FIRMS AND INTERNATIONAL TRADE

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Update: The pre-defence took place on October 30, 2013. The committee comprises professor Alexander Koch from Aarhus University (chairman), professor J. Peter Neary from University of Oxford, and professor Horst Raff from University of Kiel. I would like to thank the committee for their work and careful reading of this thesis. While the time constraint has not allowed for improvements to be made to this thesis, their comments and suggestions will be tremendously helpful going forward.

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This dissertation comprises four chapters concerned with theoretical modelling of firms’ decisions primarily in the context of international trade. The opening chapter, which represents my first serious research endeavour, is on the gains from different types of trade liberalisation in a heterogeneous-firms model. Chapter Two investigates how firm-level complementarities manifest themselves in the comparative statics both at the firm level and for the industry composition. While not concerned specifically with trade issues, the second chapter is closely related to the trade literature through the adopted approach to modelling firm heterogeneity. Building on and extending the insights of Chapter Two, the third chapter combines two prominent models of international trade and input sourcing to show how firm-level complementarities can give rise to a positive relationship between trade liberalisations and the prevalence of vertical integration. Finally, Chapter Four discusses how industry-equilibrium effects may interfere with established results on the LeChatelier principle concerning the relative size of short- and long-run price elasticities of firms’ input demand.

The first chapter, "Sunk vs. Fixed Export Costs and Gains From Trade", show how the presence of both fixed and sunk export costs means that information on the structure of trade costs (variable, fixed, and sunk) and the specific cost reductions implied by trade liberalisations is crucial for evaluating the size and, in particular, the timing of gains from trade. The full impact of a reduction in sunk export costs is realised more slowly than those of reductions in fixed or variable export costs; an important point for welfare. More generally, different implications for total export investments (the sunk cost) mean that steady-state income and thereby consumption differs systematically across various types of trade liberalization. These investment effects imply that the consumption paths, and thereby the timing of gains from trade, depend critically on the details of the liberalisation. Comparing these results to the existing literature, which largely treats fixed and sunk export costs as equivalent, suggests that distinguishing between fixed and the empirically very relevant sunk export costs is important.

The second chapter, "Monotone Comparative Statics for the Industry Composition under Monopolistic Competition" (joint work with Anders Laugesen), studies complementarities among generally-formulated activities faced by firms in a heterogeneous-firms framework of monopolistic competition. The key insight is that firm-level complementarities may manifest themselves much more clearly at the industry level than at the firm level of analysis. In particular, the chapter provides sufficient conditions for exogenous changes in the parameters of firms’ profit maximisation problem to result in first-order stochastic dominance shifts in the equilibrium distributions of all activities regardless of possible ambiguities in
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firm-level responses. The general framework we analyse has deep roots in the literature on heterogeneous-firms models of international trade. As a consequence, our results apply to many well-known models of international trade for which they provide strong, novel, and testable predictions.

The third chapter, "Trade Liberalisation and Vertical Integration" (joint work with Anders Laugesen), studies the relationship between different types of trade liberalisation and vertical integration. Final-good producers face decisions on exporting, vertical integration of intermediate-input production, and whether the intermediate-input production should be offshored to a low-wage country. Our main result is that the shares of final-good producers that pursue either vertical integration, offshoring, or exporting, i.e., the prevalences of these activities, are all increasing when intermediate- or final-goods trade is liberalised and when the cost of vertical integration is reduced. At the same time, one will observe individual firms that shift away from either vertical integration, offshoring, or exporting. The clear positive relationship between the prevalence of vertical integration and trade liberalisations is noteworthy when compared to related studies. We achieve this result in part by pointing towards a novel and plausible firm-level complementarity between offshoring and integration.

The fourth and final chapter of the dissertation, "An Industry-Equilibrium Analysis of the LeChatelier Principle" (again joint work with Anders Laugesen), show that well-established results on the celebrated LeChatelier principle do not extend into an endogenous competitive environment. For instance, firms’ labour demand may be more elastic in the short run (where capital is fixed) than in the long run even if capital and labour are either complements or substitutes in profits. A novel insight of this study is that industry-equilibrium effects introduce an asymmetry such that the LeChatelier principle may hold when the wage increases but not when the wage decreases. These results suggest that estimated labour-demand elasticities should be interpreted with care. Finally, this chapter shows that the LeChatelier principle may hold for the total industry labour demand in situations where it does not hold for the labour demand of individual firms.
DANSK RESUMÉ


Det andet kapitel, "Monotone Comparative Statics for the Industry Composition under Monopolistic Competition" (fælles arbejde med Anders Lauge-sen), studerer komplementariteter mellem generelt formulerede aktiviteter som virksomheder står overfor i en modelramme med heterogene virksomheder og monopolistisk konkurrence. En central indsigt er, at komplementariteter på virk-
somhedsniveau kan manifestere sig klarere på industriniveau end på virksomheds-
niveau. Mere specifikt giver dette kapitel tilstrækkelige betingelser for, at ek-
sogene ændringer i virksomhedernes profitmaksimeringsproblem resulterer i førsteordens stokastisk dominans-skift i ligevægtsfordelingerne af alle
de aktiviteter, virksomhederne står overfor, på trods af tvetydighed angående in-
dividuelle virksomheders reaktion. Den generelle modelramme, vi analyserer, har
dybe rødder i den del af handelsliteraturen, der beskæftiger sig med heterogene
virksomheder. Som en konsekvens heraf giver vores resulater anledning til stærke
og testbare forudsigelser for en lang række velkendte handelsmodeller.

Tredje kapitel, "Trade Liberalisations and Vertical Integration" (fælles arbe-
jde med Anders Laugesen), undersøger sammenhængen mellem forskellige typer
handelsliberaliseringer og vertikal integration. Færdigvareproducenter står over-
for beslutninger om eksport, vertikal integration af en leverandør af halvfabrikata
og offshoring af halvfabrikataproduktionen til et lavlønsland. Hovedresultatet er,
at udbredelsen af vertikal integration, offshoring og eksport stiger både, når
færdigvare- og halvfabrikatahandel liberaliseres samt, når omkostningerne ved
vertikal integration falder. Samtidig observeres virksomheder, der skifter væk fra
en af de tre aktiviteter. Den klare positive sammenhæng mellem udbredelsen
af vertikal integration og handelsliberaliseringer er bemærkelsesværdig sammen-
lignet med relaterede studier. Denne sammenhæng opnås blandt andet som følge
af en hidtil overset plausibel komplementaritet mellem offshoring og vertikal in-
tegration.

Det fjerde og sidste kapitel, "An Industry-Equilibrium Analysis of the LeChate-
lier Principle" (igen fælles arbejde med Anders Laugesen), viser, at veetablerede
resultater angående det feterede LeChatelier princip ikke kan overføres til sam-
menhænge, hvor det konkurrencemæssige miljø er endogen. For eksempel kan
virksomhedens kortsigtede (fast kapitalapparat) arbejdskraftsforspørgsel være
mere elastisk end deres langsigtede ditto, selv hvis kapital og arbejdskraft er
komplementer eller substitutter i profitfunktionen. En ny indsigt er, at indus-
 trialigevægtseffekter kan give anledning til en asymmetri, således at LeChatelier
principippet kan gælde for lønstigninger mens det ikke gælder for lønfald. Afslut-
ningsvis påpeges det, at LeChatelier principippet kan gælde for den totale arbejd-
skraftsforspørgsel i industrien i situationer, hvor det ikke gælder for individuelle
virksomheders arbejdskraftsforspørgsel.
Sunk vs. Fixed Export Costs and Gains from Trade*

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Abstract

Using a Melitz (2003) model, it is shown how the introduction of subjective discounting and sunk export costs means that information on the structure of trade costs (variable, fixed, and sunk) and the specific cost reductions implied by trade liberalisations is crucial for evaluating the size and, in particular, the timing of gains from trade. This is the case even though we condition on the effects on the steady-state share of domestic expenditure. Compared to Arkolakis, Costinot, and Rodríguez-Clare (2012b), who do not consider sunk export costs, our results suggest that distinguishing between fixed and the empirically very relevant sunk export costs is important. The full impact of a reduction in sunk export costs is realised more slowly than those of reductions in fixed or variable export costs; an important point for welfare. More generally, different implications for total export investments mean that steady-state income and thereby consumption differs systematically across various types of trade liberalisation. These investment effects imply that the consumption paths, and thereby the timing of gains from trade, depend critically on the details of the liberalisation.

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1 Introduction

Arkolakis, Costinot, and Rodríguez-Clare (2012b)—henceforth ACR—show how, in a certain class of models, the welfare effects of trade liberalisations through variable or fixed export costs can be evaluated using only the share of domestic expenditure and the elasticity of imports with respect to variable trade costs, as long as a country’s ability to serve its own market is unaffected. A remarkable implication of this result is that, given the trade elasticity and the change in the share of domestic expenditure, neither the details of the model nor the structure of trade costs matter for assessing gains from liberalising trade.

In particular, the result of ACR applies to the Melitz (2003) model when productivities are Pareto distributed. Investigating dynamics in this setup, the present paper argues that when introducing subjective discounting and sunk export costs—two empirically very appealing features—there is more to the story. This extension implies that trade liberalisations through either variable, fixed, or sunk export costs differ systematically with respect to their effects on steady-state consumption, even after conditioning on the share of domestic expenditure (as we do throughout). Further, the extension implies that trade liberalisations induce transition dynamics that differ across liberalisations and must be taken into account when evaluating welfare. The general insight of this paper is that the distinction between sunk and fixed export costs matters for welfare analysis. These nonvariable trade costs cannot be treated as equivalent. Empirical studies documenting the importance of sunk costs in the export decision of firms confirm the relevance of these considerations; see e.g. Roberts and Tybout (1997), Bernard and Wagner (2001), Campa (2004), Bernard and Jensen (2004), and Das et al. (2007).

The mechanism behind the result on steady-state consumption is straightforward. Trade liberalisations affect steady-state consumption through intra-industry reallocations of market shares and labor. Moreover, with positive discounting, trade liberalisations can also affect the total income of households which now depend on firms’ investments through the net profits required as a return on these investments. While conditioning on the share of domestic expenditure pins down the reallocation effect, trade liberalisations differ systematically with respect to their effect on average export investment. The reason is that trade liberalisations through different types of trade costs affect the extensive margin of trade and the sunk export cost to different extents. Since the sunk export cost is essentially an investment in exporting, this leads to a clear ranking of the effects on firms’ average investments, thereby on total income, and ultimately on steady-state consumption. Calibrating the model to match features of data

\[1\]The sunk export cost is an investment in exporting.
Chapter One

on U.S. manufacturing firms indicates that these differences are quantitatively important.

Further, using the calibrated model, welfare over the full transition is considered for the different types of trade liberalisation. First, it is found that, despite the significant difference in the effects on steady-state consumption, liberalisations through reductions in the variable and fixed export costs have very similar impacts on welfare. Intuitively, the higher steady-state consumption induced by reducing the fixed export cost arises due to a higher level of investment and therefore more consumption must be foregone in the short run (the net present value of investments is zero in equilibrium). The investment effects of including sunk export costs do however mean that the induced consumption paths differ considerably and therefore so does the timing of the realisation of the gains from trade. Second, the full impact of a liberalisation through the sunk export cost is not felt immediately upon impact, as incumbent exporters are unaffected (in contrast to the other two liberalisations). This means that not only is the consumption path following this type of liberalisation very different from those of the other two, the welfare effect is also significantly lower.

These implications from explicitly including sunk export costs lead us to the conclusion that distinguishing between fixed and sunk export costs is important. The size and timing of the gains of liberalising trade cannot be inferred from the domestic share of expenditure alone. The structure of trade costs and the specific changes that bring about the reduction in the share of domestic expenditure need to be taken into account as well. A brief discussion of extensions with multiple margins of adjustment (e.g. technology upgrading) and general productivity distributions accentuates that the details of the model matter as well.

Alessandria and Choi (2011) also investigate the role of sunk costs in shaping the welfare effects and consumption paths induced by trade liberalisations. Whereas the present paper focuses on comparing the effects of different types of trade liberalisation, Alessandria and Choi (2011) focus on one type of trade liberalisation (tariff reductions). Further, their framework is much more elaborate than the one used in this paper. While this allows them to conduct more rigorous quantitative analysis, the parsimonious framework we employ allows for closed form solutions for steady-state consumption and clearer interpretation of the results.

A bunch of other studies provide qualifications or further considerations to the baseline result of ACR. They themselves describe the adjustment to their formula necessary for accommodating multiple sectors and intermediate inputs. However, in these cases, details on the trade cost reductions are still irrelevant when conditioning on the industry share(s) of expenditure on domestic goods. Arkolakis et al. (2012a) and Felbermayr et al. (2012) consider how variable markups and revenue-generating tariffs, respectively, affect welfare evaluation
relative to ACR. Schröder and Sørensen (2011) set up a Melitz (2003) model with Pareto-distributed productivities and various real and tariff trade barriers. They investigate the role of efficiency in the reallocation of tariff revenues for the welfare ranking of different trade liberalisations. Felbermayr et al. (2012) and Schröder and Sørensen (2011) feature an effect on total income through the redistributed tariff revenues. In the present paper, however, the effect on total income arises from endogenous export investments (due to a sunk export cost), a channel which—to the best of our knowledge—has not yet been explored. In briefly considering extensions with more margins of adjustment, the present paper is also related to Atkeson and Burstein (2010) and Burstein and Melitz (forthcoming), who consider the interplay between different margins of adjustment and the consequences for welfare in Melitz (2003) frameworks.

The remainder of the paper is organised as follows. Section 2 outlines the model. Section 3 describes the steady state. Section 4 first derives a clear ranking of steady-state consumption following different trade liberalisations. Then the qualitative importance of these results are investigated in a calibrated version of the model. Section 5 considers welfare taking the transitions following trade liberalisations into account. Section 6 briefly discusses the effect of additional margins of adjustments and general productivity distributions on our steady-state results. Finally, Section 7 concludes.

2 Model Setup

The model is a simple extension of Melitz (2003) with sunk costs of exporting and consumers who discount future consumption. For expositional convenience we focus on the case with two countries. In the following, the wage is normalised to unity.\(^2\)

2.1 Households

The representative household has an infinite horizon and provides an inelastic labor supply of \(L\) units. The utility function is given by \(U = \sum_{t=0}^{\infty} \beta^t u(C_t)\) where \(0 < \beta < 1\) is the subjective discount factor and \(C_t\) is consumption at time \(t\). Let \(u'(C_t) > 0\) and \(u''(C_t) < 0\). Consumption at time \(t\) is a CES aggregate over a continuum of varieties of final goods each supplied by a single firm,

\[
C_t = \left[ \int_{\Omega_t} q_{\omega,t}^\alpha \, d\omega \right]^{1/\alpha},
\]

\(^2\)The Supplementary Appendix provides details for some expressions not derived or explained thoroughly in this chapter.
where $\omega$ indexes varieties, $q_{\omega,t}$ is the amount of variety $\omega$ consumed at time $t$, and $\Omega_t$ is the set of varieties available at time $t$. Letting $p_{\omega,t}$ be the price of variety $\omega$, this gives rise to the well-known demand function

$$q_{\omega,t} = \frac{I_t}{P_t} \left( \frac{p_{\omega,t}}{P_t} \right)^{-\sigma},$$

where $\sigma = 1/(1 - \alpha)$ is the elasticity of substitution among varieties, $P_t = \left( \int_{\Omega_t} p_{\omega,t}^{1-\sigma} \, d\omega \right)^{1/(1-\sigma)}$ is a price index, and $I_t$ is the income spent on final goods in period $t$. Note that the definition of $P_t$ implies that $C_t = I_t/P_t$. Consumers own the firms, so the intra-temporal budget constraint is given by

$$I_t = L + \Pi_t,$$

where $\Pi_t$ denotes the total net profits of firms. That is, total profits of firms net of investments in new firms and exporters. The investment decisions in this economy concern the creation of firms and exporters. These decisions will be made by firms in a way such that they are consistent with inter-temporal utility maximisation.

### 2.2 Production

Labor is the only input into production. The firms providing the varieties of final goods are heterogeneous with respect to productivity, $\varphi$. Production entails a constant marginal cost of $\varphi^{-1}$, and a fixed cost, $f$, which must be paid each period. When serving the foreign market, a firm incurs an iceberg trade cost of $\tau$ and must pay a fixed export cost, $f_x$, each period. Using this cost structure and the demand function (1), the indirect profits from serving the domestic and export markets are given by

$$\pi_{d,t}(\varphi) = B_t \varphi^{\sigma-1} - f$$

and

$$\pi_{x,t}(\varphi) = B_t \tau^{1-\sigma} \varphi^{\sigma-1} - f_x,$$

respectively, where $B_t = \alpha^\sigma P_t^\sigma C_t/(\sigma - 1)$.

### 2.3 Entry, Exit, and Exporting Decisions

We assume free entry and that an unbounded pool of potential entrants exists. In the period of entry, an entry cost, $F_e$, must be sunk. Upon entry, a firm draws its productivity level from an exogenous distribution with c.d.f. $G(\varphi)$. If a firm wishes to become an exporter, then a sunk export cost, $F_x$, is required.
The timing of events each period goes as follows. At the beginning of each period, a share $\delta$ of incumbent firms is hit by death shocks, which forces affected firms to shut down.\(^3\) Afterwards, new firms can choose to enter. New firms and surviving incumbents exit if their continuation value $V^c_t$ is negative. Their value function, $V_t$, is therefore given by

$$V_t(\varphi, z) = \max\{0, V^c_t(\varphi, z)\},$$

where $z \in \{0, 1\}$ denotes the export status of a firm.\(^4\) If a firm chooses not to exit, it can update its export status. The continuation value is therefore

$$V^c_t(\varphi, z) = \max_{z'} \left\{ \pi_{d,t}(\varphi) + z'\pi_{x,t}(\varphi) - 1\{z, z'\} F_x + \xi_{t,s+1}V_{t+1}(\varphi, z') \right\},$$

where $1\{z, z'\}$ is an indicator for export entry.\(^5\) The endogenous discount factor $\xi_{t,s}$ both reflects how consumers value income in period $s \geq t$ relative to period $t$ and incorporates the risk of being hit by a death shock

$$\xi_{t,s} = (1 - \delta)^{s-t} \beta^{s-t} \frac{P_t}{P_s} u'(C_s) u'(C_t).$$

This means that firms’ inter-temporal decisions are consistent with utility maximisation.\(^6\) As firms enter with export status $z = 0$, the free entry condition reads

$$F_e \geq \int V_t(\varphi, 0) \, dG(\varphi).$$

If the value of entry is negative, no firms enter. Otherwise, entry takes place until the value of entry is zero. Upon entry, a new firm is indistinguishable from incumbents with the same productivity and export status.

