable, it covers the only variables that are able to retain positive significant performance in expansions. Additionally, imposing CT-restrictions faintly increases predictive power in the overall model by 0.02%. Contrarily in recessions it reduces the predictive power to 0.68% while making it more significant at a 0.06 level, and in expansion it increases the predictive power from 0.57% to 0.73% with a p-value of 0.01.

It is evident in figure 5.3 that the term-spread predicts almost identical returns compared to the benchmark, except for when entering and leaving the financial crisis. This can also be confirmed by looking at figure 5.2. Similar to some of the other variables, it also fails to recognize the positive equity premia of the mid-nineties. Thereby it profits from imposing the non-negativity restrictions, which is ultimately the underlying reason for the better performance of the CT-restrained model up until the financial crisis. However, it is remarkably how accurate the unconstrained model explains the years 2008-2012; it is correctly forecasting both the drastic decline in the equity returns, but also the rebound period which follows, whereas the constrained version only recognizes the rebound due to the non-negativity constraint. The strong statistical performance is echoed for the utility gains; a mean-variance investor would be willing to pay a 1% fee for having the forecast available in the overall period, and a noteworthy 2.59% during recession. However, further inspection of figure 5.2 elucidates why the variable performs statistically well, while having a somewhat limited practical use. The model’s quite correct forecasts during the financial are the source of its significance, and predominantly originates from 2008-2010. Leaving this period out of the sample forecast would ultimately lead the conclusion of the variable being non-relevant. This finding is similar to the conclusion derived by Goyal and Welch (2008), who found that several variables statistical significance exclusively originated from the oil crisis of 1973. Hence deriving the conclusion that the German term spread would be a good predictor for the Danish equity premium would be deceitful and
infringe on investment criteria two, when in reality, it only successfully explains four economic turbulent years.

**Inflation (INFL)**
The unrestricted forecast is achieving an overall $R_{OOS}^2$ of 0.20% at a 0.17 significance level. The main driver is the strong performance which it exhibits during recessional periods, where it obtains a $R_{OOS}^2$ of 1.18% at a 0.06 significance level, compared to the negative $R_{OOS}^2$ of -0.43% with a p-value of 0.37 during expansions. Imposing CT restrictions changes nothing as the model does not predict any negative forecast during the OOS period in figure 5.3. The effect of CT restrictions would therefore only be evident in periods where beta has an unexpected sign, which is obviously not the case. This is further substantiated by looking at figure 5.2, which shows that the two lines move identically throughout the entire period. The utility gains reflects the statistical performance, with an overall average utility gain of 0.06%, and in expansions (recessions) the investor would lose (gain) -0.4% (1.56%). Additionally, by construction, it is possible to view a high correlation between the interest rate variables and inflation. It is possible to observe the same three OOS periods which these models consequently fails to explain, which are the dips in 1996, 2001 and 2008 where the index dramatically declines.

**Output gap (OG):**
Table 5.1 shows the unrestricted output gap was insignificant across all periods, whereas the restricted forecast is significant at the 0.10 level with $R_{OOS}^2$ of 1.51% in recessions. Clearly the model gains much predictive power in recession for the restricted forecast.

Figure 5.2 shows that OG generally performs poorly, especially for the unrestricted forecast, which actually has the lowest predictive power of all variables. The restricted forecast performs slightly better, and ends above zero at the end period.

To better understand the underlying reason why the restricted forecast differentiates itself from the unrestricted forecast, figure 5.3 will be evaluated once again. It can be seen that OG predicts positive equity premium for the first period 1990-1996, negative forecast for the period 1997-2008, and once again positive forecast for the remaining period. More specifically the highly negative equity premium forecast in 1997-2008, which the restricted forecast would alter, will generate a large difference between the restricted and unrestricted forecast. Relating back to figure 5.2, it is clear that the restricted forecast performs better in the period 1997-2008, which implies the actual equity premium was closer to zero than it was to the unrestricted forecast. For the period 1990-1996 and 2008-2014, the slope is exactly the same for both forecasts.
The expansions and recessions can be analysed more thoroughly in figure 5.2 to explain the difference in predictive power. Two events stand out as particularly important during recessions. The first one happens in the recession in 1993, and the second one in the recession of 2011-2013, where the predictive power increases. These two periods would be important contributors to the predictive power in recessions.

