Relational Bundles in the Analysis of Multiple Networks

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Basic concepts

- **Multiple network**: a configuration of different kinds of relationships between a set of social actors

- **Dyad**: pairwise (possible) relations among actors
  - the most basic level in relational analysis

- **Relational bundle**: pattern made of multiple ties
Multiple network

notation

Start with a collection $\mathbf{R} = \{R_1, R_2, \cdots, R_r\}$ of $r$ relations, measured over a set $\mathbf{N} = \{1, 2, \cdots, n\}$ of $n$ social actors.

The pair of sets $\{\mathbf{N}, \mathbf{R}\}$ represents the structure of a multiple network $X$ with two possible outcomes:

$$X_R(i, j) = \begin{cases} 1 & \text{if } R \text{ relates actors } i \text{ and } j \\ 0 & \text{otherwise} \end{cases}$$

$\implies$ i.e. either the ordered pair $(i, j) \in R$ or $(i, j) \not\in R$
Basic patterns

multiple levels

Tie Entrainment

Tie Exchange
Bundle classes

Seven bundle class patterns $\mathbf{B}$ can occur in multiple networks:

$$\mathbf{B} = \{ N, A, R, E, X, M, F \}$$

where a relational bundle $B$ is a class of $\mathbf{B}$ defined for $(i, j)$ in $R_r \in \mathbf{R}$:

$$B^\mathbf{B}_{ij} = \langle R_r \mid (i, j) \in R_r \rangle$$
Bundle class definition

minimal conditions

For $R_1, \ldots, r \in \mathbb{R}$:

$B_{ij}^N = \text{Null}$ \hspace{1cm} $(i, j) \notin R_1, \ldots, r \land (j, i) \notin R_1, \ldots, r$

$B_{ij}^A = \text{Asymmetric}$ \hspace{1cm} $(i, j) \in R_p \land (j, i) \notin R_p \land (i, j) \notin R_{r-p} \land (j, i) \notin R_{r-p}$

$B_{ij}^R = \text{Reciprocal}$ \hspace{1cm} $(i, j) \in R_p \land (j, i) \in R_p \land (i, j) \notin R_{r-p} \land (j, i) \notin R_{r-p}$
Bundle class definition

multiplexity property

\[ B_{ij}^E = \text{Tie Entrainment} \quad (i, j) \in R_1, \ldots, s \ \land \ (j, i) \notin R_1, \ldots, s, \quad \text{for } 1 < s \leq r \]

\[ B_{ij}^X = \text{Tie Exchange} \quad (i, j) \in R_p \ \land \ (j, i) \in R_q, \quad \text{for } R_p \neq R_q \]

\[ B_{ij}^M = \text{Mixed} \quad B_{ij}^A \land B_{ij}^R, \quad r = 2 \]

\[ B_{ij}^M = \text{Mixed} \quad B_{ij}^A \land B_{ij}^X, \quad r > 2 \]

\[ B_{ij}^F = \text{Full} \quad (i, j) \in R_1, \ldots, r \ \land \ (j, i) \in R_1, \ldots, r \]
Class patterns

*dyads with $r = 2$*

### Bundle Pattern

<table>
<thead>
<tr>
<th></th>
<th>k</th>
<th>l</th>
<th>m</th>
<th>n</th>
<th>o</th>
<th>p</th>
<th>q</th>
<th>r</th>
<th>s</th>
<th>t</th>
<th>u</th>
<th>v</th>
<th>w</th>
<th>x</th>
<th>y</th>
<th>z</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1(i, j)$</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$X_1(j, i)$</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
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<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$X_2(i, j)$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
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<td>1</td>
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<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$X_2(j, i)$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
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<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