### 3 Steady-state Equilibrium

The present section describes the steady-state equilibrium where all variables are constant over time. Time subscripts are therefore dropped.

\(^3\)We assume throughout that $0 < \delta < 1$. If $\delta = 1$, firms’ decisions have no intertemporal dimension which makes the distinction between fixed and sunk export costs in this setup irrelevant.

\(^4\) $z = 1$ if the firm is exporting and $z = 0$ otherwise.

\(^5\) $1\{z, z'\} = 1$ if $z = 0$ and $z' = 1$. Otherwise, $1\{z, z'\} = 0$.

\(^6\)Firm are effectively risk neutral with respect to the idiosyncratic death shock since it induces no aggregate uncertainty.
3.1 Firm Behavior

It is assumed that not all firms export in steady state, such that the least productive active firms serve only the domestic market. Let \( \phi^* \) denote the lowest productivity for which firms are profitable. This is given by

\[
\pi_d(\phi^*) = 0.
\] (5)

Firms with productivities lower than \( \phi^* \) exit the market. Firms choose to export if the export profits can cover the amortised value of the sunk export cost. Let \( \phi^*_x \) denote the productivity of the marginal exporter, which is given by

\[
\pi_x(\phi^*_x) = [1 - \beta(1 - \delta)] F_x,
\] (6)

where the right hand side is the amortised value of the sunk export cost.\(^7\) All firms with productivity above \( \phi^*_x \) export. The free entry condition can be expressed by equating the amortised cost of entry plus the amortised value of the sunk export cost times the probability of becoming an exporter with the expected per period profits,

\[
[1 - \beta(1 - \delta)](F_e + p^*_xF_x) = \int_{\phi^*}^{\infty} \pi_d(\varphi) \, dG(\varphi) + \int_{\phi^*_x}^{\infty} \pi_x(\varphi) \, dG(\varphi),
\] (7)

where \( p^*_x = \int_{\phi^*_x}^{\infty} dG(\varphi) \) denotes the share of entrants that end up exporting.

3.2 Aggregation

Let \( M_a \) denote the mass of active firms. Note that only a share \( 1 - G(\phi^*) \) of entering firms draws productivities high enough to produce. Therefore, \( M = M_a/[1 - G(\phi^*)] \) more accurately represents the mass of firms (active or not) present in the market, and we will refer to \( M \) simply as the mass of firms in the following. Each period a mass of entrants \( M_e = \delta M \) enters to replace the firms hit by the death shock.\(^8\)

Aggregate net profits \( \Pi \) comprises total domestic and export profits less the sunk costs paid by entrants and new exporters,

\[
\Pi = \int_{\phi^*}^{\infty} \pi_d(\varphi) M \, dG(\varphi) + \int_{\phi^*_x}^{\infty} \pi_x(\varphi) M \, dG(\varphi) - \delta MF_e - p^*_x\delta MF_x.
\] (8)

\(^7\)This condition can be derived from equating the sunk export cost to the net present value of future export profits, \( F_x = \sum_{s=0}^{\infty} \zeta_t \pi_x(\phi^*_x) \) and noting that in steady state, \( \zeta_t = (1 - \delta)^{s-t} \beta^s \).

\(^8\)This way \( [1 - G(\phi^*)]M_e = \delta M_a \) such that the mass of entrants with productivities above \( \phi^* \) equals the mass of active firms hit by the death shock.
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Average net profits $\bar{\pi} \equiv \Pi/M$ can be conveniently expressed by combining (7) and (8),

$$\bar{\pi} = (1 - \beta)(1 - \delta)(F_e + p_x^* F_x). \quad (9)$$

Let $R$ denote aggregate revenues and let $\bar{r} = R/M$ denote average revenues. Equating revenues and expenditure, $R = I$ and using (2), we obtain

$$I = (1 - s_\pi)^{-1} L \quad (10)$$

where $s_\pi = \bar{\pi}/\bar{r}$ denotes the share of net profits in revenues. When workers spend their labor income, $L$, on the goods produced by firms, the fraction $s_\pi$ is returned to them as net profits.\(^9\) When they spend this income, $s_\pi L$, the fraction $s_\pi$ is again returned as net profits and so on. The result is that the total income of consumers is given by the endogenous multiplier, $(1 - s_\pi)^{-1}$, times the exogenous labor income, $L$.

Finally, by combining $C = I/P$, (3), and (5), the steady-state consumption of the representative household can be expressed as

$$C = \alpha(\sigma f)^{1/\sigma} I^{1/\alpha} \varphi^*. \quad (11)$$

### 3.3 Pareto Distributed Productivities

Assume that productivity is Pareto distributed, $G(\varphi) = 1 - (\varphi_{\min}/\varphi)^k$ for $\varphi \geq \varphi_{\min}$ where $\varphi_{\min} > 0$ is the lower bound and $k > \sigma - 1$ is the shape parameter.\(^10\) As shown in Appendix A, this implies that the production cutoff and the average revenues can be expressed respectively as

$$\varphi^* = \eta \lambda^{-1/k}, \quad (12)$$

where $\lambda$ is the share of domestic expenditure and $\eta$ is a positive constant not depending on trade costs, and

$$\bar{r} = \frac{k \sigma}{\sigma - 1} [1 - \beta (1 - \delta)] F_e. \quad (13)$$

### 4 Trade Liberalisations and Steady-State Consumption

This section considers the effect on steady-state consumption of trade liberalisation through a reduction of either the variable $(\tau)$, fixed $(f_x)$ or sunk $(F_x)$ export cost. We let the initial steady-state be the same in all three cases.

\(^9\)With positive subjective discounting, $\beta < 1$, the cross section of net firm profits is larger than the time series of discounted net profits. As firms consider the time series in their entry decision, these are the profits that are driven to zero by free entry. The cross section of net profits is thus positive.

\(^10\) $k > \sigma - 1$ is needed to ensure that expected profits upon entry are bounded.
4.1 Analytical Results

Let \( C_0 \) and \( C \) denote steady-state consumption before and after a trade liberalisation, respectively. Combining (11) and (12), the change in consumption \( \dot{C} = C/C_0 \) can be expressed as

\[
\dot{C} = \dot{I}^{1/\alpha} \dot{\lambda}^{-1/k},
\]

(14)

where \( \dot{I} = I/I_0 \) and \( \dot{\lambda} = \lambda/\lambda_0 \) denote the changes in total income and the share of domestic expenditure, respectively. Steady-state consumption can be affected by a trade liberalisation through two channels: (i) intra-industry reallocations of market shares and (ii) changes in the total income of consumers. The former of these channels (the reallocation effect) is represented in (14) by the change in the share of domestic expenditure, \( \dot{\lambda} \). Intuitively, the reallocation of market shares induced by a trade liberalisation is a reallocation from non-exporters towards exporters. With Pareto-distributed productivities, the extent of this reallocation is exactly captured by the share of domestic expenditure, \( \lambda \). From (10) it is clear that the effect on total income is determined by the change in the share of net profits in revenues, \( s_\pi = \bar{\pi}/\bar{r} \). Since \( \bar{r} \) is constant in trade liberalisations, see (13), total income is only affected through the term \( p^*_x F_x \) in average net profits; see (9).

**Assumption 1.** The trade liberalisations considered have the same effect on the steady-state share of domestic expenditure, \( \lambda \).

Assumption 1 is maintained in the following to facilitate comparison between the trade liberalisations and is not only a natural baseline for doing so. ACR show that, conditional on the effect on the share of domestic expenditure, the \( \tau \)- and \( f_x \)-liberalisations have identical effects on steady-state consumption in the original Melitz (2003) model when productivities are assumed to be Pareto distributed.

Note that if we fix \( F_x \equiv 0 \) in our model, we are left with the original Melitz (2003) model with subjective discounting and Pareto distributed productivities. This only differs from the ACR treatment of the Melitz (2003) model through subjective discounting but, as discussed by Burstein and Melitz (forthcoming), the ACR formula still applies in this case. This can be seen by noting that with \( F_x \equiv 0 \), the effect on average export profits, \( p^*_x F_x \), of either a \( \tau \)- or a \( f_x \)-liberalisation is trivially zero. Hence, the change in total income is zero. Then (14) reduces to \( \dot{C} = \dot{\lambda}^{-1/k} \), which is exactly the ACR formula. In the following we therefore refer to the case with \( F_x \equiv 0 \) as the ACR benchmark. Further, as argued by Burstein and Melitz (forthcoming), this version of the model does not feature transition dynamics since the mass of entrants is unchanged. Thus, for the ACR benchmark, \( \dot{C} = \lambda^{-1/k} \) does not only describe the change in steady-state consumption.
consumption, but also the change in welfare. Obviously, in the ACR benchmark, steady-state consumption and welfare effects of the $\tau$- and $f_x$-liberalisations are identical due to Assumption 1. Further, the timing of gains from trade is identical due to immediate adjustment to the new steady-state.

However, things change when we introduce sunk export costs. In this case, the different liberalisations yield different effects on steady-state consumption. This is because they have different effects on the average export investment, $p_x^*F_x$, and thereby on the total income, as we shall now see. In deriving their formula, ACR impose the restriction that the share of net profits in revenues, $s$, is constant. From (9) and (13), this restriction is obviously not satisfied when we let $F_x > 0$. Further, due to changes in the mass of entrants, trade liberalisations induce transition dynamics which need to be taken into account when evaluating welfare. Welfare effects taking the transition into account are considered in Section 5. For now, we focus on steady-state consumption.

Since we compare trade liberalisations on the basis of identical effects on the share of domestic expenditure, they obviously have identical reallocation effects. The ranking of the effects on steady-state consumption following the three trade liberalisations is therefore determined by their effects on total income. This boils down to the effects on average export investment, $p_x^*F_x$, as argued above. To determine the effects on the average export investment, the following lemma will be useful.

**Lemma 1.** The $\tau$-, $f_x$-, and $F_x$-liberalisations have identical effects on the average export overhead, defined as

$$\xi \equiv p_x^*(f_x + [1 - \beta(1 - \delta)]F_x).$$

**Proof.** See Appendix B.

Intuitively, average revenues from exporting are proportional to the average export overhead due to CES preferences. Therefore, for the liberalisations to have the same effects on the share of export revenues in total revenues (the share of domestic expenditure), they must have the same effects on average export overhead. With this result in hand, we are ready to state the main result regarding steady-state consumption.

**Proposition 1.** Under Assumption 1 and Pareto-distributed productivities, the effects on steady-state consumption of the $\tau$-, $f_x$-, and $F_x$-liberalisations can be ranked according to

$$\hat{C}_{f_x} > \hat{C}_\tau > \hat{C}_{F_x}. \quad (15)$$

\[11\] Note that due to CES preferences, the total income does not feed back into firm-level variables such as the cutoffs, $\varphi^*_\tau$ and $\varphi^*_f$, nor relative variables such as the share of domestic expenditure, $\lambda$. Thus, the reallocation effect can be taken as given when examining the effects on total income.
Proof. Note that the average export investment can be written as

\[ p^x_F = \frac{F_x}{f_x + [1 - \beta(1 - \delta)]F_x}. \tag{16} \]

By Lemma 1 all three liberalisations have the same effect on \( \xi \). Further, the \( f_x \)-liberalisation increases the fraction on the right hand side of (16) while the \( \tau \)-liberalisation leaves it constant and the \( F_x \)-liberalisation reduces it. From (14), the ranking in (15) now follows from a corresponding ranking of \( p^x_F \) and the fact that the effect on \( p^x_F \) determines the effect on \( s_\pi \) and hence on \( I \).

The \( f_x \)-liberalisation achieves its effect on the share of domestic expenditure through the extensive margin of trade to a larger extent than does the \( \tau \)-liberalisation. Therefore, the probability of becoming an exporter, \( p^*_x \), is larger following the \( f_x \)-liberalisation and it has a larger effect on the average export investment, \( p^*_xF_x \), than the \( \tau \)-liberalisation. Next, the \( F_x \)-liberalisation also works through the extensive margin of trade to a larger extent than does the \( \tau \)-liberalisation. However, the direct effect on the investment per exporter, \( F_x \), means that the average export investment, \( p^*_xF_x \), is smaller following the \( F_x \)-liberalisation than following the \( \tau \)-liberalisation. Through the implied ranking of total income, the ranking of steady-state consumption levels follows. Note that the \( f_x \)- and \( \tau \)-liberalisations always have positive effects on average export investment, and thus steady-state consumption following these liberalisations is larger than in the ACR benchmark without sunk export cost. The \( F_x \)-liberalisation on the other hand has an ambiguous effect on \( p^*_xF_x \), such that steady-state consumption can be either larger or smaller than following a liberalisation with the same effect on \( \lambda \) in the ACR benchmark.

We stress that the three liberalisations having different effects on steady-state consumption, even after conditioning on \( \lambda \), is due to the endogeneity of \( s_\pi \) and hence \( I \). Aside from the sunk export cost, the presence of subjective discounting is also important. In the case without subjective discounting, \( \beta \equiv 1 \), no net profits are earned since no return is required on investments, see (9). Therefore, \( I = L \) is constant.\(^{12}\) In this case, all liberalisations have identical effects on steady-state consumption due to Assumption 1.

Interestingly, \( \beta < 1 \) and \( F_x > 0 \) imply that the trade liberalisations have effects on total incomes but not that they are different. For this to be the case, we must have \( f_x > 0 \) as well. If we fix \( f_x \equiv 0 \), then Lemma 1 implies that the

\(^{12}\)Strictly speaking, \( \beta \equiv 1 \) renders the intertemporal maximisation problem of households in the present model meaningless. However, interpreting households as myopic and letting firms maximise the expected intertemporal profits, means that the steady-state expressions above survive.
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τ- and the $F_x$-liberalisations have equal effects on $p_x^* F_x$ and they end up having positive but identical effects on total income.\(^{13}\)

The results of the present section accentuate that if there are indeed both fixed and sunk costs of exporting, this is important for evaluating steady-state consumption effects of trade liberalisations. In this case, treating the nonvariable export costs as either sunk or fixed, will make you unable to properly evaluate and compare steady-state consumption effects of trade liberalisation through variable and nonvariable export costs. Further, Proposition 1 means that the effect of episodes of trade liberalisation on steady-state consumption cannot be inferred from the effect on the share of domestic expenditure alone. One needs to obtain information on which trade costs have changed and by how much. Analogously, to compare steady-state consumption to the level which would prevail under autarky, information on the amount of investment in exporting, $p_x^* F_x$, is needed.

4.2 Calibration

We now describe the calibration used in the next subsection to quantify the importance of the results on steady-state consumption and in Section 5 for calculating welfare taking the transition into account.

As Alessandria and Choi (2011), we set the subjective discount factor, $\beta$, to 0.96 which is quite standard. The elasticity of substitution among varieties, $\sigma$, is set to 5 in line with estimates by Broda and Weinstein (2006). This implies an $\alpha$ of 0.8. The shape parameter of the Pareto distribution, $k$, is set to 5.68 in order to get $k/(\sigma - 1) = 1.42$ as in Melitz and Redding (2013).\(^{14}\) The share of firms hit by the death shock each period, $\delta$, is set to 0.023 to match the labour share of establishments shutting down of 2.3 percent reported by Davis et al. (1996) for U.S. manufacturing.\(^{15}\) The number of consumers, $L$, and the lower bound of the Pareto, $\varphi_{\text{min}}$, are both set to 1, which simply corresponds to choosing the units of measurement. Since scaling up $f_x$, $f_x^*$, $F_e$, and $F_x$, by a common factor simply determines the normalisation of firm size, we just set $f = 1$. $F_e$ is chosen such that 17 percent of entrants are unsuccessful in line with numbers reported by Bartelsman et al. (2004) for U.S. manufacturing. The iceberg trade cost, $\tau$, and (steady-state) export overhead, $f_x + [1 - \beta (1 - \delta)] F_x$, are chosen such that

\(^{13}\)Compared to the $\tau$-liberalisation, the larger effect on $p_x^* F_x F_x$ of the $F_x$-liberalisation exactly balances with its direct effect on $F_x$ in this case. The identical effects on total income imply that their effects on steady-state consumption do not differ. However, the effect on steady-state consumption is not the same as in the ACR benchmark, since there is a positive effect on total income.

\(^{14}\)Melitz and Redding (2013) note that Axtell (2001) shows that the firm size distribution is well approximated with $k/(\sigma - 1)$ close to 1, but that it has to be larger than one for profits upon entry to be bounded.