During expansions, there is a steep declining trend from 1990-1995, which gradually weakens the predictive power to below -1%. This period would severely deteriorate the overall predictive power of expansions, even though the two latest expansions increase the predictive power to some degree. The utility gains associated by relying on the output gap are low but positive, in contrast to the highly negative and insignificant $R^2_{00s}$. Similar to SVAR, the output gap has benefitted from the weight invested in the risky asset in periods where the variable performs well. This is evident in recessions where the statistical performance is small and positive with high utility gain.

5.2.1.1 Partial Conclusion using Individual Predictor Variables

In sum, unrestricted individual predictors seem to have had poor predictive performance relative to the historical average. This was evident in both expansions and recessions, where only a few variables outperformed the benchmark, but the general tendency points to higher predictability during recessions. This is in accordance to the findings of Henkel et al. (2011), who finds the historical average benchmark adequate during ‘normal’ periods, but that bivariate regression usually outperforms during recessions. Furthermore the performance of the bivariate regressions seems to have been substantially improved over the last seven years.

Supporting the economic plausibility of several variables; their forecast seems to be linked to the developments in the business cycle in figure 5.3. This could be explained by high surplus consumption at the end of an expansion, where investors would demand low risk premiums to invest in the market. Contrarily, surplus consumption fell during the course of a recession, thereby increasing investor risk aversion (see formula 2.26) and demanding high risk premiums. This was especially evident in the recession of 1993 and 2011 for Rf, STR, TS, as their forecast reached a local minimum when leaving the expansion, and reached a local maximum through the recession.

Enforcing CT-restrictions profoundly changed the findings, where many variables were able to outperform the historical average, although not always at a significant level. This was mostly driven by the increase in performance in recessions, where 9 variables outperformed the unconditional mean model, relative to 5 in expansions.

The overall picture of figure 5.3 points to the fact why the non-negativity restrictions were able to improve the predictive power of the bivariate regressions, as there were several negative observations in the data. These negative observations were removed from the data, and sat equal to zero.
Obviously, this meant, that an unrestricted model with a prevailing number of negative forecasts was destined to perform worse relative to its restricted model, if the actual equity premium was non-negative. This claim was specifically evident for some periods, such as the recession of 1993 and the period of 1995-1997. Some variables enjoyed the benefit of having the non-negativity restriction imposed, which supports the findings in table 5.1, where these variables were moderately improved. DP, DY and P/E showed to be bad predictor variables in the unrestricted model, which is in accordance with the study of Goyal and Welch (2008), but also underperformed when imposing the CT-restrictions, and thus in contrast to Campbell and Thompson (2008) which showed significant predictive power.

Overall the utility gains mirrored the statistical performance, except for the unrestricted SVAR (and to some extent OG) which had positive utility gains, while dramatically underperforming the historical average model. Therefore statistical significance might not always be the most important criteria for a mean-variance investor with constant relative risk aversion. Although the CT restrictions improved the overall statistical performance as expected, utility gains did not change, as the portfolio was not permitted to comprise negative weights in the market portfolio.

In addition to the statistically significant variables SP500, STR and TS - SVAR will be evaluated according to the investment criteria put forth in section 4.2.4.

First of all, the chosen variables fulfil investment criteria 1 because they have positive utility gains. Second of all, SVAR, SP500 and TS are all variables which get most of their predictive power in the financial crisis, as they predicted the upcoming crisis remarkably better than the historical average. Since the strong OOS performance depends on few periods, thereby violating criteria 2, using these variables for ex-ante forecasting might be associated with a high degree of uncertainty, as they do not possess consistent predictive power. This is also a violation of investment criteria 3, which further substantiates the inability of these bivariate regressions. However, it could be argued that the restricted SP500 actually also has a marginal positive and constant drift throughout the entire OOS period, although the majority of the performance originates from an outlier, i.e., the financial crisis. Arguably the STR variable does not strictly rely on consistently positive slopes, but there is a clear positive trend, hence STR is the only variable arguably fulfils all the criteria.