**N A A A A R R E E X X M M M M F**

**e.g. pattern m:**  
$X_1(j, i) = 1 \quad \Rightarrow \quad X\{1\}(i, j)$

**r:**  
$X_1(i, j) = X_2(i, j) = 1 \quad \Rightarrow \quad X\{1 \rightarrow 2\}(i, j)$
Bundle isomorphic patterns

dyads with single, 2-, 3-, and $r$-types of tie

For a directed network $X$ the potential number of bundle classes is

$$h = 2^{2r}$$

<table>
<thead>
<tr>
<th>PROPERTY</th>
<th>single</th>
<th>2-types</th>
<th>3-types</th>
<th>$r$-types</th>
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<tbody>
<tr>
<td>Null</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Asymmetric</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>$2r$</td>
</tr>
<tr>
<td>Reciprocal</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>$r$</td>
</tr>
<tr>
<td>Tie Entrainment</td>
<td>–</td>
<td>2</td>
<td>8</td>
<td>$r(r + 1) - 4$</td>
</tr>
<tr>
<td>Tie Exchange</td>
<td>–</td>
<td>2</td>
<td>6</td>
<td>$r(r - 1)$</td>
</tr>
<tr>
<td>Mixed</td>
<td>–</td>
<td>4</td>
<td>39</td>
<td>$h - (r(2r + 3) - 2)$</td>
</tr>
<tr>
<td>Full</td>
<td>–</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>4</strong></td>
<td><strong>16</strong></td>
<td><strong>64</strong></td>
<td><strong>$h$</strong></td>
</tr>
</tbody>
</table>
Bundle occurrences

The potential number of occurrences for relational bundles in \( X \) corresponds to the amount of possible unordered pair of nodes

\[
\binom{n}{2} \quad \text{or} \quad n(n - 1)/2
\]

...the amount of null dyads is deducted from \( \binom{n}{2} \) by subtracting the total number of the other bundle occurrences
Example: Multiple network $X_1$
Example: Bundle census for $X_1$

<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>$\to$ C (30, 46)</td>
<td>$\leftrightarrow$ C (46, 63)</td>
<td>$\to\leftrightarrow$ C F (46, 5)</td>
<td>$\to\leftrightarrow$ C F (75, 76)</td>
<td>$\to\leftrightarrow$ C F (14, 71)</td>
</tr>
<tr>
<td>$\to$ C (50, 64)</td>
<td>$\leftrightarrow$ C (46, 72)</td>
<td>$\to\leftrightarrow$ C F (46, 59)</td>
<td>$\leftrightarrow$ C F K (20, 6)</td>
<td>$\to\leftrightarrow$ C F K (23, 89)</td>
</tr>
<tr>
<td>$\to$ C (50, 71)</td>
<td>$\leftrightarrow$ C (5, 75)</td>
<td>$\to\leftrightarrow$ F K (9, 5)</td>
<td>$\to\leftrightarrow$ C F K (46, 48)</td>
<td>$\leftrightarrow$ C F K (46, 90)</td>
</tr>
<tr>
<td>$\to$ C (50, 90)</td>
<td>$\leftrightarrow$ C (59, 90)</td>
<td>$\leftrightarrow$ C F K (9, 75)</td>
<td>$\leftrightarrow$ C F K (46, 48)</td>
<td>$\to\leftrightarrow$ C F K (58, 89)</td>
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<tr>
<td>$\to$ C (71, 90)</td>
<td>$\leftrightarrow$ C (63, 90)</td>
<td></td>
<td>$\to\leftrightarrow$ C F K (58, 89)</td>
<td>$\to\leftrightarrow$ C F K (72, 90)</td>
</tr>
<tr>
<td>$\to$ F (6, 90)</td>
<td>$\leftrightarrow$ C (84, 90)</td>
<td></td>
<td>$\leftrightarrow$ C F K (72, 90)</td>
<td>$\leftrightarrow$ C F K (76, 9)</td>
</tr>
<tr>
<td>$\to$ F (20, 50)</td>
<td>$\leftrightarrow$ F (12, 5)</td>
<td></td>
<td></td>
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<tr>
<td>$\to$ F (20, 90)</td>
<td>$\leftrightarrow$ F (23, 58)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\to$ F (35, 90)</td>
<td>$\leftrightarrow$ F (5, 59)</td>
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<td></td>
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</tr>
<tr>
<td>$\to$ F (64, 14)</td>
<td>$\leftrightarrow$ F (59, 60)</td>
<td></td>
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<tr>
<td>$\to$ F (72, 5)</td>
<td>$\leftrightarrow$ F (59, 75)</td>
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</tr>
<tr>
<td>$\to$ K (48, 90)</td>
<td>$\leftrightarrow$ F (64, 71)</td>
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<tr>
<td>$\to$ K (60, 5)</td>
<td>$\leftrightarrow$ F (64, 75)</td>
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<td>$\to$ K (60, 76)</td>
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<tr>
<td>$\to$ K (72, 14)</td>
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</tbody>
</table>