\(^{15}\)In our model firms shutting down have the same employment on average as surviving firms.
the share of exports in exporting firms’ revenues, \( \frac{\tau}{1+\tau} \), is equal to 0.14 and the share of exporters in active firms equals 0.18 as reported by Bernard et al. (2007) for U.S. manufacturing. What remains to be determined is the share of the sunk cost in the export overhead, \( \frac{[1-\beta(1-\delta)]F_x}{F_x+|[1-\beta(1-\delta)]_F_x} \), or effectively to pin down the relative size of \( F_x \) and \( f_x \). Empirical estimates for determining this are not clear cut, so the present section provides quantitative results for a range of values. However, as a reference point and to fix a value for use in solving for the transition in Section 5, note that using data from Das et al. (2007), Alessandria and Choi (2011) estimate that the mean startup export costs are about 1.5 times the mean annual export profits. We match this with \( \frac{[1-\beta(1-\delta)]F_x}{F_x+|[1-\beta(1-\delta)]_F_x} = 0.25 \). In the ACR benchmark, this value is of course zero.

### 4.3 Quantitative Results

Using the calibration described above, we calculate the impact on steady-state consumption of decreasing the share of domestic expenditure, \( \lambda \), by one percentage point through either of the three types of liberalisation. The results are depicted in Figure 1 for a range of values of the ex-ante share of sunk costs in the export overhead. The results for the \( f_x \)- and \( F_x \)-liberalisations are not given for the full range, as either of the corresponding costs must be of sufficient size ex-ante in order to be able to bring about the decrease in \( \lambda \) while remaining nonnegative.

Also depicted in Figure 1 is the consumption growth in the ACR benchmark. Note that when the ex-ante share of the sunk cost in the export overhead is zero, we are in the ACR benchmark and the \( \tau \)- and \( f_x \)-liberalisations have identical effects. At the other end of the spectrum, when the fixed export cost is zero, the \( \tau \)- and \( F_x \)-liberalisations have the same effects on steady-state consumption. As discussed above, in this case a positive effect on total income is present such that the change in steady-state consumption is higher than in the ACR benchmark.

With nonnegligible sunk and fixed export costs, the figure shows that the effects of the three liberalisations differ significantly both amongst themselves and when compared to the ACR benchmark. For example, with the ex-ante share of the sunk cost in the export overhead at 0.25, the change in steady-state consumption is about 20 percent larger for the \( f_x \)- and 41 percent smaller for the \( F_x \)-liberalisation when they are compared to the \( \tau \)-liberalisation. Further, the \( \tau \)-liberalisation induces a change in steady-state consumption that is 7 percent larger than in the ACR benchmark.\(^{17}\)

\(^{16}\)Section 5 also considers welfare evaluation with \( \frac{[1-\beta(1-\delta)]F_x}{F_x+|[1-\beta(1-\delta)]_F_x} \) set to 0.5.

\(^{17}\)The change in steady-state consumption induced by the \( F_x \)-liberalisation is lower than in the ACR benchmark for a broad range of values. This is due to a negative effect on total income. The possibility of a negative effect on total income following an \( F_x \)-liberalisation makes
Another quantitative exercise on steady-state consumption is a comparison to that under autarky. With our calibration, the effect of trade on steady-state consumption ranges from 1.66 percent to 2.13 percent depending on whether the export overhead consists only of the fixed export cost or only of the sunk export cost. The reason is that in the former case, trade has not affected the share of net profits in revenues, while in the latter case it has increased the share of net profits in revenues. Of course, it may be anywhere in between, and details on the structure of trade costs are needed to determine the exact value. With the share of the sunk cost in export overhead at our reference value of 0.25, steady-state consumption is 1.78 percent larger than under autarky.

\[ \text{Change in steady-state consumption} \]

\[ \text{Ex-ante share of sunk cost in export overhead} \]

Figure 1: Steady-state consumption effects of the different types of trade liberalisation compared to the ACR benchmark.

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a reduction in steady-state consumption theoretically possible. Thus, it may turn out that the change in the share of domestic expenditure is not only an inaccurate measure of the steady-state consumption effects of trade liberalisations, in extreme cases it may even get the sign wrong. However, this does not seem to be the case for plausible parameter values. Further, even if the change in steady-state consumption is negative, the change in welfare is another story due to transition dynamics.

\[ 18 \text{In the ACR benchmark the increase is 1.66 percent as it corresponds to the case without the sunk export costs.} \]
5 Welfare and Transition Dynamics

An implication of a non-zero effect on total income that we have not considered so far is transition dynamics. With a Pareto distribution, the total revenues, $R = I$, are proportional to the mass of firms, $M$, since average revenues, $\bar{r}$, are constant. Thus, the effect on total income feeds directly into the mass of firms, which behaves much like an irreversible capital stock due to the sunk cost of entry.\(^{19}\) In general, liberalisations with non-zero income effects affect entry, and thus induce transition dynamics.

In solving for the transition, we need to parametrise the utility function. We choose the standard form with constant elasticity of intertemporal substitution, $u(C_t) = C_t^{1-\theta}/(1 - \theta)$ and set $\theta = 2$. Further, as mentioned above, we set the ex-ante share of the sunk cost in export overhead to 0.25, but end the section with a discussion of the results when this value is set to 0.5. We let each of the three liberalisations considered reduce the steady-state share of domestic expenditure by one percentage point as in the previous section. Figure 2 depicts the paths of consumption following each of the three types of liberalisation. Also depicted is the consumption path that would occur in the ACR benchmark. That is, immediate transition to the new steady-state with consumption predicted by the ACR formula.

Consider first the $f_x$- and $\tau$-liberalisations. While the former induces higher steady-state consumption, consumption in the short run is lower. The reason is that the higher level of steady-state consumption is achieved through higher investment in exporting, which requires foregoing more consumption in the short run. For evaluating the welfare implications, we calculate the relative equivalent variation (REV) in steady-state consumption that would induce the same welfare as either liberalisation. Both the $f_x$- and the $\tau$-liberalisations correspond to an immediate and permanent increase in steady-state consumption of 0.195 percent. This is also the REV of the ACR benchmark as given by the ACR formula.\(^{20}\) Intuitively, the main difference among these two liberalisations and the ACR benchmark is the induced investment in exporting. In equilibrium, the investments undertaken along the transition have zero net present value. Therefore, for welfare purposes they are irrelevant. While the investments have zero net present value, the higher endogenous discounting prevailing in the short run slightly skews firms’ decisions away from exporting. This means that the share of domestic expenditure does not reach the new steady-state value immediately.\(^{21}\)

\(^{19}\)The death shock plays the role of deterministic depreciation since there is no uncertainty at the aggregate level.

\(^{20}\)While the two liberalisations and the ACR benchmark have similar REV (indistinguishable with the significant digits provided here) they are not exactly equal. More on this below.

\(^{21}\)Details on the adjustment in $\lambda$ are provided in the Supplementary Appendix.
As a consequence, both liberalisations have slightly lower REV than the ACR benchmark. This effect is stronger under the $f_x$- than under the $\tau$-liberalisation since the former requires more short-run investment. Quantitatively this is however negligible and the REV of both the $\tau$- and the $f_x$-liberalisations is very close to that of the ACR benchmark.

In contrast to the REV, the steady-state consumption and the consumption paths induced by the $f_x$- and $\tau$-liberalisations are quite different as should be clear by now. That is, the timing of the realisation of the gains from trade differs substantially. In particular, the effect on steady-state consumption of the $f_x$-liberalisation is 20 percent larger than that of the $\tau$-liberalisation, while the initial effect on consumption is 54 percent smaller. Such differences in investment and consumption paths arising from the sunk export cost could be important for welfare purposes in setups where investments are not carried out efficiently or where policymakers are shortsighted. Further, the finding that the welfare effect of the liberalisation through variable costs of exporting ($\tau$) is not much different compared to the ACR benchmark without a sunk export cost is interesting in relation to Alessandria and Choi (2011). They find that the welfare effect of reducing variable export costs (a tariff) does depend on the presence of a sunk export cost. Our results suggest that this is not a consequence of the sunk export cost itself but rather of interaction with other features of their model. Finally,
while we find that the sunk cost of exporting does have significant implications for the path of consumption, in line with Alessandria and Choi (2011), we do not observe overshooting relative to the new steady-state level. This is primarily due to firms exiting endogenously which means that our model does not feature an overhang of firms.

Consider next the $F_x$-liberalisation. From Figure 2, it seems as though the welfare effect of the $F_x$-liberalisation is somewhat smaller than those of the other two and the ACR benchmark. Indeed, the REV is 0.093 percent. The reason that the $F_x$-liberalisation has less than half the REV of the other two liberalisations is that the full impact of the cost reduction is not felt immediately. Incumbent exporters at the time of liberalisation do not benefit from the reduction in the sunk export cost, since they have entered in the past paying the old cost. The full benefit of a reduction in the sunk cost therefore materialises slowly, as old exporters are hit by the death shock and new exporters enter at the reduced cost. Contrarily, the cost reductions implied by the $f_x$- and $\tau$-liberalisations benefit all exporters immediately.\footnote{Assume (counterfactually) that in the first period following an $F_x$-liberalisation, consumers receive a one-time income equal to the reduction in the sunk export cost times the mass of incumbent exporters. In this case the full benefit of the reduction in $F_x$ can be said to materialise immediately. Computing the REV in this scenario (0.194) reveals that more than 99.8 percent of the difference towards the other two liberalisations have disappeared. Thus, the overwhelming reason that the $F_x$-liberalisation has much lower REV than the other two is indeed the described delay in the realisation of the cost reduction.} This goes to show that when considering liberalisations through nonvariable export costs, it is important to distinguish between whether it is the fixed or sunk cost that is reduced. Not only are the implications for steady-state consumption different, so are those for the effect on welfare. Moreover, these differences are quantitatively important.

Setting the ex-ante share of the sunk cost in export overhead to 0.5 instead yields very similar results for welfare. The REV of the $\tau$-, $f_x$-, and $F_x$- liberalisations are 0.195, 0.194, and 0.093 percent in this case, respectively. The main difference to the scenario above is the size of the sunk costs and thus the induced export investments. As mentioned above, all investments have zero net present value over the transition, and thus the effect on welfare is negligible.\footnote{With a higher sunk cost of exporting, more investment is needed in the short run. Therefore the endogenous discounting is slightly higher over the transition, and firms’ decisions with respect to exporting is skewed a little further. However, this effect is still quite small and only noticeable for the $f_x$-liberalisation with the significant digits provided here.} However, the differences in the timing of gains from trade between the $\tau$- and $f_x$-liberalisation are much larger with a higher sunk export cost. In this case, the effect of the $f_x$-liberalisation on steady-state consumption is 37 percent larger than that of the $\tau$-liberalisation while the immediate effect on consumption is 127 percent lower (following the $f_x$-liberalisation, consumption drops below the...}
initial steady-state level at first).

6 Extensions

The present section briefly discusses how extending the model to additional margins of adjustment or a general distribution of productivities affect the analytical results on steady-state consumption from Section 4.\footnote{Derivations and proofs of the results stated below can be found in the Supplementary Appendix.}

6.1 Additional Margins of Adjustment

Recently, many heterogeneous-firms trade models have been developed in which the firms face multiple margins of adjustment. Among these are Antràs and Helpman (2004), Helpman et al. (2004), Helpman et al. (2010), Bustos (2011), Caliendo and Rossi-Hansberg (2012), Kasahara and Lapham (2013), and many others.\footnote{Further references are provided by Bache and Laugesen (2013) who conduct a general analysis of the comparative statics under firm-level complementarities such as those common in the trade literature.} Additional margins of adjustment are likely to interact with the export decision, thereby accentuating the need to take the structure of the trade costs and the details of the model into account when evaluating trade liberalisations. As an example, consider allowing for technology upgrading at the firm level.

Particularly, and analogous to Bustos (2011), firms can now scale up their productivity by a given factor, $\mu > 1$. Maintaining the higher productivity implies an additional fixed cost of $f_u$ and we assume that upgrading involves an investment, $F_u$ (a sunk cost). While steady-state consumption and average revenue can still be expressed as in (11) and (13) respectively, average net profits are now given by

$$\bar{\pi} = (1 - \beta)(1 - \delta)(F_e + p_x^* F_x + p_u^* F_u),$$

where $p_u^*$ denotes the share of entrants that become technology upgraders. We restrict attention to the same case of firm sorting as Bustos (2011). That is, the most productive firms export and upgrade technology, less productive firms export but do not upgrade technology, even less productive firms neither export nor upgrade technology, and the least productive firms exit. Note that, when this is the case, the marginal technology-upgrading firm is an exporter.

Consider first the $f_x$- and $F_x$-liberalisations. For these two liberalisations we obtain the same ranking of the effects on steady-state consumption as in (15). Firm-level decisions such as producing, exporting, and technology upgrading are
affected by the $f_x$- and $F_x$-liberalisations solely through the export overhead, $f_x + [1 - \beta(1 - \delta)]F_x$. The same is true for the share of domestic expenditure. Therefore, under Assumption 1, the $f_x$- and $F_x$-liberalisations have the same reallocation effect. Further, they are going to have the same effect on the share of entrants that exports, $p_x^e$, and the share that upgrades technology, $p_u^e$. But then their effect on average net profits, $\bar{\pi}$, differs only through the direct effect of $F_x$ on the average export investment, $p_x^e F_x$. Thus, the $f_x$-liberalisation has a larger effect on total income than the $F_x$-liberalisation. Hence, it also has a larger effect on steady-state consumption.

Next, consider the $\tau$-liberalisation. When the marginal technology upgrader is an exporter, trade liberalisations directly affect the decision to upgrade technology. The $\tau$-liberalisation induces exporters to increase production more than do the other two liberalisations. Since technology upgrading is worth more with a larger production volume, this implies that the $\tau$-liberalisation induces more technology upgrading than do the $f_x$- and $F_x$-liberalisations. Compared to the case without technology upgrading, the $\tau$-liberalisation now achieves a given effect on the share of domestic expenditure through reallocation towards technology-upgrading exporters to a greater extent and through reallocation towards exporters in general to a lesser extent. This means that the $\tau$-liberalisation induces less export investment than earlier but at the same time induces more investment in technology upgrading compared to the $f_x$- and $F_x$-liberalisations. The change in total income compared to the other two liberalisations is therefore ambiguous and depends on the magnitudes of the investments in exporting and upgrading, $F_x$ and $F_u$. Finally, due to spill-overs on technology upgrading, the $\tau$-liberalisation induces a larger reallocation effect than the other two liberalisations. In total, the spill-overs on the technology-upgrading decision imply that the effect of the $\tau$-liberalisation on steady-state consumption can be either greater, smaller, or in between those of the other two types of liberalisation.\textsuperscript{26}

6.2 General Distribution of Productivities

It turns out that, when productivities are not Pareto distributed, it is still possible to rank the effects on steady-state consumption of the $f_x$- and the $F_x$-liberalisations according to $\hat{C}_{f_x} > \hat{C}_{F_x}$. Again, the intuition is that with CES

\textsuperscript{26}We could have considered many other extensions of the Melitz (2003) model with additional margins of adjustment such as those mentioned in the beginning of this section. However, following the intuition provided in this section the $f_x$-liberalisation is likely to induce higher steady-state consumption than the $F_x$-liberalisation in many of these multi-margin extensions of the model. On the other hand, the above example also clearly illustrates why we cannot say anything in general about the effect on steady-state consumption of the $\tau$-liberalisation compared to the other two liberalisations, as it interacts with other decisions in a different way.
preferences, an identical effect on $\lambda$ means that these two liberalisations have the same effects on average export overhead and on reallocation. Thus, as the $F_x$-liberalisation by construction reduces $F_x$, the $f_x$-liberalisation induces a higher level of average export investment, $p^*_x F_x$. Effectively, the $f_x$-liberalisation has the larger effect on steady-state consumption through a larger effect on average net profits and hence total income.

For general distributions, the $\tau$-liberalisation cannot be unambiguously ranked relative to the other two liberalisations. First of all, without the Pareto distribution, Assumption 1 does not imply that the $\tau$-liberalisation has the same reallocation effect as the other two.\footnote{Note that ACR need the Pareto distribution for their results to apply to the Melitz (2003) model.} Besides the direct implication for steady-state consumption, this also makes the effect on total income very hard to rank. In general, because both the reallocation effect and the effect on total income may differ, the total effect on steady-state consumption relative to the other liberalisations is ambiguous.

7 Concluding Remarks

We have showed that distinguishing between fixed and the empirically relevant sunk export costs in the composition of nonvariable export costs is indeed important for welfare analysis of trade liberalisations even when conditioning on the share of domestic expenditure. Contrary to the case without sunk export costs analysed by ACR, details on the structure of, and change in, trade costs matter. First off, sunk export costs mean that different liberalisations have systematically different effects on steady-state consumption. Further, trade liberalisations through the fixed or sunk export costs differ significantly with respect to their effect on welfare. The introduction of sunk export costs also implies that even though liberalisations through fixed and variable trade costs induce similar increases in welfare, the timing of consumption differs considerably. The importance of this difference may be accentuated if investments are not conducted efficiently or if policymakers have short time horizons. Overall, our results lead us to the conclusion that fixed and sunk costs cannot be treated as equivalent representations of nonvariable export costs. That the importance of sunk costs in the export decisions of firms has been confirmed empirically emphasises the relevance of this consideration.

Finally, we showed how the ranking of the effects on steady-state consumption levels may depend on the margins of adjustment that are available to firms. The steady-state consumption effects of a given change in the share of domestic expenditure depend not only on which type of trade cost are reduced, it also
depends on the spillovers on other margins of adjustment such as technology upgrading and the associated investments.