Preceding analysis motivates the conclusion that simple individual predictive regressions might not be the most optimal way to predict equity premium. This is due to the fact that relying solely on one source of information creates unstable forecasts that cannot capture the full picture in the market relative to the historical average. This is also partly due to the inability of the models to react to changes in the markets, as parameter instability and model uncertainty are subject to business-cycle
fluctuations. This could be the theoretical foundation for the more complicated models to outperform the bivariate regression, which will be discussed in the next section.

5.2.2 Monthly Equity Premium Predictability using Multiple Predictor Variables
This section will analyse more complicated models using several predictor variables from the bivariate regression, and more specifically analyse their performance.

Table 5.2 shows the monthly out-of-sample equity premium forecast for the Danish market based on multiple predictor variables for different forecasting methods, relative to the historical average for the period 1990:1-2014:12. Group A reports the unrestricted forecasts, and group B reports the forecasts with the economically motivated restrictions imposed by Campbell and Thompson (2008). The results from the table are also divided into an overall model, an expansion period and a recession period, using the pre-defined business cycles calculated using the BBQ Measure (Harding and Pagan, 2002).

Table 5.2 group A reports the unrestricted forecast. It is already apparent that using multiple variables for forecasting shows promising results. Only the diffusion index seems to entail underperformance across all periods, whereas the sum-of-parts method only underperforms in recessions. The POOL-AVG and complete subset on the other hand outperforms across all periods. In general, p-levels are below or near conventional significant levels, except for the diffusion index. Group B reports the restricted forecasts, but imposing the CT-restrictions only seems to influence the performance of the diffusion index and the complete subset.

To facilitate the evaluation of forecasting methods more closely, figure 5.4 and 5.5 will be introduced. Figure 5.4 shows the cumulative differences in squared forecast errors for the monthly out-of-sample equity premium forecast for the Danish Market based on multiple predictor variables for different forecasting methods relative to the historical average. Recessions are depicted with grey vertical bars in the time series.

Figure 5.5 shows the monthly out-of-sample equity premium forecast for the Danish Market based on multiple predictor variables for different forecasting methods relative to the historical average. Recessions are depicted with grey vertical bars in the time series.
Table 5.2 Monthly Out-of-Sample Equity Premium Forecast Results in the Danish Market Based on Multiple Predictor Variables, 1990:1-2014:12

<table>
<thead>
<tr>
<th>Method</th>
<th>Overall R² (%)</th>
<th>p-value</th>
<th>Δ (ann %)</th>
<th>Expansion R² (%)</th>
<th>p-value</th>
<th>Δ (ann %)</th>
<th>Recession R² (%)</th>
<th>p-value</th>
<th>Δ (ann %)</th>
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<td>0.01</td>
<td>1.91</td>
<td>0.82</td>
<td>0.03</td>
<td>1.73</td>
<td>1.90</td>
<td>0.04</td>
<td>2.46</td>
</tr>
<tr>
<td>K=8</td>
<td>0.79</td>
<td>0.01</td>
<td>1.58</td>
<td>0.32</td>
<td>0.04</td>
<td>1.56</td>
<td>1.53</td>
<td>0.05</td>
<td>1.57</td>
</tr>
<tr>
<td>K=9</td>
<td>0.23</td>
<td>0.02</td>
<td>1.13</td>
<td>-0.28</td>
<td>0.05</td>
<td>1.29</td>
<td>1.04</td>
<td>0.07</td>
<td>0.50</td>
</tr>
<tr>
<td>K=10</td>
<td>-0.38</td>
<td>0.02</td>
<td>0.61</td>
<td>-0.91</td>
<td>0.07</td>
<td>0.99</td>
<td>0.47</td>
<td>0.09</td>
<td>-0.73</td>
</tr>
</tbody>
</table>

The first column measures the $R^2_{OOS}$, defined as the mean squared forecast error (MSFE) reduction from using the different forecasting methods with multiple predictor variables relative to the historical average benchmark forecast. Second column reports the p-values from conducting a hypothesis test using the MSFE-adjusted statistic from Clark and West (2007), where the null tests if the historical average benchmark outperforms the proposed forecasting method in terms of MSFE, against the alternative that the proposed forecasting method outperforms the historical average. The last column reports the percentage increase in utility gain for a mean-variance investor, by using the forecasting method with multiple predictor variables relative to the historical average. All three columns are divided into group A and B, which shows the unrestricted forecasts and the Campbell and Thompson (2008) restricted forecasts, respectively, and also evaluated in different business cycles according to the BBQ measure proposed by Harding and Pagan (2002).
**Figure 5.4** Cumulative Differences in Squared Forecast Errors, monthly Out-of-Sample Equity Premium Forecast in the Danish Market Based on Multiple Predictor Variables, 1990:1-2014:12