$n = 26; \quad X_1 \binom{n}{2} = 325$

Bundles: 41 then Null dyads: 284
Relational bundles and signed networks

With networks having two types of relations having opposite signs, the bundle classes serve for the valence assignment of ties

\[ p \text{ or } n \text{ iff } B = \{ B_{ij}^A, B_{ij}^R, B_{ij}^X \} \]

\[ a \text{ iff } B = \{ B_{ij}^E, B_{ij}^M, B_{ij}^F \} \]

\[ o \text{ iff } B = \{ B_{ij}^N \} \]

for \( p \): positive, \( n \): negative, \( a \): ambivalent, and \( o \): absent relations

- tie entrainment is an ambivalent relation
- tie exchange is a transaction of \( p \) and \( n \) that is determined by the relational content of the ties
Bundle patterns properties

- In dyadic patterns we distinguish *asymmetric* and *reciprocal* ties
  two radical different types of behaviour

- In multiple networks these properties are present too:

  - **Tie Entrainment**
    - *Asymmetric character*

  - **Tie Exchange**
    - *Mutual character* ?

  - **Mixed pattern**
    - *Mutual character*
Consequences of asymmetry and mutuality

- Bundles with mutual character characterize *strong bonds* among the actors involved, whereas asymmetric patterns assume *weak* bonds...

- Bundles of similar character allow identifying particular *systems* inside the network

- A system of strong bonds is likely to be the locus of social influence processes like contagion

- Hierarchical structures made of asymmetric bundle patterns may be an active part of the multiple network
Example: Strong bonds system in $X_1$

with actors attributes
**Typology of ties and relational bundles**

Tönnies’ categorization of social ties: *affective* (*Gemeinschaft*), and *instrumental* (*Gesellschaft*) may serve to characterize bundles

For positively valuated affective and instrumental ties:

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**Entrainment:**

<table>
<thead>
<tr>
<th></th>
<th>instrumental</th>
<th>affective</th>
</tr>
</thead>
<tbody>
<tr>
<td>instrumental</td>
<td>asymm</td>
<td>asymm</td>
</tr>
<tr>
<td>affective</td>
<td>asymm</td>
<td>asymm</td>
</tr>
</tbody>
</table>

**Exchange:**

<table>
<thead>
<tr>
<th></th>
<th>instrumental</th>
<th>affective</th>
</tr>
</thead>
<tbody>
<tr>
<td>instrumental</td>
<td>mutual</td>
<td>mutual ?</td>
</tr>
<tr>
<td>affective</td>
<td>mutual ?</td>
<td>mutual</td>
</tr>
</tbody>
</table>
Typology of ties and bundles II

Borgatti et al (2009) propose another typology of ties for SNA:

1. **Similarities**: location, membership, attribute
2. **Social relations**: kinship, affective, cognitive, other role
3. **Interactions**
4. **Flows**

- Similarity ties may suggest a weak bond among actors
  - bundle patterns blur with undirected relations

- Interactions (taked to) and flows (resources) are instrumental ties
  - same characterization as in previous typology

Typology of ties and bundles II (cont.)

- Social relations
  - affective: hence same characterization as previous typology
  - cognitive: ties that represent awareness
  - other roles: implies symmetric labelling (*friend*), symmetric counterpart (*teacher/student*), or maybe just asymmetric (?)
    - reciprocation may be established by the actors
  - kinship: is either symmetric labelling (*siblings*) or symmetric counterpart (*mother/daughter*)
    - reciprocation is established by definition
Important issues & further research

- Categorization of ties is not always clear
  - e.g. does competitor of imply a social role or a cognitive relation?

- Treatment of a mixture of directed and undirected relations
  - for bonds with two and more levels

- Composition and closure of relational bundles

- Statistical analysis of bundle configurations and structural mechanisms driving social behaviour
Thank you!

cran.r-project.org/web/packages/multiplex