A $\varphi^*$, $\lambda$, and $\bar{r}$ under the Pareto

From (3) and (5) we get

$$B_t = f(\varphi^*)^{1-\sigma}$$

while from (4) and (6), we get

$$B_t = (f_x + [1 - \beta(1 - \delta)]F_x)\tau^\sigma - 1(\varphi^*_x)^{1-\sigma}.$$  \hspace{1cm} (18)

Combining these, we get

$$\varphi^*_x = \tau \varphi^* \left( \frac{f_x + [1 - \beta(1 - \delta)]F_x}{f} \right)^{\frac{1}{\sigma-1}}.$$  \hspace{1cm} (19)

Substituting the expressions for $B_t$ in (17) and (18) back into the profit functions (3) and (4), respectively, and plugging these profit functions into the free entry condition (7), we obtain the condition

$$[1 - \beta(1 - \delta)]F_e = \int_{\varphi^*}^\infty \left( \frac{(\varphi^*_x)^{1-\sigma} - 1}{f} \right) k\varphi^{-k-1}_{\min} d\varphi$$

where $G(\varphi) = 1 - (\varphi_{\min}/\varphi)^k$ has been used. Integrating and rearranging, we get

$$[1 - \beta(1 - \delta)]F_e = \frac{\sigma-1}{k-(\sigma-1)} f \left( \frac{\varphi_{\min}}{\varphi^*} \right)^k \left( 1 + \frac{f_x + [1 - \beta(1 - \delta)]F_x}{f} \right) \left( \frac{\varphi^*_x}{\varphi^*} \right)^k.$$  \hspace{1cm} (20)

Now, noting that the revenue from domestic and export sales can be expressed as $r_d(\varphi) = \sigma(\pi_d(\varphi) + f)$ and $r_x(\varphi) = \sigma(\pi_d(\varphi) + f)$, as shown in the Supplementary Appendix, the share of domestic expenditure can be expressed as

$$\lambda = \frac{\int_{\varphi^*}^\infty \sigma(\pi_d(\varphi) + f) dG(\varphi)}{\int_{\varphi^*}^\infty \sigma(\pi_d(\varphi) + f) dG(\varphi) + \int_{\varphi^*_x}^\infty \sigma(\pi_x(\varphi) + f_x) dG(\varphi)}.$$  \hspace{1cm}

Repeating analogous steps of substituting for profit functions and integrating as above, we get

$$\lambda = \left( 1 + \frac{f_x + [1 - \beta(1 - \delta)]F_x}{f} \right)^{-1}.  \hspace{1cm} (20)$$
Combining (19) and (20), we get after rearranging that

\[ \varphi^* = \varphi_{\text{min}} \left( \frac{\sigma - 1}{k - (\sigma - 1)} \frac{f}{[1 - \beta(1 - \delta)]F_e} \right)^{1/k} \lambda^{-1/k}, \tag{21} \]

which corresponds to (12).

Average revenues can be expressed as

\[ \bar{r} = \int_{\varphi^*}^{\infty} \sigma(\pi_d(\varphi) + f) \, dG(\varphi) + \int_{\varphi^*}^{\infty} \sigma(\pi_x(\varphi) + f_x) \, dG(\varphi). \]

Substituting for profit functions and integrating once again yields,

\[ \bar{r} = \frac{\sigma k}{k - (\sigma - 1)} f(\varphi_{\text{min}})^k \left( 1 + \frac{f_x + [1 - \beta(1 - \delta)]F_e}{\varphi^*} \right). \]

Combining this expression with (19), we obtain

\[ \bar{r} = \frac{\sigma k}{\sigma - 1} [1 - \beta(1 - \delta)] F_e, \]

which corresponds to expression (13).

**B Proof of Lemma 1**

Rearranging (20) and using that with the Pareto, \( p_x^* = (\varphi_{\text{min}} / \varphi^*_x)^k \), we get

\[ \lambda = \left( 1 + \xi f^{-1} \left( \frac{\varphi^*_x}{\varphi_{\text{min}}} \right)^k \right)^{-1}. \tag{22} \]

Now, since the three liberalisations have the same effect on \( \lambda \) and, by (21) this means that they have the same effect on \( \varphi^* \) as well, (22) implies that they must have the same effects on average export overhead, \( \xi \).

**References**


Firms and International Trade


Supplementary Appendix for Chapter One

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Abstract

This Appendix provides details and derivations left out of the main part of the first chapter. Section 1 provides details on a few expressions from the baseline model used in the main part of the chapter. Section 2 discusses the adjustment of the share of domestic expenditure along the transitions induced by the liberalisations considered in the chapter. Sections 3 and 4 derive the steady-state results discussed in the chapter when firms are allowed to upgrade technology and the distribution function is kept general, respectively. Finally, Section 5 describes the algorithm employed in solving numerically for transition paths.
1 Baseline Model

This section provides details on the few expressions from the paper’s Sections 2 and 3 that were not thoroughly explained or derived.

1.1 Demand

For future reference we restate some expressions given in the paper. Aggregate consumption reads

$$C_t = \left[ \int_{\Omega_t} q_{\omega,t}^\alpha d\omega \right]^{1/\alpha}.$$  

This gives rise to the following demand function for variety \( \omega \),

$$q_{\omega,t} = \frac{I_t}{P_t^{1-\sigma}p_{\omega,t}^{-\sigma}},$$  \hspace{1cm} (1)

where \( \sigma = 1/(1 - \alpha) > 1 \), \( I_t \) is the income spent in period \( t \), and \( P_t = \left[ \int_{\Omega_t} p_{\omega,t}^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}} \) is a price index. Note that \( C_t = I_t/P_t \) holds. These expressions arise from standard analysis of the CES preferences.

1.2 Firm Profits, and Revenues

A firm with productivity \( \varphi \) has constant marginal cost at \( 1/\varphi \).\(^1\) For ease of exposition, we drop time subscripts in this section. Indexing firms by their productivity and using (1), profits from serving the domestic market are given by

$$\pi_d(\varphi) = p_d q_d - \frac{1}{\varphi} q_d - f = \frac{I}{P^{1-\sigma}}(p_d^{1-\sigma} - \varphi^{-1} p_d^{-\sigma}) - f,$$  \hspace{1cm} (2)

where \( p_d \) and \( q_d \) denote optimal price and quantity for the domestic market. The profit optimising price is given by \( p_d = (\alpha \varphi)^{-1} \). Plugging this into (2), we get

$$\pi_d(\varphi) = \frac{\alpha}{\sigma - 1} CP^\sigma \varphi^{-1 - \sigma} - f,$$ \hspace{1cm} (3)

where we have used \( \sigma - 1 = \alpha/(1 - \alpha) \) and \( C = I/P \). Further, using (1) and \( p_d = (\alpha \varphi)^{-1} \), we get the expression for revenues from domestic sales

$$r_d(\varphi) = p_d q_d = \sigma \frac{\alpha}{\sigma - 1} CP^\sigma \varphi^{-1 - \sigma} = \sigma(\pi_d(\varphi) + f).$$ \hspace{1cm} (4)

\(^1\)The wage is normalised to unity.
Repeating the same steps for the export market, which a firm serves with effective productivity \(\varphi/\tau\), and incurs fixed costs \(f_x\), gives the export market equivalents of (3) and (4) as

\[
\begin{align*}
\pi_x(\varphi) &= \frac{\alpha^\sigma}{\sigma - 1} CP^{\sigma} \tau^{1-\sigma} \varphi^{\sigma-1} - f_x \tag{5} \\
r_x(\varphi) &= \sigma \frac{\alpha^\sigma}{\sigma - 1} CP^{\sigma} \tau^{1-\sigma} \varphi^{\sigma-1} = \sigma (\pi_x(\varphi) + f_x). \tag{6}
\end{align*}
\]

### 1.3 Steady-state Relation between \(\varphi^*\) and \(\varphi_x^*\)

As noted in the paper, \(\pi_d(\varphi^*) = 0\) and \(\pi_x(\varphi_x^*) = [1 - \beta(1 - \delta)]F_x\) has to hold in steady-state. Using, (3) and (5) this gives rise to

\[
\frac{\alpha^\sigma}{\sigma - 1} CP^{\sigma} = (\varphi^*)^{1-\sigma} \tag{7}
\]

and

\[
\frac{\alpha^\sigma}{\sigma - 1} CP^{\sigma} = \tau^{\sigma-1} (\varphi_x^*)^{1-\sigma} (f_x + [1 - \beta(1 - \delta)]F_x). \tag{8}
\]

Combining (7) and (8) yields,

\[
\varphi_x^* = \tau \varphi^* \left( \frac{f_x + [1 - \beta(1 - \delta)]F_x}{f} \right)^{\frac{1}{\sigma - 1}}. \tag{9}
\]

### 1.4 Steady-state Consumption

This subsection shows how to obtain the expression for steady-state consumption provided in Subsection 3.2 of the paper by combining \(C = I/P\), (3), and \(\pi_d(\varphi^*) = 0\). Note first that (3) and \(\pi_d(\varphi^*) = 0\) results in (7). Using \(P = I/C\),

\[
\frac{\alpha^\sigma}{\sigma - 1} C^{1-\sigma} I^{\sigma} (\varphi^*)^{\sigma-1} - f = 0
\]

from which rearranging gives

\[
C = \left( \frac{\alpha^\sigma}{f/(\sigma - 1)} \right)^{1/(\sigma - 1)} I^{\sigma/(\sigma - 1)} \varphi^*. \tag{10}
\]

Using \(\alpha = (\sigma - 1)/\sigma\) yields the expression in the paper.

## 2 Adjustment of \(\lambda\) over Transition

Figure 1 below shows how the share of domestic expenditure adjusts for each of the three liberalisations along with the ACR benchmark (immediate transition to new steady-state value).²

²Evaluation of \(\lambda\) along the transition is discussed in Subsection 5.6 after the algorithm for solving for transition paths has been described.
Consider first the $\tau$- and $f_x$ liberalisations. Both of these increase the endogenous discounting in the short run due to foregone consumption implied by investments in new firms and exporters. Thus, as exporting implies an investment, this slightly skews the decisions of firms away from exporting relative to domestic production. Further, for the $f_x$-liberalisation this effect is stronger than the $\tau$-liberalisation since it induces more investment. This means that the share of domestic expenditure does not drop to the new steady-state value immediately as is seen in Figure 1 and that the immediate drop is smaller following the $f_x$-liberalisation than following the $\tau$-liberalisation.\(^3\) This discrepancy between the adjustment in the share of domestic expenditure is the source of the slight differences in REV of the two liberalisations (the investment effects are unimportant for welfare as described in the paper). We stress that in our calibrated model, this discrepancy is very slight. The adjustment of the share of domestic expenditure over the transition is less than 0.01 percentage point and thus less than one

\(^3\)That $\lambda$ for the two liberalisations actually falls below the new steady-state value at some point is due to the export threshold being increasing along the transition. That is, when the skewing implied by higher discounting disappears, $\lambda$ is actually smaller than the new steady-state due to past exporters who have entered with a productivity below the new steady-state export threshold and who have not been hit by the death shock yet.
hundreth of the total adjustment in $\lambda$ from the initial to the new steady-state (which is 1 percentage point). Therefore the welfare discrepancy between the two liberalisations is negligible and cannot be detected with the significant digits provided for the REV in the paper.

The $F_x$-liberalisation features a decumulation of firms over the transition. Due to a negative income effect, the mass of firms is lower in the new compared to the initial steady state. This means that in the short run there are relatively many firms, which intensifies competition and thus discourages investment in exporting. On the other hand, the low discounting implied by the decumulation of firms encourages investment in exporting. In total, the adjustment in $\lambda_t$ over the transition is very slight (well above 99 percent of the total adjustment in $\lambda$ happen immediately), and the main source of the difference in REV compared to the other two liberalisations is the fact that the $F_x$-liberalisation does not benefit incumbent exporters (as discussed in the paper).

3 Steady-state Results with Technology Upgrading

Now, extend the above model to include the possibility of technology upgrading along the lines of Bustos (2011), but with a sunk cost of upgrading. Note that expressions derived in Section 1 of this appendix are still valid under this extension.

3.1 Firm Profits and Revenues with Upgraded Technology

In case technology is upgraded, productivity is scaled up by the factor $\mu > 1$. If a firm has upgraded technology, the total profits and revenues at the domestic market are given by

$$\pi_{du}(\phi) = \frac{\alpha^\sigma}{\sigma-1}CP^\sigma(\mu\phi)^{\sigma-1} - f - f_u$$

$$r_{du}(\phi) = \sigma\frac{\alpha^\sigma}{\sigma-1}CP^\sigma(\mu\phi)^{\sigma-1} = \sigma(\pi_{du}(\phi) + f + f_u),$$

where $f_u$ is the fixed cost of technology upgrading, while the counterparts for the export market are given by

$$\pi_{xu}(\phi) = \frac{\alpha^\sigma}{\sigma-1}CP^\sigma\tau^{1-\sigma}(\mu\phi)^{\sigma-1} - f_x$$

$$r_{xu}(\phi) = \sigma\frac{\alpha^\sigma}{\sigma-1}CP^\sigma\tau^{1-\sigma}(\mu\phi)^{\sigma-1} = \sigma(\pi_{xu}(\phi) + f_x).$$

These expressions are analogous to (3)–(6).

3.2 The Technology Upgrading Decision

Note that we assume that the threshold for technology upgrading is higher than that for exporting. Thus, when firms make their decision to invest in technology
upgrading, they compare the discounted future increments in domestic and export profits from upgrading technology with the sunk cost of upgrading, $F_u$. That is, $\varphi^*_u$ is given by

$$F_u = \sum_{s=t}^{\infty} [\pi_{xu}(\varphi^*_u) - \pi_x(\varphi^*_u) + \pi_{du}(\varphi^*_u) - \pi_{du}(\varphi^*_u)] \beta^{s-t}(1-\delta)^{s-t},$$

we know that all firms with $\varphi > \varphi^*_u$ upgrades technology. Rewriting the infinite sum and rearranging while using (3), (5), (11), and (13) gives

$$f_u + [1 - \beta(1-\delta)]F_u = \frac{\alpha^*_{\infty}}{\sigma-1} CP^\sigma(\mu^{\sigma-1} - 1)(1 + \tau^{1-\sigma})(\varphi^*_u)^{\sigma-1}.$$  

Using this along with (7), we get the relation between $\varphi^*_u$ and $\varphi^*$,

$$\varphi^*_u = \varphi^* \left( \frac{f_u + [1 - \beta(1-\delta)]F_u}{f(\mu^{\sigma-1} - 1)(1 + \tau^{1-\sigma})} \right)^{\frac{1}{\sigma-1}},$$

which with the pareto distribution gives the probability of upgrading technology as

$$p^*_u = \left( \frac{\varphi_{\min}}{\varphi^*} \right)^k \left( \frac{\varphi^*}{\varphi^*_u} \right)^k = p^* \left( \frac{f_u + [1 - \beta(1-\delta)]F_u}{f(\mu^{\sigma-1} - 1)(1 + \tau^{1-\sigma})} \right)^{-\frac{k}{\sigma-1}},$$

where $p^* = (\varphi_{\min}/\varphi^*)^k$ is the share of successful entrants. Similarly, using (9), we get

$$p^*_x = \left( \frac{\varphi_{\min}}{\varphi^*} \right)^k \left( \frac{\varphi^*_u}{\varphi^*_x} \right)^k = p^* \tau^{-k} \left( \frac{f_x + [1 - \beta(1-\delta)]F_x}{f} \right)^{-\frac{k}{\sigma-1}}.$$  

### 3.3 Entry

Extending the free-entry condition from the paper to take technology upgrading into account, we get

$$[1 - \beta(1-\delta)](F_e + p^*_xF_x + p^*_u F_u) = \int_{\varphi^*_u}^{\varphi^*_x} \pi_d(\varphi) dG(\varphi) + \int_{\varphi^*_u}^{\varphi^*_x} \pi_x(\varphi) dG(\varphi) + \int_{\varphi^*_u}^{\infty} \pi_{du}(\varphi) + \pi_{xu}(\varphi) dG(\varphi).$$  

Subtracting the average reinvestments, $\delta(F_e + p^*_xF_x + p^*_u F_u)$ from both sides gives us the following expression for average net profits,

$$\bar{\pi} = (1 - \beta)(1-\delta)(F_e + p^*_xF_x + p^*_u F_u).$$  

Note that using the relationships between $\pi_d$ and $r_d$ in (4), between $\pi_x$ and $r_x$ in (6), between $\pi_{du}$ and $r_{du}$ in (12), and between $\pi_{xu}$ and $r_{xu}$ in (14), the condition (18) can be written as

$$\bar{\tau} = \sigma([1 - \beta(1-\delta)](F_e + p^*_xF_x + p^*_u F_u) + p^* f + p^*_x f_x + p^*_u f_u).$$
3.4 Production threshold, Average Revenues, and the Share of Expenditure on Domestic Goods

To arrive at the expression for the production threshold and average revenues, we go through the same steps as for the case without technology upgrading described in the paper. First, plugging profit functions (3), (5), (11), and (13) along with the distribution function \( G(\varphi) = 1 - (\varphi_{\min}/\varphi)^k \) into (18) and integrating gives

\[
[1 - \beta(1 - \delta)] F_e = \frac{\sigma - 1}{k - (\sigma - 1)} \int \frac{f(\varphi_{\min})}{\varphi^k} \left( 1 + \tau^{-k} \left( \frac{f_x + [1 - \beta(1 - \delta)]F_x}{f} \right)^{1 - \frac{k}{\sigma - 1}} \right)^{1 - \frac{k}{\sigma - 1}} \phi \left( \frac{f_x + [1 - \beta(1 - \delta)]F_x}{f} \right)^{1 - \frac{k}{\sigma - 1}} \right], \tag{21}
\]

when using (7) and the relationships between thresholds in (9) and (15). From (21) we can immediately obtain

\[
\varphi^* = \varphi_{\min} \left[ \frac{\sigma - 1}{k - (\sigma - 1)} \int \frac{f(\varphi)}{\varphi^k} \left( 1 + \tau^{-k} \left( \frac{f_x + [1 - \beta(1 - \delta)]F_x}{f} \right)^{1 - \frac{k}{\sigma - 1}} \right)^{1 - \frac{k}{\sigma - 1}} \right]^{1/k}. \tag{22}
\]

Combining (16), (17), (20), and (22) yields

\[
\bar{r} = \frac{\sigma k}{\sigma - 1} [1 - \beta(1 - \delta)] F_e. \tag{23}
\]