Black lines depict the cumulative differences in squared forecast errors for the unrestricted forecasting methods with multiple predictor variables in the heading, relative to the historical average, whereas the red lines shows the forecast relative to the historical average with the Campbell and Thompson (2008) restrictions imposed. Recessions are depicted with grey vertical bars in the time series, and are based on the BBQ measure proposed by Harding and Pagan (2002).
Figure 5.5 Monthly Out-of-Sample Equity Premium Forecast (in percent) in the Danish Market Based on Individual Predictor Variables, 1990:1-2014:12

Black lines depict the equity premium forecast (in percent) for the unrestricted forecasting methods with multiple predictor variables in the heading, relative to the historical average, whereas the red lines show the forecast relative to the historical average with the Campbell and Thompson (2008) restrictions imposed. Recessions are depicted with grey vertical bars in the time series, and are based on the BBQ measure proposed by Harding and Pagan (2002).
Figure 5.6 Monthly Out-of-Sample Equity Premium Forecast (in percent) in the Danish Market Based on Complete Subset Regression, 1990:1-2014:12

The lines depict the equity premium forecast (in percent) for the complete subset regression forecasting method with a fixed number of different k predictors, relative to the historical average dotted line, with Campbell and Thompson (2008) restrictions imposed. The red line shows the subset regression with k=10, and the green line shows the optimal number of k=5. Blue lines illustrate the remaining k predictors. Recessions are depicted with grey vertical bars in the time series, and are based on the BBQ measure proposed by Harding and Pagan (2002).
Combination forecast (POOL-AVG)

POOL-AVG seems to have relatively good predictive power in all periods. Table 5.2 shows that POOL-AVG is significant at the 0.01 level in all three periods, with a $R^2_{OS}$ of 0.69% in the overall, 0.59% in expansion and 0.84% in recession. Imposing restrictions on the combination forecast (Group B), slightly weakens the forecast in the three periods, as the $R^2_{OS}$ decreases to 0.66%, 0.58% and 0.79% with p-values of 0.01. Looking at figure 5.4, the variation between the restricted and unrestricted forecast is incredibly small, only changing positions slightly in some periods such as the financial crisis (2008-2009), where the unrestricted forecast is able to capture the actual negative equity premium in a few periods. Furthermore, it is evident in figure 5.5 that the model follows the historical average close for half of the out-of-sample period. For the remaining 12 years, the forecast starts distinguishing itself from the historical average, and generally forecast an equity premium above the historical average. This is also the period in figure 5.4, where the combination forecast gains a significant part of the predictive power. In addition, the model has been superior in the last seven years, and it could signal that the model has benefitted from gaining additional information from all the individual variables in a period where many variables perform relatively well.

Looking at figure 5.4 and figure 5.5 in connection, it is clear that the combination forecast is stable with low volatility, relative to other variables e.g. SVAR. The reason behind this is intuitive; as mentioned in section 4.1.4; the combination forecast has a diversification effect by including more variables. Furthermore, if the correlation between the forecasts is low, the combination forecast should benefit even more in terms of reduced variance.

Relating back to the discussion for bivariate regression variables, the combination forecast could both benefit and impede from including the individual forecasts, as the predictive power of the individual variables varied significantly. This could give rise to calculating weights based on the predictive power quantified by CDSFE. It was mentioned in 4.1.4 that this often yields approximately the same weights as the simple average, and the simple actually often dominates the more complex methods.

The utility gain follows the statistical performance closely, meaning there is no substantial difference between the economic and the statistical performance. This is further evidence that the variable has a stable performance.

**Sum-of-Parts (SOP)**

The statistical performance of this method seems to be aligned with the unconditional mean benchmark by only varying marginally in explanatory power at non-significant levels, thus overall explaining 0.03% more than the benchmark and failing to reject the null at a 0.15 significance level. The forecast only exhibits negative performance during the course of recessions with $R^2_{OOS}$ of -0.45%, but is accompanied by extreme negative utility gains of -6.99%. Figure 5.4 supports these key
numbers as it can be seen that the performance is highly unstable. Although experiencing small periods of steep positive slopes e.g. from 1995-1997 and 2003-2007, it still fails to realize the financial crisis and its implications for the market. Thus one of the main drivers for its underperformance is the steep decline in CDSFE relative to the benchmark in 2009. The aim of introducing the SOP model was to accommodate deficiency from estimation error. Since there are no (OLS) parameters to be estimated, which can be confirmed from looking at figure 5.5, where the forecasted returns, although not the performance, seem quite stable compared to bivariate forecasts. It is again important to emphasise that relying exclusively on the statistical performance could seriously neglect the dimensions and implications faced by the investor. On the face of it, using this methodology might not statistically seem to be a bad decision, however, the utility gains reveal that relying on the method can be associated with highly negative utility gains and potentially be expensive.

**Diffusion index (DI)**

The diffusion index seems to have poor predictive power across all periods, especially for the unrestricted model, with $R^2_{OOS}$ for the three periods of -0.12%, -0.07% and -0.21%, and p-values of 0.57, 0.47 and 0.66 respectively. Imposing restrictions alters the findings slightly, as the $R^2_{OOS}$ for the three periods is 0.05%, -0.08% and 0.27%, with p-values of 0.34, 0.49 and 0.22 respectively. Looking at figure 5.4, the performance of the diffusion index seems inferior to the benchmark. Clearly the only distinction between the restricted and unrestricted is in the 1993 recession, which makes the restricted relatively better, and the two lines remain parallel after that. In addition it is definitely not able to capture the full picture of the financial crisis, where the slope in figure 5.4 is relatively flat. A prominent question could be why the diffusion index performs poorly compared to the combination forecast, when the diffusion index should estimate the most relevant co-variation amongst the variables. This can be explained by two relevant points. First of all, the diffusion index only tracks the relevant co-variation between the individual predictors themselves, i.e., it disregards the relationship between the individual variables and the equity premium, which was mentioned in section 4.5.1. This implies, that the diffusion index would only be a good forecasting technique if the co-variation between the variables are significantly able to forecast the dependent variable, which is clearly not the case. The second point relates to specific individual variables. It was mentioned under the bivariate equity premium section, that some variable performed well in expansions, and other variables performed well in recessions. Without taking into account the actual equity premium relationship, it becomes increasingly difficult to estimate the most accurate latent factor, as the co-movement between the variables seem to have a contradictory effect on each other in expansion versus recession. This also confirms why there is low and similar predictive power in both expansion and recession,
which is rarely the case for most other models. Low utility gains across all periods further substantiate the poor predictive power of the model, thus the diffusion index seems to be a poor predictor model.

**Complete Subset Regressions (CS)**

The 10 different complete subset forecast have their commonality in their significant positive explanatory power, except k = 10, and hence this methodology clearly stands out from the others. This is apparent from looking at table 5.2 as well as figure 5.4. The table reports remarkably positive $R_{OOS}^2$ ranging from 0.74% to 2.23%, although with k=10 being an outlier with negative performance of −0.56%. All k’s entails significant performance, but k = 5 exhibit the best performance. The forecast made by all 10 different k models can be seen in figure 5.6. The best k model is in contrast to the conclusion reached by Elliot et al (2013), as they find an optimal value of k equal to 2 or 3 through Monte Carlo simulation and empirical analysis of the US equity market. Nevertheless, the positive tendency is continued when decomposing the OOS period into recessions and expansions. The incremental explanatory power compared to the benchmark ranges from -1.48% to 1.74% in the expansionary period with p-values ranging from 0.01 to 0.08. In recessional periods, forecasts are remarkably better with no negative performance ranging from 0.79% to 2.96% with identical p-values. Interestingly, when imposing CT restrictions nearly all versions of the forecast have lower explanatory power, except for k = 9 or 10. Additionally, expansionary periods have for the high dimensions (k > 7) faintly better $R_{OOS}^2$ and p-values, in contrast to the smaller dimensions of k where the opposite applies. Holistically, there is a clear tendency in the data; increasing number of included in the short regressions results in consequently higher p-values. In addition, explanatory power increases alongside the number of short regressions included in the model until it reaches the maximum in k = 5, i.e. 252 short regressions, and then decreases for the remaining k values.