Next, we can express the share of domestic expenditure as

\[
\lambda = \frac{\int_{\varphi_*}^{\varphi_u^*} r_d(\varphi) M dG(\varphi) + \int_\varphi^{\varphi_u} r_{du}(\varphi) M dG(\varphi)}{\int_{\varphi_*}^{\varphi_u^*} r_d(\varphi) M dG(\varphi) + \int_{\varphi_*}^{\varphi_u^*} r_x(\varphi) M dG(\varphi) + \int_{\varphi_u^*}^{\varphi_*} [r_{du}(\varphi) + r_{ux}(\varphi)] M dG(\varphi)}
\]

Plugging in (4), (6), (12), and (14) together with \( G(\varphi) \) gives

\[
\lambda = \frac{\int_{\varphi_*}^{\varphi_u^*} \varphi^{\sigma - 2 - k} d\varphi + \mu^{-1} \int_{\varphi_u^*}^{\varphi_*} \varphi^{\sigma - 2 - k} d\varphi}{\int_{\varphi_*}^{\varphi_u^*} \varphi^{\sigma - 2 - k} d\varphi + \tau^{1 - \sigma} \int_{\varphi_*}^{\varphi_u^*} \varphi^{\sigma - 2 - k} d\varphi + \mu^{-1} \int_{\varphi_u^*}^{\varphi_*} (1 + \tau^{1 - \sigma}) \int_{\varphi_u^*}^{\varphi_*} \varphi^{\sigma - 2 - k} d\varphi}
\]

upon cancelling out terms. Integrating and rearranging yields

\[
\lambda = \frac{1 + (\mu^{-1} - 1)(\varphi_u^*/\varphi_x^*)^{\sigma - 1}}{1 + \tau^{1 - \sigma} (\varphi_x^*/\varphi_u^*)^{\sigma - 1} + (\mu^{-1} - 1)(1 + \tau^{1 - \sigma}) (\varphi_u^*/\varphi_x^*)^{\sigma - 1}}.
\]

Inserting for \( \varphi_x^* \) and \( \varphi_u^* \) using (9) and (15) respectively, we get

\[
\lambda = \frac{1 + (\mu^{-1} - 1) \left( \frac{f_u + [1 - \beta(1 - \delta)]F_u}{f(\mu^{-1} - 1)(1 + \tau^{1 - \sigma})} \right)^{1 - \frac{k}{\sigma - 1}}}{1 + \tau^{-k} \left( \frac{f_x + [1 - \beta(1 - \delta)]F_x}{f} \right)^{1 - \frac{k}{\sigma - 1}} + (\mu^{-1} - 1)(1 + \tau^{1 - \sigma}) \left( \frac{f_u + [1 - \beta(1 - \delta)]F_u}{f(\mu^{-1} - 1)(1 + \tau^{1 - \sigma})} \right)^{1 - \frac{k}{\sigma - 1}}}. \tag{24}
\]
Combining (22) and (24) yields

\[ \varphi^* = \varphi_{\min} \left( \frac{\sigma^{-1}}{k-(\sigma-1)} \left[ \frac{f}{[1-\beta(1-\delta)]F_x} \right] \right)^{1/k} \lambda^{-1/k} \left[ 1 + \left( \mu^{\sigma-1} - 1 \right) \left( \frac{f_u + [1-\beta(1-\delta)]F_u}{f(\mu^{\sigma-1} - 1)(1+\tau^{1-\sigma})} \right)^{1-\frac{k}{\sigma-1}} \right]^{1/k}. \]

Note that the reallocation effect is now no longer determined solely by \( \lambda \) as \( \varphi^* \) also depends explicitly on \( \tau \).

3.5 Showing that \( \hat{C}_{f_x} > \hat{C}_{F_x} \)

From (24) and (25), it is seen that, if the \( f_x \)- and the \( F_x \)-liberalisations have identical effects on \( \lambda \), they must have identical effects on \( \varphi^* \) and on \( f_x + [1-\beta(1-\delta)]F_x \). It follows that their reallocation effects are identical, and thus from (10) that their relative effect on steady-state consumption is determined by their relative effect on total income.

As was the case without technology upgrading, the average revenue, given by (23), is unaffected by trade costs. That is, the effects on the total income of the \( f_x \)- and \( F_x \)-liberalisations are determined by their effects on the average net profits given by (19). Now, as the \( f_x \)- and \( F_x \)-liberalisations have the same effects on \( \varphi^* \) and the export overhead, \( f_x + [1-\beta(1-\delta)]F_x \), they have the same effects on \( p_x^* \) and \( p_u^* \); see (16) and (17). Therefore, by (19), the \( f_x \)-liberalisation has a larger effect on \( \bar{\pi} \), and thereby a larger effect on total income. The ranking of consumption effects follows.

3.6 Showing that the Ranking of \( \hat{C}_\tau \) Relative to \( \hat{C}_{f_x} \) and \( \hat{C}_{F_x} \) is Ambiguous

First off, note that when the \( \tau \)-liberalisation has the same effect on \( \lambda \) as the other two liberalisations, then by (25) it has a larger effect on the production threshold, \( \varphi^* \). Hence, it has a larger reallocation effect; see (10).

Next step is to note that, if the \( \tau \)-liberalisation has the same effect on \( \lambda \) as the other two liberalisations, it has a lower effect on the average export overhead, \( \xi \).\(^4\)

Further, \( p_u^* \) is larger following the \( \tau \)-liberalisation than following the other two. The last point can be seen from plugging (25) into (16) to get

\[ p_u^* = \varphi_{\min}^{k} \gamma^{-k} \lambda \left[ \frac{\left( \frac{f_u + [1-\beta(1-\delta)]F_u}{f(\mu^{\sigma-1} - 1)(1+\tau^{1-\sigma})} \right)^{-\frac{k}{\sigma-1}}}{1 + \left( \mu^{\sigma-1} - 1 \right) \left( \frac{f_u + [1-\beta(1-\delta)]F_u}{f(\mu^{\sigma-1} - 1)(1+\tau^{1-\sigma})} \right)^{1-\frac{k}{\sigma-1}}} \right]^{1/k}. \]

\(^4\)This is shown in the next subsection.
where the expression in the square brackets is decreasing in $\tau$.

Consider first the case of $F_x \equiv 0$. Then the only effect on total income is through average upgrading investment. Therefore, since $p^*_u$ is larger following the $\tau$-liberalisation, it has a larger effect on total income than the $f_x$-liberalisation. Second, consider the case of $F_u \equiv 0$, such that the only effect on total income is through average export investment, and let $f_x \equiv 0$. Then we have that the share of the sunk cost in export overhead is one, and since the $F_x$-liberalisation has a larger effect on average export overhead, it has a larger effect on average export investment. Hence, the $F_x$-liberalisation has a larger effect on total income than the $\tau$-liberalisation. Of course, we may be anywhere in between these two extremes and it follows that the effect on total income of the $\tau$-liberalisation may be larger, smaller, or in between those of the other two liberalisations.

Finally, these ambiguities imply that the $\tau$-liberalisation may lead to an effect on steady-state consumption that is either larger, smaller or in between those of the other two liberalisations, partly depending on the degree of love-of-variety which influences the relative importance of the income effect.

### 3.7 $\xi$ lowest following a $\tau$-liberalisation

To show this, consider the following expression for $\lambda$ in (24)

$$\lambda = \frac{1 + \Theta_1}{1 + \frac{\xi}{p^*f} + \Theta_2}$$

where, using (17),

$$\frac{\xi}{p^* f} = \frac{p^*_u [f_x + [1 - \beta (1 - \delta)] F_x]}{p^* f} = \tau^{-k} \frac{(f_x + [1 - \beta (1 - \delta)] F_x)}{f} \tau^{-\frac{k}{\mu - 1}}$$

and $\Theta_1$ and $\Theta_2$ are defined as

$$\Theta_1 = (\mu^\sigma - 1) \left( \frac{f_u + [1 - \beta (1 - \delta)] F_u}{f (\mu^\sigma - 1) (1 + \tau^{1 - \sigma})} \right) \tau^{-\frac{k}{\mu - 1}}$$

$$\Theta_2 = (\mu^\sigma - 1) (1 + \tau^{1 - \sigma}) \left( \frac{f_u + [1 - \beta (1 - \delta)] F_u}{f (\mu^\sigma - 1) (1 + \tau^{1 - \sigma})} \right) \tau^{-\frac{k}{\mu - 1}}$$

and thus the expression in (26) is equivalent to (24).

It should be evident that the $f_x$- and $F_x$-liberalisations decrease $\lambda$ solely by increasing $\xi/(p^* f)$. However, while the $\tau$-liberalisation also increases $\xi/(p^* f)$, it also affects $\Theta_1$ and $\Theta_2$. Assume that the combined effect through $\Theta_1$ and $\Theta_2$ on $\lambda$ of a $\tau$-liberalisation was negative. Then both the combined effect through
\( \Theta_1 \) and \( \Theta_2 \) and the effect through \( \xi/(p^*f) \) of the \( \tau \)-liberalisation tend to reduce \( \lambda \). But then the \( \tau \)-liberalisation must increase \( \xi/(p^*f) \) less than the \( f_x \) - and the \( F_x \)-liberalisations for them to have the same effect on \( \lambda \) which the paper assumes throughout. Since the \( \tau \)-liberalisation also implies that \( p^* \) is less than that following the other two liberalisations,\(^5\) this means that the average export overhead, \( \xi \), must be lowest following the \( \tau \)-liberalisation. What remains to be shown is that following a \( \tau \)-liberalisation, the combined effect on \( \lambda \) through \( \Theta_1 \) and \( \Theta_2 \) is in fact negative.

To do so, differentiate (26) with respect to \( \tau^{1-\sigma} \),

\[
\frac{d \lambda}{d \tau^{1-\sigma}} = \frac{(1 + \frac{\xi}{p^*f} + \Theta_2) \frac{d \Theta_1}{d \tau^{1-\sigma}} - (1 + \Theta_1) \frac{d \Theta_2}{d \tau^{1-\sigma}}}{(1 + \frac{\xi}{p^*f} + \Theta_2)^2} - \frac{1 + \Theta_1}{(1 + \frac{\xi}{p^*f} + \Theta_2)^2} \frac{d \xi}{d \tau^{1-\sigma}}
\]

Combined effect through \( \Theta_1 \) and \( \Theta_2 \)

As \( \tau^{1-\sigma} \) is increased following a \( \tau \)-liberalisation, we now need to show that the first fraction on the right hand side in (27) is negative. First, differentiate \( \Theta_1 \) with respect to \( \tau^{1-\sigma} \)

\[
\frac{d \Theta_1}{d \tau^{1-\sigma}} = \left( \frac{k}{\sigma-1} - 1 \right) \Theta_1(1 + \tau^{1-\sigma})^{-1}
\]

Second, differentiate \( \Theta_2 \) with respect to \( \tau^{1-\sigma} \)

\[
\frac{d \Theta_2}{d \tau^{1-\sigma}} = \frac{k}{\sigma-1} \Theta_2(1 + \tau^{1-\sigma})^{-1}
\]

Next, consider the numerator in the first fraction on the right hand side in (27)

\[
(1 + \frac{\xi}{p^*f} + \Theta_2) \frac{d \Theta_1}{d \tau^{1-\sigma}} - (1 + \Theta_1) \frac{d \Theta_2}{d \tau^{1-\sigma}} = (1 + \frac{\xi}{p^*f} + \Theta_2) \left( \frac{k}{\sigma-1} - 1 \right) \Theta_1(1 + \tau^{1-\sigma})^{-1} - (1 + \Theta_1) \frac{k}{\sigma-1} \Theta_2(1 + \tau^{1-\sigma})^{-1}
\]

Rearranging and using the fact that \( \Theta_2 = \Theta_1(1 + \tau^{1-\sigma}) \) yields

\[
(1 + \frac{\xi}{p^*f} + \Theta_2) \frac{d \Theta_1}{d \tau^{1-\sigma}} - (1 + \Theta_1) \frac{d \Theta_2}{d \tau^{1-\sigma}} = \frac{k}{\sigma-1} \Theta_1(1 + \tau^{1-\sigma})^{-1} \left( \frac{\xi}{p^*f} \tau^{\sigma-1} - 1 \right) - \Theta_1(1 + \tau^{1-\sigma})^{-1} \left( 1 + \frac{\xi}{p^*f} + \Theta_2 \right)
\]

---

\(^5\)This follows directly from \( \varphi^* \) being higher following the \( \tau \)-liberalisation, which is shown in the paper.
Finally, if we show that $\xi \tau^{\sigma-1}/(p^* f) < 1$ then we have shown that following the $\tau$-liberalisation the combined effect through $\Theta_1$ and $\Theta_2$ on $\lambda$ is negative and we are done. That this is indeed the case can be seen from

$$\frac{\xi}{p^* f} \tau^{\sigma-1} = \frac{p_x^*}{p_x^* \tau^{\sigma-1}} f_x + \frac{1 - \beta (1 - \delta)}{f} = \left( \frac{f_x + [1 - \beta (1 - \delta)] F_x}{f \tau^{1-\sigma}} \right)^{1 - \frac{k}{\sigma - 1}} \tag{28}$$

where we have used (17). Since we have assumed that not all firms export, we must have that $\varphi^* < \varphi_x^*$ and thus $p^* > p_x^*$ which by (17) implies that $f_x + [1 - \beta (1 - \delta)] F_x > f \tau^{1-\sigma}$. But by (28) this means that $\xi \tau^{\sigma-1}/(p^* f) < 1$ and we are done.\(^6\)

### 4 General Distribution of Productivities

Let us drop technology upgrading and instead let $G(\varphi)$ be a general distribution. The expressions derived in Section 1 still hold in this case. Note that using the relationship between $\pi_d$ and $r_d$ in (4) and between $\pi_x$ and $r_x$ in (6), the free entry condition from the paper gives rise to

$$\bar{r} = \sigma \left( [1 - \beta (1 - \delta)] F_e + p^* f + p_x^* (f_x + [1 - \beta (1 - \delta)] F_x) \right). \tag{29}$$

#### 4.1 The Share of Domestic Expenditure

The share of domestic expenditure can be expressed as

$$\lambda = \frac{\int_{\varphi^*}^{\varphi} r_d(\varphi) M \, dG(\varphi)}{\int_{\varphi^*}^{\varphi} r_d(\varphi) M \, dG(\varphi) + \int_{\varphi^*}^{\varphi} r_x(\varphi) M \, dG(\varphi)}. \tag{30}$$

Substituting in for (4) and (6) and cancelling out terms gives

$$\lambda = \frac{\int_{\varphi^*}^{\varphi} \varphi^{\sigma-1} dG(\varphi)}{\int_{\varphi^*}^{\varphi} \varphi^{\sigma-1} dG(\varphi) + \tau^{1-\sigma} \int_{\varphi^*}^{\varphi} \varphi^{\sigma-1} dG(\varphi)}. \tag{31}$$

Now, $\bar{r} = R/M$ can be expressed as

$$\bar{r} = \int_{\varphi^*}^{\varphi} r_d(\varphi) \, dG(\varphi) + \int_{\varphi^*}^{\varphi} r_x(\varphi) \, dG(\varphi).$$

Using (30),

$$\bar{r} \lambda = \int_{\varphi^*}^{\varphi} r_d(\varphi) \, dG(\varphi).$$

\(^6\)Remember that $k > \sigma - 1$ in order to ensure bounded expected profits of entry.
Further, by inserting for \( r_d(\varphi) \) while using (7), we get
\[
\bar{r}\lambda = \sigma f \int_{\varphi^*}^{\infty} \left( \frac{\varphi}{\varphi^*} \right)^{\sigma-1} dG(\varphi).
\] (32)

4.2 Ranking of Steady-state Consumption Effects

Now, \( \varphi^*_x \), \( \lambda \), \( \bar{r} \), and \( \varphi^* \) are determined by (9), (31), and (32) along with the free-entry condition expressed in terms of average revenue, \( (29) \). From these equations it can be confirmed that \( \lambda \), \( \bar{r} \), \( \varphi^* \), and \( \varphi^*_x \) only depend on \( f_x \) and \( F_x \) through the export overhead, \( f_x + [1 - \beta (1 - \delta)]F_x \). Thus, when the \( f_x \)- and the \( F_x \)-liberalisations have the same effect on the share of domestic expenditure, \( \lambda \), they have the same effect on \( \bar{r} \), \( \varphi^* \), and \( \varphi^*_x \).

Since the two liberalisations have the same effects on \( \varphi^* \), they also have the same reallocation effects. Thus, their relative effect on steady-state consumption is determined by their relative effect on total income. Since the two liberalisations have the same effects on \( \varphi^*_x \) and thereby on \( p^*_x \), the \( f_x \)-liberalisation has a larger effect on average export investment and thereby on average profits, \( \bar{\pi} = (1 - \beta)(1 - \delta)(F_e + p^*_xF_x) \). As argued above, the two liberalisations have the same effects on average revenues, and therefore it follows that the \( f_x \)-liberalisation has the larger effect on \( s_\pi = \bar{\pi}/\bar{r} \) and hence on total income \( I = (1 - s_\pi)^{-1}L \). Thus, it has a larger effect on steady-state consumption.

Without the Pareto, we cannot say how the \( \tau \)-liberalisation affects reallocation and income relative to the other two liberalisations. The reason is mainly that with the way \( \tau \) enters explicitly in (31) we cannot be sure how the \( \tau \)-liberalisation affects \( \varphi^* \) and the average export overhead with a general \( G(\varphi) \).

5 Solving for the Transition

This appendix describes the algorithm employed to solve numerically for the transition following the liberalisations as described in the paper. The first subsection describes the idea of the algorithm, and the next subsections describe the detailed calculations carried out. The calculations are simplified by utilising conjectures about the transition, which are of course checked to be true ex post.