Turning to figure 5.4 depicting performance for the best k-variate subset regression (k=5), it is possible to see a clear trend of almost exclusively positive slopes throughout the forecasting period. However, it is noteworthy that up until 2005 this method has accumulated more errors compared to the benchmark, which is clearly offset by the bad performance in the initial two years in the forecasting period. Moreover, the constrained and unconstrained model seems to move closely together, only differing from each other in a few months until 2009, which is driven by the zero-bound constraint. One could expect the theoretically motivated constraints to improve the model, but the unconstrained model actually succeeds in forecasting present negative equity premiums in the period as depicted in figure 5.1.

Inspecting figure 5.4 substantiates this claim, as the forecast is accurately forecasting the development in the financial crisis. This is evident in both the initial negative equity premiums, as well as the rebound period – which is the only period incorporated into the CT constrained version.
As mentioned there are various advantages to this model. First of all, it allows for incorporation of nuanced information flows from all theoretically relevant variables. Meanwhile it is doing so without deficiency from parameter estimation error, since the number of parameters being estimated is limited to the number of $k$, which would be the “classical” obstacle for the forecaster. The intuitive explanation to the models distinctive performance the last 7 years of the sample could originate from the individual predictors performance in the same period, similar to combination forecast. As the predominant fraction of predictor variables also exhibits distinct positive performance in this period this, their accurate information flows are incorporated into the model. Moreover the variance of the forecast will also increase as the dimensionality of the model is increased, as expressed in equation 3.22, which is also quite apparent from figure 5.6. This is beneficial for the model only up until the variance of the forecast will have declining correlation with the actual equity premiums, and will thus in this context be optimal for around $k=5$. Nonetheless, relying on the generic advice of Elliot et al. (2013) of setting $k$ equal to 2 or 3 would not have been neither statistically nor economically optimal on the Danish market.

5.2.2.1 Partial Conclusion using Multiple Predictor Variables

Overall the models using multiple predictor variables perform statistically better than the individual predictor regressions in section 5.2.1. Similar to bivariate approach, the performance was remarkably better in recession versus expansion, and seems to have been considerably improved over the last seven years. Common for all models is the ability to rely on several variables and thereby diversify the amount of information from the sample. This turned out to be one of the main arguments for using the more complicated proposed models to reduce model uncertainty and parameter instability. Specifically, combination forecast accomplishes a stabilization effect by including an equally-weighted portfolio of information sources, sum-of-parts reduces estimation error, diffusion index filters noise from the data, and finally complete subset achieves stabilization, as well as reducing estimation error, contingent on the pre-determined $k$-factor.

Especially two models stand out as being particular good, namely the combination forecast and the complete subset model with $k = 5$. Since the combination forecast is simply a special case of the complete subset ($k = 1$), strictly speaking one could argue that the prevailing multivariate forecasting method is the complete subset approach. One major drawback of the complete subset is the difficulty in estimating the most optimal value of $k$ in real-time, since the evaluation happens ex-post. Imposing the CT-restrictions had limited effect on the results, presumably due to the stabilising effect of using multiple variables.

Referring to the investment criteria in section 4.2.4, the first criteria excludes the SOP and diffusion index methodologies due to negative utility gains, even though the statistical performance SOP is
positive and almost significant at the 0.10 level. As the diversified nature of these forecasts naturally accommodates outliers, as expected, criteria 2 does not seem to be violated in any of the remaining methods, although the complete subset gains predictive power in a more volatile manner compared to the combination forecast.

For the final criteria of consistent and positive upward drift, the combination forecast fulfils this criterion to perfection, as there is a slight positive upward momentum throughout the entire OOS period in figure 5.4. Arguably the complete subset method fulfils the criteria as well, but in an inferior manner relative to the POOL-AVG model. It is interesting to look at the selection of the OOS period, since the first two years have a dominating effect on the CDSFE measure and could potentially create a conflict for criteria 3.

In sum, a prime example of fulfilling all the proposed criteria to excellence is the combination forecast. The complete subset fairly fulfils the criteria as well, while enjoying a higher amount of explanatory power, making it the dominant forecasting model.

5.3 Model Critique

This section will look more critically at other possible methods that could have been implemented in order to improve or evaluate the proposed models differently.