5.1 The Algorithm

First, the number of time periods, \( T \), is chosen. It is assumed that the economy has reached a steady-state at time \( T \).\(^7\) A guess for \( \{P_t, C_t\}_{t=1}^{T} \) is then pro-

\(^7\)The endogenous variables are not "forced" to end up at their new steady-state values. Everything is just assumed to be constant from period \( T \) onwards. However, solving for the
vided. Formulating the equilibrium conditions as $2T$ equations in $2T$ unknowns \( \{P_t, C_t\}_{t=1}^T \), we construct an error vector which measures how far the guess for \( \{P_t, C_t\}_{t=1}^T \) is from satisfying the equilibrium conditions. Using a numerical nonlinear-equation solver (SolveNLE in ox) the values of \( \{P_t, C_t\}_{t=1}^T \) which satisfy the equilibrium conditions are then obtained.

In evaluating the equilibrium conditions, the following conjectures are utilised.

1. There is positive entry in all periods.
2. No firms leave the export market except when hit by the death shock.
3. The threshold for producing, \( \varphi^* \), increases in the first period compared to the initial steady state and subsequently increases each period over the rest of the transition.
4. The threshold for starting to export, \( \varphi_{x}^* \), decreases in the first period compared to the initial steady state and subsequently increases each period over the rest of the transition.

Conjecture 1 implies that the value of entry must be zero in all periods. It will become clear below how the remaining conjectures help simplify the calculations, thereby improving the convergence properties of the algorithm.

In some instances (for example the \( F_x \)-liberalisation considered in the paper), conjectures 3 and 4 do not hold but have to be replaced by the following conjectures

3*. The threshold for producing, \( \varphi^* \), increases in the first period compared to the initial steady state and subsequently decreases over the rest of the transition.
4*. The threshold for exporting, \( \varphi_{x}^* \), decreases in the first period compared to the initial steady state and subsequently decreases over the rest of the transition.

The implications for the calculations carried out from using conjectures 3 and 4 are discussed in Subsection 5.5. In solving for the transition following any trade liberalisation, it is of course checked which conjectures end up being true and the appropriate version of the algorithm is applied.

### 5.2 Discount Factors and thresholds

Using the guess for \( \{C_t, P_t\}_{t=1}^T \) the discount factors firms apply to profits between period \( t \) and period \( s \geq t \),

\[
\zeta_{t,s} = (1 - \delta)^{s-t} \beta^{s-t} \frac{P_t u'(C_s)}{P_s u'(C_t)},
\]

transition path implies that the period \( T \) values of all endogenous variables end up very close to the new steady-state values.
can easily be evaluated. Further, the values of \( B_t = \alpha^o P_t C_t / (\sigma - 1) \) also follow directly. Due to conjecture 3, the marginal firm in period \( t \), which has productivity \( \varphi^* \) will earn zero profits. (In period \( t + 1 \) it exits, so it produces in period \( t \) only if doing so is not associated with nonnegative profits.) Using \( \pi_{d,t}(\varphi^*_t) = 0 \) gives us \( \varphi^*_t = (f/B_t)^{1/(\sigma - 1)} \). Next, due to conjecture 2, exporters continue to export as long as they are not hit by the death shock. That means that the export-entry threshold in period \( t \) satisfies

\[
F_x = \frac{\zeta_t x}{1 - \beta/(1-\delta)} \pi_{x,t} (\varphi^*_t) + \sum_{s=t}^{T-1} \zeta_t, s \pi_{x,s} (\varphi^*_t).
\]

Utilising \( \pi_{x,s}(\varphi) = B_s t^1 - \sigma \varphi^{\sigma - 1} - f_x \), we get

\[
\varphi^*_t = \tau \left( \frac{F_x + f_x \left[ \frac{\zeta_t, t}{1 - \beta/(1-\delta)} + \sum_{s=t}^{T-1} \zeta_t, s \right]}{\frac{\zeta_t, t B_t}{1 - \beta/(1-\delta)} + \sum_{s=t}^{T-1} \zeta_t, s B_s} \right)^{\frac{1}{\sigma - 1}}.
\]

### 5.3 Mass of Entrants, Firms, and Exporters

The mass of firms in each period can be obtained from

\[
M_t = (1 - \delta) M_{t-1} + M^e_t
\]

(33)

where \( M_0 \) is the old steady-state value of \( M \). The mass of active firms in period \( t \) is given by \( (1 - G(\varphi^*_t)) M_t \) due to conjecture 3. Further, due to conjecture 4, the mass of exporters in period \( 1 \), \( M^e_1 \) is given by \( (1 - G(\varphi^*_1)) M_1 \). Moreover, conjectures 2 and 4 imply that for \( t \geq 2 \), the mass of exporters is given by

\[
M^e_t = (1 - \delta) M^e_{t-1} + (1 - G(\varphi^*_t)) M^e_t.
\]

(34)

Now we only need \( M^e_t \). This can be inferred from the guess for \( P_t \), using

\[
P_t^{1 - \sigma} = \int_{\varphi^*_1}^{\infty} (\alpha \varphi)^{\sigma - 1} M_t \, dG(\varphi) + \tau^{1 - \sigma} \int_{\varphi^*_1}^{\infty} (\alpha \varphi)^{\sigma - 1} (1 - \delta)^{t-1} M_1 \, dG(\varphi) + \tau^{1 - \sigma} \sum_{s=2}^{t} \int_{\varphi^*_s}^{\infty} (\alpha \varphi)^{\sigma - 1} (1 - \delta)^{t-s} M^e_s \, dG(\varphi).
\]

Using the Pareto for \( G \) and (33) we get

\[
\frac{1}{k \varphi^*_{\text{min}}} \alpha^{1 - \sigma} P_t^{1 - \sigma} = \left( \varphi^*_t \right)^{-k/(\sigma - 1)} \left[ (1 - \delta) M_{t-1} + M^e_t \right] + \tau^{1 - \sigma} \left( \varphi^*_t \right)^{-k/(\sigma - 1)} (1 - \delta)^{t-1} \left[ (1 - \delta) M_0 + M^e_t \right] + \tau^{1 - \sigma} \sum_{s=2}^{t} \left( \varphi^*_s \right)^{-k/(\sigma - 1)} (1 - \delta)^{t-s} M^e_s.
\]
For $t = 1$ we get
\[
M_t^e = \frac{k-(\sigma-1)}{k\varphi_{\min}^k} \alpha^{1-\sigma} P_1^{1-\sigma} \\
(\varphi_1^{-1})^{k+(\sigma-1)} + \tau^{1-\sigma}(\varphi_{x,1}^{-1})^{k+(\sigma-1)} - (1-\delta) M_0.
\] (35)

For $t \geq 2$ it is easiest to use
\[
\frac{k-(\sigma-1)}{k\varphi_{\min}^k} \alpha^{1-\sigma}[P_t^{1-\sigma} - (1-\delta) P_{t-1}^{1-\sigma}] = (1-\delta) M_{t-1}[(\varphi_t^*)^{-k+(\sigma-1)} - (\varphi_{t-1}^*)^{-k+(\sigma-1)}] \\
+ M_t^e[(\varphi_t^*)^{-k+(\sigma-1)} + \tau^{1-\sigma}(\varphi_{x,t}^*)^{-k+(\sigma-1)}]
\]
which yields
\[
M_t^e = \frac{k-(\sigma-1)}{k\varphi_{\min}^k} \alpha^{1-\sigma}[P_t^{1-\sigma} - (1-\delta) P_{t-1}^{1-\sigma}] - (1-\delta) M_{t-1}[(\varphi_t^*)^{-k+(\sigma-1)} - (\varphi_{t-1}^*)^{-k+(\sigma-1)}]
\]
\[
(\varphi_t^*)^{-k+(\sigma-1)} + \tau^{1-\sigma}(\varphi_{x,t}^*)^{-k+(\sigma-1)}.
\]

Using this equation together with (33), (34), and (35) allows you to sequentially compute $\{M_t, M_t^e, M_t^x\}_{t=1}^T$.

### 5.4 2T Equilibrium Conditions

The first $T$ equilibrium conditions that will form half the entries in the error vector is the value of entry at each point in time (has to be zero in all periods due to conjecture 1). These conditions are
\[
F_e = \frac{\zeta_{t,T}}{1-\beta(1-\delta)} \int_{\varphi_T^*}^\infty \pi_d,T(\varphi) \, dG(\varphi) + \sum_{s=t}^{T-1} \zeta_{t,s} \int_{\varphi_s^*}^\infty \pi_d,s(\varphi) \, dG(\varphi)
\]
\[
+ \frac{\zeta_{t,T}}{1-\beta(1-\delta)} \int_{\varphi_{x,t}^*}^\infty \pi_{x,T}(\varphi) \, dG(\varphi) + \sum_{s=t}^{T-1} \zeta_{t,s} \int_{\varphi_{x,s}^*}^\infty \pi_{x,s}(\varphi) \, dG(\varphi) - \int_{\varphi_{x,t}^*}^\infty F_x \, dG(\varphi)
\]
which, using the Pareto can be rewritten into
\[
\frac{k-(\sigma-1)}{\sigma-1} F_e = \frac{\zeta_{t,T}}{1-\beta(1-\delta)} \left( \frac{\varphi_{\min}}{\varphi_T^*} \right)^k f + \sum_{s=t}^{T-1} \zeta_{t,s} \left( \frac{\varphi_{\min}}{\varphi_s^*} \right)^k f + EX_t
\]
where
\[
EX_t = \left( \frac{\varphi_{\min}}{\varphi_{x,t}^*} \right)^k \left( F_x + f_x \left[ \frac{\zeta_{t,T}}{1-\beta(1-\delta)} + \sum_{s=t}^{T-1} \zeta_{t,s} \right] \right).
\]

For $t = T$ we immediately obtain
\[
\frac{k-(\sigma-1)}{\sigma-1} F_e = \frac{1}{1-\beta(1-\delta)} \left( \frac{\varphi_{\min}}{\varphi_T^*} \right)^k f + \left( \frac{\varphi_{\min}}{\varphi_{x,T}^*} \right)^k \left( F_x + f_x \frac{1}{1-\beta(1-\delta)} \right)
\] (36)
while for $1 \leq t \leq T - 1$ we can use
\[ \frac{k-(\sigma-1)}{\sigma-1}(1 - \zeta_{t,t+1})F_e = \left(\frac{\varphi_{t}}{\varphi_{t-1}}\right)^k f + EX_t - \zeta_{t,t+1}EX_{t+1}. \] (37)

The conditions (36) and (37) together give us the first $T$ entries of the error vector.

The next $T$ entries follow from the market clearing condition that total labour income and net profits equal expenditure. For $t \geq 2$ this gives us
\[ P_tC_t = L + \Pi_t \]
\[ = L + \frac{1}{\sigma}P_tC_t - \left(\frac{\varphi_{t}}{\varphi_{t-1}}\right)^k M_t f - M_t^e f_x - M_t^e F_e \]
\[ - \left(\frac{\varphi_{t}}{\varphi_{t-1}}\right)^k M_t^e F_x \] (38)

where we have used that operating profits are a share $1/\sigma$ of revenues. For $t = 1$ we have to take into account that some incumbents invest in exporting as well,
\[ P_1C_1 = L + \frac{1}{\sigma}P_1C_1 - \left(\frac{\varphi_{t}}{\varphi_{t-1}}\right)^k M_1 f - M_1^e f_x - M_1^e F_e \]
\[ - \left(\frac{\varphi_{t}}{\varphi_{t-1}}\right)^k M_1^e F_x - \left[\left(\frac{\varphi_{t}}{\varphi_{t-1}}\right)^k - \left(\frac{\varphi_{t}}{\varphi_{t-1}}\right)^k\right] (1 - \delta)M_0 F_x, \] (39)

where $\varphi_{t,0}^*$ denotes the export entry threshold in the old steady state. The conditions (38) and (39) give rise to the last $T$ entries of the error vector.

### 5.5 Alternative conjectures

Now, consider the case with the conjectures 3 and 4 replaced by conjectures $3^*$ and $4^*$. First off, under conjecture $3^*$ the threshold for exiting must now be calculated differently. The reason is that future profits are increasing and therefore a firm is willing to accept (small) losses at any given point. Further, if a firm does not exit in period $t$ it never exits except if hit by the death shock. The productivity of the marginal firm at time $t$ now satisfies
\[ 0 = \frac{\zeta_{t,T}}{1 - \beta (1 - \delta)} \pi_{d,T}(\varphi_t^*) + \sum_{s=t}^{T-1} \zeta_{t,s} \pi_{d,s}(\varphi_t^*) \]

using $\pi_{d,t} = B_t \varphi^{\sigma-1} - f$, we get,
\[ \varphi_t^* = f^{\frac{1}{\sigma-1}} \left( \frac{\zeta_{t,T}}{1 - \beta (1 - \delta)} + \sum_{s=t}^{T-1} \zeta_{t,s} B_s \right)^\frac{1}{\sigma-1}. \]
Supplementary Appendix for Chapter One

Next, under conjecture 4*, the decision to start exporting this period must be compared to starting next period, thus $\varphi_{x,t}^*$ must satisfy

$$
\frac{\zeta_{t,T}}{1 - \beta (1 - \delta)} \pi_{x,t}(\varphi_{x,t}^*) + \sum_{s=t}^{T-1} \zeta_{s,t} x_s(\varphi_{x,s}^*) - F_x
= \zeta_{t,t+1} \frac{\zeta_{t+1,T}}{1 - \beta (1 - \delta)} \pi_{x,t}(\varphi_{x,t}^*) + \sum_{s=t+1}^{T-1} \zeta_{t+1,s} x_s(\varphi_{x,t}^*) - F_x.
$$

Cancelling out terms gives

$$
\pi_{x,t}(\varphi_{x,t}^*) = (1 - \zeta_{t,t+1}) F_x,
$$

which states that for a firm to start exporting the export profits of the current period must provide sufficient return on the investment. Now using $\pi_{x,t}(\varphi) = B_t \tau^{1 - \sigma} \varphi^{\sigma - 1} - f_x$, we get

$$
\varphi_{x,t}^* = \tau \left( \frac{f_x + (1 - \zeta_{t,t+1}) F_x}{B_t} \right)^{\frac{1}{\sigma - 1}}.
$$

The dynamics of the mass of firms, $M_t$, are still given by (33). However, due to conjecture 4*, the mass of exporters can simply be inferred from $M_t^e = (1 - G(\varphi_{x,t}^*)) M_t$. However, under conjecture 3, we need to keep track of the mass of active firms, $M_t^a$, separately. Note that $M_t^a = (1 - G(\varphi_{x,t}^*))((1 - \delta) M_0 + M_t^e)$ while for $t \geq 2$,

$$
M_t^a = (1 - \delta) M_{t-1}^a + (1 - G(\varphi_{x,t}^*)) M_t^e.
$$

Again, $M_t^e$ is inferred from the guess for $P_t$. The calculations are slightly modified however,

$$
P_t^{1 - \sigma} = \int_{\varphi_{x,t}^*}^{\infty} (\alpha \varphi)^{\sigma - 1} (1 - \delta)^t M_0 dG(\varphi) + \sum_{s=1}^{t} \int_{\varphi_{x,s}^*}^{\infty} (\alpha \varphi)^{\sigma - 1} M_t^e (1 - \delta)^{t-s} dG(\varphi)
+ \int_{\varphi_{x,t}^*}^{\infty} (\alpha \varphi)^{\sigma - 1} \tau^{1 - \sigma} ((1 - \delta) M_{t-1} + M_t^e) dG(\varphi).
$$

For $t = 1$ we again arrive at (35) while for $t \geq 2$ we get

$$
M_t^e = \frac{k - (\sigma - 1)}{k \varphi_{\min}^{\sigma - 1}} \alpha^{1 - \sigma} [P_t^{1 - \sigma} - (1 - \delta) P_{t-1}^{1 - \sigma}] - (1 - \delta) M_{t-1} \tau^{1 - \sigma} [(\varphi_{x,t}^*)^{-k + (\sigma - 1)} - (\varphi_{x,t-1}^*)^{-k + (\sigma - 1)}]
/ (\varphi_{x,t}^*)^{-k + (\sigma - 1)} + \tau^{1 - \sigma} (\varphi_{x,t}^*)^{-k + (\sigma - 1)}.
$$
Next, the expression for the value of entry is
\[ F_e = \frac{\zeta_{t,T}}{1 - \beta(1 - \delta)} \int_{\varphi_t^*}^{\infty} \pi_{d,T}(\varphi) \, dG(\varphi) + \sum_{s=t}^{T-1} \zeta_{t,s} \int_{\varphi_s^*}^{\infty} \pi_{d,s}(\varphi) \, dG(\varphi) \]
\[ + \frac{\zeta_{t,T}}{1 - \beta(1 - \delta)} \int_{\varphi_{s,t}^*}^{\infty} \pi_{x,T}(\varphi) \, dG(\varphi) + \sum_{s=t}^{T-1} \zeta_{t,s} \int_{\varphi_{s,t}^*}^{\infty} \pi_{x,s}(\varphi) \, dG(\varphi) \]
\[ - \int_{\varphi_{s,t}^*}^{\infty} F_x \, dG(\varphi) - \sum_{s=t+1}^{T} \zeta_{t,s} \int_{\varphi_{s,t}^*}^{\varphi_{s,t}^*} F_x \, dG(\varphi). \]
Integrating and rearranging we again get (36) for \( t = T \). Further, for \( 1 \leq t \leq T - 1 \) we now have
\[ \frac{k - (\sigma - 1)}{\sigma - 1} (1 - \zeta_{t,t+1}) F_e = \left( \frac{\phi_{\min}}{\varphi_{t+1}^*} \right)^k \left( \frac{\phi_{\min}}{\varphi_t^*} \right)^k - \left( \frac{\phi_{\min}}{\varphi_{t+1}^*} \right)^k \left( \frac{\phi_{\min}}{\varphi_{t-1}^*} \right)^k \left( \frac{\zeta_{t,T}}{1 - \beta(1 - \delta)} + \sum_{s=t}^{T-1} \zeta_{t,s} \right) \]
\[ + \left( \frac{\phi_{\min}}{\varphi_{x,t}^*} \right)^k (f_x + (1 - \zeta_{t,t+1}) F_x). \]
Finally, the condition on income reads
\[ P_tC_t = L + \frac{1}{\sigma} P_tC_t - M_t f - M_t^x f_x - M_t^e F_e - \left( \frac{\phi_{\min}}{\varphi_{t}^*} \right)^k M_t^e F_x \]
\[ - \left( \frac{\phi_{\min}}{\varphi_{x,t}^*} \right)^k - \left( \frac{\phi_{\min}}{\varphi_{x,t-1}^*} \right)^k \right] (1 - \delta) M_{t-1} F_x \]
where \( \varphi_{x,0}^* \) is the initial steady-state value for the export threshold.

5.6 Evaluating \( \lambda_t \) along the Transition

With the conjectures 3 and 4, \( \lambda_t \) must solve
\[ \lambda_t P_tC_t = \int_{\varphi_t^*}^{\infty} r_{d,t}(\varphi) M_t \, dG(\varphi). \]
Substituting for (4), integrating and rearranging gives
\[ \lambda_t = \left( \frac{\alpha P_t \varphi_t^*}{\varphi_t^*} \right)^{\sigma - 1} \frac{k}{k - (\sigma - 1)} \left( \frac{\phi_{\min}}{\varphi_t^*} \right)^k M_t. \]
With the conjectures 3* and 4* \( \lambda_t \) are found from
\[ (1 - \lambda_t) P_tC_t = \int_{\varphi_{x,t}^*}^{\infty} r_{x,t}(\varphi) M_t \, dG(\varphi), \]
which ultimately gives
\[ \lambda_t = 1 - \left( \frac{\alpha P_t \varphi_{x,t}^*}{\tau} \right)^{\sigma - 1} \frac{k}{k - (\sigma - 1)} \left( \frac{\phi_{\min}}{\varphi_{x,t}^*} \right)^k M_t. \]
References

Monotone Comparative Statics for the Industry Composition under Monopolistic Competition*

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Abstract

We let heterogeneous firms face decisions on an arbitrary number of complementary activities in a monopolistically-competitive industry. The key insight is that firm-level complementarities may manifest themselves much more clearly at the industry level than at the firm level of analysis. The response of an individual firm to exogenous changes in the parameters of its profit maximisation problem is ambiguous due to indirect effects through changes in industry competition. Only in special cases are firm-level comparative statics monotone. Turning to the industry composition, we provide sufficient conditions for first-order stochastic dominance shifts in the equilibrium distributions of all activities regardless of the ambiguities prevailing at the firm level. Our results apply to many well-known models of international trade and provide strong, novel, and testable predictions. A technical contribution is to apply powerful supermodular optimisation techniques in a context of monopolistic competition.

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1 Introduction

More than two decades ago, Milgrom and Roberts (1990a) argued that strategy and structure in modern manufacturing firms reflect widespread complementarities among many diverse activities undertaken by firms. Drawing on Topkis (1978), they emphasised how such complementarities—and the corresponding mathematical property of supermodularity—make firms’ decisions exhibit monotone comparative statics. That is, the optimal levels of the activities are monotonic in parameters of the profit maximisation problem that influence the set of available activities or the attractiveness of these activities all else equal. Since this seminal contribution, the monotonicity theorems developed by Topkis (1978), and later by Milgrom and Shannon (1994) and Athey (2002), have played important roles in the comparative statics of firms. One reason is their virtue of focusing on the properties of the optimisation problem that are essential for obtaining monotone comparative statics and doing away with superfluous assumptions. For example, these monotonicity theorems allow activities to be discrete choice variables and the profit function to be nonconcave, nondifferentiable, and discontinuous at some points. When applying these monotonicity theorems, many studies of firm-level complementarities in organisational economics assume that the competitive environment of the firm is exogenous.1 While being convenient and serving as a natural baseline, such partial analysis is not sufficient when studying exogenous shocks that affect all firms in an industry. In this case, firms are not only directly affected by the exogenous changes but also indirectly affected through changes in the competitive environment. In studies of games with strategic complementarities or substitutes, where monotonicity theorems have also been applied, such an indirect effect is central.2

The present paper shows that the presence of an indirect effect has a number of interesting implications for the comparative statics in a setting of monopolistic competition among heterogeneous firms. Our main result is that firm-level complementarities can manifest themselves much more clearly in the comparative statics for the industry composition than at the firm level. Tests of firm-level complementarities conducted at the industry level may therefore be a promising empirical strategy. A technical contribution is to reveal that monotonicity theorems can indeed be very useful in the analysis of firms in monopolistic competition; a workhorse market structure in many diverse fields of economics. Our study is closely related to the recent literature on heterogeneous firms in international trade for which our results are highly relevant. Before going into details

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regarding this part of our contribution, let us describe our setup and results further. Specifically, we put forward a model of monopolistic competition where heterogeneous firms each make a combined decision on an arbitrary number of activities. Firms endogenously enter and exit the industry. The demand level of the industry reflects the intensity of competition and adjusts to ensure a zero expected value of entry. Importantly, the activities faced by firms are complementary with each other and with the demand level of the industry.\textsuperscript{3}

At the firm level, we investigate how the decisions of individual firms respond to exogenous increases in industry-wide parameters of the profit maximisation problem. These parameter changes do not only have a nonnegative direct effect (given the demand level) on the equilibrium decisions of individual firms but also an indirect effect operating through changes in the demand level. Only when the indirect effect is nonnegative and thus aligned with the direct effect can we be sure that the firm-level comparative statics are monotone. We provide sufficient conditions for this to be the case by restricting the nature of the exogenous changes such that the demand level increases. The indirect effect is however generally ambiguous and, as a consequence, so are the comparative statics at the firm level. These results show how an endogenous competitive environment refines the comparative statics obtained in the above-mentioned studies of firm-level complementarities in organisational economics.

The centrepiece of our contribution is the provision of sufficient conditions for monotone comparative statics for the industry composition. By this we mean that exogenous increases in industry-wide parameters of the profit maximisation problem lead to first-order stochastic dominance shifts in the equilibrium distribution of any activity across firms. This implies that the share of active firms undertaking at least a given level of any activity increases and so does the average level of any activity. The equilibrium distribution of any activity does not only depend on the levels chosen by firms conditional on being active (the level effect) but also on the endogenous selection of which firms are active (the selection effect). The presence of the selection effect implies that monotone comparative statics at the firm level are neither necessary nor sufficient for monotone comparative statics for the industry composition. We are therefore able to identify sufficient conditions for monotone comparative statics for the industry composition regardless of the ambiguities prevailing at the firm level. These conditions ensure that the level and the selection effects induced by changes in the demand level cancel out. The indirect effect—the source of ambiguity at the firm level—is therefore neutral at the industry level. This is in some sense a knife-edge result.

\textsuperscript{3}Since our focus is on the implications of complementarities, we simply assume their existence rather than investigate how they may arise. Topkis (1995) and Mrazova and Neary (2013) consider conditions for complementarities to arise and provide examples of specific models.
We also show how weaker conditions ensure monotone comparative statics for the industry composition when the demand level is known to either rise or fall. An implication of our results is that one may observe that many, or even all, firms reduce their levels of a given activity while the equilibrium distribution of this activity shifts towards higher levels thereby increasing the average level of this activity.\footnote{Even though this result may seem like a theoretical curiosity, it holds in some models of international trade. We provide one example in Section 6.4.}

Since the seminal work by Melitz (2003), models with monopolistic competition among heterogeneous firms have become mainstream in the literature on international trade. A vast number of these recent trade models feature firm-level complementarities and this paper provides a unifying framework for these. The framework is sufficiently general to encompass, at least symmetric-market versions of, well-known contributions such as Melitz (2003), Antràs and Helpman (2004), Helpman et al. (2004), Melitz and Ottaviano (2008), Arkolakis (2010), Helpman and Itskhoki (2010), Helpman et al. (2010), Amiti and Davis (2011), Arkolakis and Muendler (2011), Bernard et al. (2011), Bustos (2011), Caliendo and Rossi-Hansberg (2012), Mayer et al. (2011), and Kasahara and Lapham (2013). By applying monotonicity theorems to obtain comparative statics, our mathematical techniques differ substantially from the standard approach in these studies. The monotonicity theorems allow us to focus on the complementarities at the heart of these models without relying on specific functional forms. Our results reveal that, in these trade models, the common assumption of Pareto-distributed firm-specific productivities represents the knife-edge case mentioned above and therefore leads to monotone comparative statics for the industry composition. Further, we show how one can relax the assumption of Pareto-distributed productivities and still get monotone comparative statics for the industry composition in cases where the demand level is known to rise or fall. Trade liberalisations belong to the latter category. Because our results provide strong, novel, and testable implications for many recent and well-known models from international trade, our main result that firm-level complementarities may manifest themselves much more clearly at the industry level than at the firm level is relevant for both theorists and empiricists.

Bernard et al. (2003) and Arkolakis et al. (2012) have noted the discrepancy between firm- and industry-level effects of trade liberalisation on firm markups using models with heterogeneous firms and endogenous selection. But, to the best of our knowledge, we are the first to provide a general and thorough analysis of how firm-level complementarities can imply monotone comparative statics for the industry composition despite ambiguities in firm-level responses. In illustrating the merits of the mathematical tools behind monotonicity theorems for
analysing the above-mentioned trade models, this paper is also related to Mrazova and Neary (2013). These two authors emphasise the role of supermodularity (complementarity) in shaping the sorting pattern of firms in a given equilibrium. Our approach differs by not only focusing on a given equilibrium but rather conducting comparative statics across equilibria. Finally, Costinot (2009) examines the role of log-supermodularity in generating comparative advantage. In the working-paper version, Costinot (2007) considers applications to specific heterogeneous firms setups. Here, comparative statics with respect to the productivity dispersion are conducted whereas we consider comparative statics with respect to parameters directly affecting the maximisation problem of the firms. We emphasise that, although the present paper has close ties to the international-trade literature, nothing in the formulation of our framework limits the relevance or application of our results to trade-related issues.

The remainder of the paper is organised as follows. Section 2 briefly reviews the central mathematical results from Topkis (1978, 1995) and Milgrom and Shannon (1994) that we draw upon in our analysis as these may be unfamiliar to some readers. Section 3 develops our framework and presents our central assumption of complementarities. Section 4 emphasises the ambiguities in firm-level responses and provides special cases with monotone comparative statics. Next, Section 5 characterises the equilibrium distributions of activities and derives sufficient conditions for monotone comparative statics for the industry composition. Section 6 presents concrete applications of both our framework and results. Finally, Section 7 offers some concluding remarks.

2 Mathematical Preface

Let $X \subseteq \mathbb{R}^n$ and $T \subseteq \mathbb{R}^m$ be partially ordered sets with the component-wise order. For two vectors $x', x'' \in \mathbb{R}^n$, we let $x' \lor x''$ denote the component-wise maximum and $x' \land x''$ denote the component-wise minimum. The set $X$ is a lattice if for all $x', x'' \in X$, $x' \lor x'' \in X$ and $x' \land x'' \in X$. The set $S \subseteq X$ is a sublattice of $X$ if $S$ is a lattice itself. For two sets $S', S'' \subseteq \mathbb{R}^n$, we say that $S''$ is higher than $S'$ and write $S' \leq_s S''$ if for all $x' \in S'$ and all $x'' \in S''$, $x' \lor x'' \in S''$ and $x' \land x'' \in S'$. If a set becomes higher, then we say that the set is increasing.

Let $X$ be a lattice. The function $h : X \times T \to \mathbb{R}$ is supermodular in $x$ on $X$ for each $t \in T$ if for all $x', x'' \in X$ and $t \in T$,

$$h(x', t) + h(x'', t) \leq h(x' \land x'', t) + h(x' \lor x'', t).$$

5 For $x' = (x'_1, \ldots, x'_n) \in \mathbb{R}^n$ and $x'' = (x''_1, \ldots, x''_n) \in \mathbb{R}^n$, $x' \leq x''$ if $x'_i \leq x''_i$ for $i = 1, \ldots, n$ and $x' < x''$ if $x' \leq x''$ and $x' \neq x''$.

6 That is, $x' \lor x'' = (\max\{x'_1, x''_1\}, \ldots, \max\{x'_n, x''_n\})$ and $x' \land x'' = (\min\{x'_1, x''_1\}, \ldots, \min\{x'_n, x''_n\})$. 53
Supermodularity of \( h \) in \( x \) implies that the return from increasing several elements of \( x \) together is larger than the combined return from increasing the elements separately.\(^7\) This follows from the fact that a higher value of one subset of the elements in \( x \) increases the value of increasing other subsets of elements. Supermodularity thus implies that the elements of the vector \( x \) are (Edgeworth) complements. If \( h \) is smooth, supermodularity is equivalent with \( \partial^2 h / \partial x_i \partial x_j \geq 0 \) for all \( i, j \) where \( i \neq j \). By (1), it follows that any function \( h \) is trivially supermodular in \( x \) when \( x \) is a single real variable. The function \( h(x,t) \) has increasing differences in \((x,t)\) if for \( x' \leq x'' \), \( h(x'',t) - h(x',t) \) is monotone nondecreasing in \( t \). Increasing differences mean that increasing \( t \) raises the return from increasing \( x \) and vice versa. If \( h \) is smooth, increasing differences are equivalent with \( \partial^2 h / \partial x_i \partial t_j \geq 0 \) for all \( i, j \). The following monotonicity theorem is due to Topkis (1978).

**Theorem 1.** Let \( X \subseteq \mathbb{R}^n \) be a lattice, \( T \subseteq \mathbb{R}^m \) be a partially ordered set, \( S \) be a sublattice of \( X \), and \( h : X \times T \to \mathbb{R} \). If \( h(x,t) \) is supermodular in \( x \) on \( X \) for each \( t \in T \) and has increasing differences in \((x,t)\) on \( X \times T \), then \( \arg \max_{x \in S} h(x,t) \) is monotone nondecreasing in \((t, S)\).

If the set of maximisers only contains one element, this unique maximiser is nondecreasing in \((t, S)\). In the remainder of this paper, we restrict attention to cases where the set of maximisers is a nonempty and complete sublattice.\(^8\) This implies that the set of maximisers has greatest and least elements and Theorem 1 implies that these greatest and least elements are nondecreasing functions of \((t, S)\). We follow the convention of focusing on the greatest element in the set of maximisers, effectively treating the maximiser as unique.\(^9\) The monotone comparative statics result of Theorem 1 could also be obtained under the weaker assumption that \( h \) is quasisupermodular in \( x \) and exhibits the single crossing property in \((x,t)\).\(^{10}\) However, the properties supermodularity and increasing differences more accurately represent the standard notion of complementarity, are well-known, and are easy to characterise for smooth functions. Therefore we use these assumptions throughout the main part of the paper while keeping in mind that all our results will also hold under quasisupermodularity and the single crossing property.

\(^7\)To see this, rewrite (1) into \([h(x',t) - h(x' \land x'',t)] + [h(x'',t) - h(x' \land x'',t)] \leq h(x' \lor x'',t) - h(x' \land x'',t)\).

\(^8\)General sufficient conditions for this are found in Milgrom and Shannon (1994).

\(^9\)We share this approach with Bagwell and Ramey (1994) and Holmstrom and Milgrom (1994). All results also hold when one focuses on e.g. the least element.

\(^{10}\)See Appendix A for formal definitions and a theorem.
3 The Model

After paying a sunk entry cost of \( f_e \) units of the numéraire, atomistic firms enter an industry characterised by monopolistic competition. Upon entry into this industry, a firm receives a productivity level, \( \theta \in \mathbb{R}_+ \), and a vector of characteristics, \( \gamma \in \mathbb{R}^p \).\(^{11}\) Individual firms are fully characterised by the pair \((\theta, \gamma)\) which is a realisation of the random variables \((\Theta, \Gamma)\). We let \( \Theta \) and \( \Gamma \) be independently distributed with c.d.f. \( F \) and \( G \), respectively. Let productivity be a continuous variable and \( F \) be \( C^1 \) on the interior of its support. In line with labelling \( \theta \) as productivity, assume that firms with higher \( \theta \) are able to earn weakly higher profits.

After observing its realisation \((\theta, \gamma)\), a firm has to choose whether to start producing or to exit the industry. If a firm chooses to produce, it has to make a decision, \( x = (x_1, \ldots, x_n) \), where \( x_i \) denotes the chosen level of activity \( i \). An activity refers to any variable at the discretion of the firm. The level of an activity can be either discrete or continuous. We let \( x \in X \) where \( X \subseteq \mathbb{R}^n \) is the set of all conceivable, but not necessarily available, decisions. The set \( X \) is assumed to be a lattice which, loosely speaking, means that undertaking a higher level of any activity may require, but importantly, cannot prevent undertaking a higher level of another activity. Restricting attention to lattices allows complementarities to take effect. The profitability of the decisions in \( X \) is influenced by a vector of exogenous industry-wide parameters, \( \beta \in B \) with \( B \subseteq \mathbb{R}^m \). Further, the actual choice set of all firms is restricted to a set of available decisions, \( S \subseteq X \), with \( S \) being a sublattice of \( X \). Our comparative statics are going to focus on changes in \((\beta, S)\) which determines the attractiveness (all else equal) and availability of activities.