Even though several methods accomplishes all three investment criteria, this is not necessarily equivalent to predictability in the future, as they are still subject to biases such as those discussed in section 4.3, which could potentially ruin their ex-ante usefulness.

The first thing which could have been done differently would have been to implement additional economic variables with a causal link to the development in the Danish equity premium. The findings in this paper showed two non-Danish economic variables to be the most significant individual predictor variables, which could give rise to implementing additional foreign variables. Some of these could be other interest rates, or another Industrial Production Index in the European countries.

Another possibility would be implementing longer horizon for the prediction of the equity premium. It was mentioned in Chapter 3 that longer horizon strategies often yields dominant results to the shorter horizon, which would improve the statistical significance. In addition to an increase in performance, it would also give the possibility to include more variables, such as GDP (only reported on a quarterly basis), which could be used as; a) a predictor variable, b) an alternative to the industrial production index in determining the business cycles, but also c) as an alternative output gap.

Utility gains are increasingly used in the empirical literature, but they rarely contain an arduous way to evaluate their performance. McCracken and Valente (2012) introduce a test statistic for the utility gain performance, which they show can account for estimation error and provide powerful tests for
evaluating the validity in an empirically relevant sample. By implementing this procedure, the utility gains performance would be more critically assessed, similar to the statistical test statistic.

As mentioned earlier, one major critique for the complete subset regression is the fact that the best value for a fixed number of predictors would need to be estimated in real-time. Elliot et al (2013) implements a procedure which recursively estimates the best value of k, hence making both the comparison to other models easier, but also the real-time usage of the model. Implementing this procedure would have made the evaluation more realistic, but as discussed in section 4.1.6 this is out of the scope of this dissertation. All of these methods could ultimately be used to improve or build-upon the proposed models in this paper, thus providing material for further research.

Chapter 6: Conclusion

The overall aim of this thesis was to examine stock predictability of the Danish stock market using different theoretically proposed forecasting methodologies. The risk and return of the aggregate equity index was allowed to fluctuate contingent on the fundamentals of the economy, and on this basis of time-varying risk premium it would be hypothetically possible to suggest stock market predictability. Under this assumption, a theoretical relationship between 10 predictor variables and the equity premium was established. An in-depth empirical analysis was carried out using bivariate and multivariate forecasting techniques. It was established that a successful predictor of future stock returns should adhere to three investment criteria: 1) positive utility gain, 2) no outliers in CDSFE measure and 3) a consistent and upward trend in CDSFE measure.

Unrestricted bivariate forecasts proved to have poor predictive power relative to the historical average, which was apparent across all periods. Imposing the Campbell and Thompson (2008) restrictions greatly changed these findings, since several variables outperformed the unconditional mean model, even though not always at a significant level. This was primarily driven by an increase in performance during recessions, but also the inability for several variables to capture the positive equity premiums of the mid-nineties.

The only variable which fully fulfilled the investment criteria was the short-term German interest rate (STR), which significantly explained 2% more than the benchmark in the overall model, and the investor would be willing to pay 2.51% fee for having the forecast available.

Turning to the multivariate forecasting approaches, they generally outperformed the bivariate forecasts. Combination forecast and complete subset accomplished a stabilization effect, but complete subset also reduced estimation error.
Referring to the investment criteria, the combination forecasts and complete subset regression were the only methods fulfilling all criteria. However, the complete subset enjoyed a higher amount of explanatory power at 2.21% relative to the historical average and a utility gain of 2.38%. The performance is similar to the bivariate regression using the short-term German interest rate; however, it is on the basis of the stable performance the complete subset regression dominates in the overall experiment.

Generally the dominating origin of predictability was found in recessions for both bivariate and multivariate approaches, similar to conclusions found in Rapach et al. (2013), and the models appear to have improved particularly over the last seven years. In retrospect simple individual predictive regressions might not be the most optimal way to predict equity premium. This was partly due to the inability of the bivariate models to react to changes in the markets, as parameter instability and model uncertainty are subject to business-cycle fluctuations, but also from solely on one source of information.

The findings in this thesis displayed statistical and economic supremacy of the multivariate approach over the simple bivariate method. Overall this motivates the conclusion that there is evidence of predictability of the Danish equity premium in the period 1990-2014 using both bivariate and multivariate methods.
References