Firm profits also depend on a common endogenous aggregate statistic. We will think of and refer to this variable, \( A \in \mathbb{R}_+ \), as the demand level and let firm profits be nondecreasing in \( A \). In line with monopolistic competition among atomistic firms, individual firms perceive \( A \) as exogenous.\(^{12}\) To get a sense of what \( A \) could be, consider a model where identical consumers’ preferences are additively separable across varieties of a differentiated good. In this case, the inverse marginal utility of income enters into the profit function of firms through

\(^{11}\)The distinction between what we call productivity, \( \theta \), and firm characteristics, \( \gamma \), is made since we impose some assumptions on \( \theta \) that we will not impose on \( \gamma \). \( \theta \) could in principle represent any firm characteristic that conforms to the assumptions we impose on \( \theta \). If one wishes to analyse a situation with a single source of firm heterogeneity, one can simply ignore \( \gamma \) in the following.

\(^{12}\)This setup also encompasses the case of perfect competition. To see this, let all firms share the same characteristics \((\theta, \gamma)\), \( f_e = 0 \), and \( A \) could simply be the endogenous price level. For our industry-level analysis to be interesting, firm heterogeneity is however central.
the demand function and would constitute the demand level.\textsuperscript{13} Note that $A$ could also comprise endogenous variables such as factor prices as long as all endogenous variables outside the control of the firm can be combined into the single variable $A$.

### 3.1 Profits, Complementarities, and the Optimal Decision

Profits, $\pi$, of a firm with characteristics $(\theta, \gamma)$ depend on the decision, $x$, the demand level, $A$, and the industry parameters, $\beta$. Formally,

$$\pi = \pi(x, \theta, \gamma; A, \beta), \quad (2)$$

where the semicolon separates firm-specific from industry-wide variables. The following assumption summarises the complementarities that are central to our analysis.

**Assumption 1.** For all $(\theta, \gamma, A, \beta)$, the profit function, $\pi(x, \theta, \gamma; A, \beta)$, is supermodular in $x$ on $X$ and has increasing differences in $(x, \theta)$, $(x, A)$, and $(x, \beta)$ on $X \times \mathbb{R}_+$, $X \times \mathbb{R}_+$, and $X \times B$, respectively.

Supermodularity in $x$ implies that the activities are complementary with each other. Further, the assumption of increasing differences implies that productivity, the demand level, and the elements in $\beta$ are all complementary to the $n$ activities.\textsuperscript{14} While $\theta$, $A$, and $\beta$ thus influence firms’ decisions in similar ways, they play very different roles in our model. Productivity, $\theta$, is a firm characteristic that helps us characterise the equilibrium sorting of firms, $A$ is an endogenous demand level to be determined in equilibrium, and $\beta$ is a vector of exogenous parameters with respect to which we conduct comparative statics.

Proper ordering of activity levels and parameters is crucial for profits to satisfy Assumption 1.\textsuperscript{15} Even after proper ordering, Assumption 1 may not apply to all conceivable activities that firms can face. However, if one can redefine or reduce the decision of firms to a form where the corresponding profit function satisfies our assumptions, then our results can be applied to the activities that constitute that decision.\textsuperscript{16} Consequently, we do not necessarily require that all possible

\textsuperscript{13}See e.g. Section 6.3.

\textsuperscript{14}Note that $\beta$ only contains those parameters that comply with Assumption 1. As other parameters are kept constant throughout our analysis, we simply abstract from these.

\textsuperscript{15}If a function is supermodular in $(x_1, x_2)$, then it is not supermodular in $(-x_1, x_2)$. Such a reordering trick is useful if one activity is a substitute for all others (Milgrom and Roberts, 1995). If a function has increasing differences in $(x_1, x_2, \beta)$, then it does not have increasing differences in $(x_1, x_2, -\beta)$.

\textsuperscript{16}For an example of how to obtain a supermodular profit function in a specific case with a core group of complementary activities and a group of additional activities, see Milgrom et al. (1991).
activities faced by firms are complementary. Finally, note that (2) does not have to represent a certain payoff to firms. In case of uncertainty after a firm has realised its characteristics and made its decision, the objective function (2) could be interpreted as expected profits. For an illustrative example, see Athey and Schmutzler (1995).

Faced with the profit function in (2), a firm makes the optimal decision, $x^*$, under the constraint that $x \in S$ while taking $\theta, \gamma, A$, and $\beta$ as given. Formally we have that\footnote{Our focus on the greatest element among maximisers in case of nonuniqueness allows us to treat $x^*$ as unique. Therefore, we use "=" and not "$\in$" in (3).}

$$x^*(\theta, \gamma; A, \beta, S) = \arg \max_{x \in S} \pi(x, \theta, \gamma; A, \beta).$$

\textbf{Lemma 1.} The optimal decision, $x^*(\theta, \gamma; A, \beta, S)$, is monotone nondecreasing in $(\theta, A, \beta, S)$.

Lemma 1 follows readily from Assumption 1 and Theorem 1 and is simply the manifestation of the complementarities discussed above. Importantly, these comparative statics are partial in nature since the endogeneity of $A$ is ignored. In the analysis of equilibrium comparative statics, Lemma 1 is nevertheless used repeatedly. The profits obtained under the optimal decision are denoted by $\pi^*$ which is defined as

$$\pi^*(\theta, \gamma; A, \beta, S) \equiv \pi(x^*(\theta, \gamma; A, \beta, S), \theta, \gamma; A, \beta).$$

Remember that more productive firms earn higher profits and firms earn higher profits the higher is the demand level, i.e., $\pi^*$ is nondecreasing in $(\theta, A)$.

\subsection*{3.2 Entry and the Equilibrium Distributions of Activities}

Firm profits upon entry are bounded below by zero because when $\pi^*$ happens to be negative, the firm exits the industry and forfeits the sunk cost of entry. The expected profits upon entry, $\Pi$, are thus given by

$$\Pi(A, \beta, S) \equiv \int \int \max\{0, \pi^*(\theta, \gamma; A, \beta, S)\} \, dG(\gamma) \, dF(\theta),$$

and are assumed to be finite. We assume free entry and an unbounded pool of potential entrants. In equilibrium, we therefore have that the expected profits upon entry are equal to the cost of entry,

$$\Pi(A, \beta, S) = f_e.$$
We assume the existence and uniqueness of an equilibrium with an $A$ that satisfies this free-entry condition.\footnote{For a similar assumption and also discussion thereof, see Mrazova and Neary (2013). We specify neither the forces of adjustment, such as entry of new firms or competition over inputs, nor the process that leads to (4) being satisfied.} Then (4) pins down the endogenous demand level, $A$, as a function of $\beta$ and $S$. The optimal decisions of firms conditional on being active, the equilibrium demand level, and the distributions of productivities and characteristics together give rise to endogenous distributions of the activities among active firms. We denote the c.d.f. of the equilibrium distribution of activity $i$ by $H_i(x_i; \beta, S)$.\footnote{Note that this c.d.f. does not condition on $\gamma$. It is the distribution of the activity among all active firms in the industry.} These distributions reflect the industry composition and are the focus of our industry-level analysis in Section 5. Before we consider the industry level, we proceed with an analysis of firm-level comparative statics.

### 4 Firm-Level Comparative Statics

We now investigate the equilibrium responses of individual firms to increases in the industry-wide parameters $(\beta, S)$.\footnote{In this analysis, the special properties of $\theta$ relative to $\gamma$ are irrelevant and one can abstract from $\theta$ for the remainder of this section.} Both increases in $\beta$ and increases in $S$ provide firms with an incentive to increase their levels of all activities, all else equal. Increases in $\beta$ do so by increasing the relative attractiveness of undertaking higher levels of the activities while increases in $S$ do so by shifting upwards the set of available decisions. These incentives can be brought about in two distinct ways. By increasing $\beta$ the relative attractiveness of higher levels of activities is increased both if profits associated with higher levels increase and if profits associated with lower levels decrease. Analogously, the set of available decisions is shifted upwards both if higher levels of activities become available or if lower levels become unavailable. While these two approaches to increasing $\beta$ and $S$ are not mutually exclusive, their distinct effects on firm profits will prove important.

**Definition 1.** Increases in $(\beta, S)$ are carried out with the carrot (stick) if expected profits upon entry, $\Pi$, increase (decrease) for a given demand level.

Let us define the equilibrium decision of a firm conditional on being active as

$$\tilde{x}^*(\theta, \gamma; \beta, S) \equiv x^*(\theta, \gamma; A(\beta, S), \beta, S).$$

From the right hand side of (5), it is clear that changes in $(\beta, S)$ have a direct effect on firm decisions but, since the whole industry is affected, such changes also have an indirect effect through changes in the demand level. This dichotomy
is important, so let us be more formal. When we consider a change from \((\beta', S')\) to \((\beta'', S'')\), where either \(\beta\) or \(S\) could remain unchanged, we make the following decomposition of the total effect on \(\tilde{x}^*\).

\[
\Delta \tilde{x}^* = \left[ x^*(\theta, \gamma; A', \beta'', S'') - x^*(\theta, \gamma; A', \beta', S') \right] + \left[ x^*(\theta, \gamma; A'', \beta'', S'') - x^*(\theta, \gamma; A', \beta'', S'') \right]
\]

where \(A' = A(\beta', S')\) and \(A'' = A(\beta'', S'')\). The direct effect on \(\tilde{x}^*\) is thus the change in the decision prompted by changes in \((\beta, S)\) when \(A\) is held constant. The indirect effect on \(\tilde{x}^*\) stems from \(A\) not being constant but rather being a function of the conditions affecting firms’ decisions.

It follows readily from Lemma 1 that an increase in \((\beta, S)\) has a nonnegative direct effect on the equilibrium decision, \(\tilde{x}^*\). Regardless of whether the carrot or the stick is employed in increasing \((\beta, S)\), firms get an incentive to increase their levels of all activities. The inherent complementarities among activities ensure that this is manifested in an increase in the optimal decisions, all else equal. This is the central result in the analyses of complementary activities in the organisational-economics papers cited in the introduction.

### 4.1 The Indirect Effect on \(\tilde{x}^*\)

Whereas the direct effect of an increase in \((\beta, S)\) on \(\tilde{x}^*\) is unambiguously nonnegative, the sign of the indirect effect depends on whether the carrot or the stick is employed.

**Lemma 2.** The indirect effect on the equilibrium decision, \(\tilde{x}^*\), is nonpositive (nonnegative) if \((\beta, S)\) is increased by use of the carrot (stick).

In general, an increase in \((\beta, S)\) can thus have either a positive or a negative indirect effect on \(\tilde{x}^*\). To see this, note two things. By Lemma 1, the sign of the indirect effect has the same sign as the change in the demand level. Second, the demand level responds to satisfy the free-entry condition (4). Whether this results in a lower or higher \(A\) depends crucially on whether the increase in \((\beta, S)\) increases or decreases expected profits upon entry given the demand level. That is, whether the increase in \((\beta, S)\) is brought about by the carrot or the stick.\(^{21}\)

\(^{21}\)It is quite common in the literature to analyse introductions of new activities, meaning changes from \(S'\) to \(S''\) such that \(S' \subset S''\) and \(S' \leq S''\). Since all of the decisions available under \(S'\) are also available under \(S''\), such introductions imply a carrot approach. The same can be said about reductions in trade costs in many models of international trade. Such trade liberalisations can be represented by an increase in \(\beta\); see Section 6.1.
4.2 The Total Effect on $\tilde{x}^*$

By combining the insights of the two preceding sections, we obtain the following proposition.

**Proposition 1.** The total effect on the equilibrium decision, $\tilde{x}^*$, is nonnegative (ambiguous) for all firms if $(\beta, S)$ is increased by use of the stick (carrot).

The direct effect on the equilibrium decisions of some firms may very well be zero.\(^{22}\) In this case, it is both necessary and sufficient to have a nonnegative indirect effect on $\tilde{x}^*$, in order for the total effect to be unambiguously nonnegative for all firms. In cases where $(\beta, S)$ is increased by use of the carrot, additional or more attractive activities, that is new opportunities, may actually turn out to be a detrimental threat for some firms as these opportunities become available to all firms and thus depress the demand level. Since firms and activities differ in their characteristics, either of the direct or the indirect effects may dominate the other for a given activity in a given firm.\(^{23}\) Despite the fewer or less attractive activities that may result from using the stick, firms are provided with an incentive to increase the levels of their activities that is only reinforced by the fact that these apparently worse conditions affect all firms and therefore ease competition ($A$ increases).

Proposition 1 highlights that analysing complementary activities in industry equilibrium refines the comparative statics obtained in some earlier studies. In their analyses of complementary activities, Milgrom and Roberts (1990a, 1995), Milgrom et al. (1991), Holmstrom and Milgrom (1994), Athey and Schmutzler (1995), and Topkis (1995) assume that all variables that affect profits, but are outside the control of the firm, are exogenous.\(^{24}\) As insightful as their analyses are, this means that they do not capture the feedback through the indirect effect. It is exactly this feedback from the endogenous demand level which implies that the decisions of individual firms do not generally exhibit monotone comparative statics in $(\beta, S)$ in spite of the complementarities imposed by Assumption 1.\(^{25}\)

\(^{22}\)This is e.g. the case for a firm that, given $A$, maintains its decision following an increase in $S$.

\(^{23}\)When the sign of the total effect is ambiguous, it may vary both across activities within a given firm and across different firms for a given activity. We provide an example of this in Section 6.1.

\(^{24}\)Our nonnegative direct effect on $\tilde{x}^*$ is completely equivalent to the firm-level monotone comparative statics obtained in these earlier studies. These papers do not feature an indirect effect and therefore the total effect is always nonnegative.

\(^{25}\)The reason why we do not obtain monotone comparative statics in $\beta$ is that our assumption of the profit function having increasing differences in $(x, \beta)$ is partial. The assumption holds for a given $A$ which by no means ensures increasing differences in $(x, \beta)$ once we recognise the endogeneity of $A$. Similarly, the conditions for monotone comparative statics in $S$ only hold for a given $A$. 

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Only in special cases are the comparative statics monotone for all firms.

The main arguments above have not depended on firm heterogeneity being nondegenerate. Thus, the mechanisms described are still at play if the distributions $F$ and $G$ are degenerate such that all firms share the same characteristics and make the same decision, $\tilde{x}^\ast$. One qualification to our results must however be given. With homogeneous firms, an increase in $S$ will unambiguously increase $\tilde{x}^\ast$. To see this, note first that if the initial $\tilde{x}^\ast$ becomes unavailable, then only higher decisions are possible. Second, if the initial $\tilde{x}^\ast$ does not become unavailable, we know, by the definition of the strong set order ($\leq_s$) in Section 2, that possible lower decisions available ex post were also available ex ante. By a revealed preference argument, possible lower decisions cannot be optimal after $S$ has increased. The total effect of an increase in $\beta$ remains ambiguous.\(^{26}\)

5 Industry-Level Comparative Statics

This section considers how the equilibrium distributions of activities respond to changes in $(\beta, S)$. First, let us formalise our notion of monotone comparative statics for the industry composition.

**Definition 2.** The industry composition exhibits monotone comparative statics when increases in $(\beta, S)$ induce first-order stochastic dominance (FOSD) shifts in the equilibrium distributions of all activities. That is, $H_i(x_i; \beta, S)$ is nonincreasing for all levels, $x_i$, of all activities, $i = 1, \ldots, n$.

Consequently, monotone comparative statics for the industry composition imply that the share of active firms that undertake at least any given level of any activity increases. The equilibrium distributions of the activities thus shift towards higher values of all activities and the average level of any activity increases.

Without endogenous exit, such that all firms choose to be active upon entry, comparative statics for the industry composition follow directly from the firm-level comparative statics.\(^{27}\) In this case, the industry composition exhibits monotone comparative statics if the total effect on $\tilde{x}^\ast$ is nonnegative for all firms. If the total effect on $\tilde{x}^\ast$ is ambiguous, then so are the comparative statics for the industry composition in general. In this section, we therefore analyse the more interesting case where some firms endogenously shut down upon entry. With endogenous exit, the equilibrium distributions of the activities are both affected

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\(^{26}\)A good example, which fits into our framework, is the Krugman (1979) model. As shown by Zhelobodko et al. (2012), the total output of firms (an activity) may either increase or decrease following an increase in market size ($\beta$) depending on how the "relative love of variety" varies with consumption.

\(^{27}\)This includes the case without firm heterogeneity.
by the levels of activities undertaken by firms conditional on being active (level effect) and by the endogenous selection of which firms are active (selection effect). The selection effect introduces a discrepancy between the firm-level comparative statics (the level effect) and the comparative statics for the industry composition. On the one hand, monotone comparative statics at the firm level do not ensure monotone comparative statics for the industry composition. On the other hand, it is possible that the industry composition exhibits monotone comparative statics regardless of ambiguity at the firm level.

5.1 The Equilibrium Sorting of Firms

To characterise the equilibrium distributions of activities, we consider the sorting of firms based on productivity. Remember that firms with higher productivity levels are able to earn higher profits. Thus, given \( \gamma \), the self-selection of firms into being active or exiting obeys a threshold rule with respect to productivity; all firms with productivities above this threshold are active and all firms with productivities below exit. Denoting this threshold by \( \theta_a \), we have that

\[
\theta_a(\gamma; A, \beta, S) \equiv \inf\{\theta : \pi^*(\theta, \gamma; A, \beta, S) > 0\}.
\]

For the reasons discussed above, we focus on the case with endogenous exit and assume that no draw of firm characteristics, \( \gamma \), is sufficiently favourable to ensure that a firm is able to produce profitably regardless of its productivity draw. That is, \( \theta_a(\gamma; A, \beta, S) > \theta_0 \) for all \( \gamma \) where \( \theta_0 \geq 0 \) is the lowest possible productivity draw.

To characterise the sorting of active firms into the activities, consider the cross-section of firms in a given equilibrium. In this case, \( (A, \beta, S) \) are given. By Lemma 1, we therefore know that higher productivity firms choose weakly higher levels of all activities, conditional on \( \gamma \).\(^{28}\) Let \( \theta_i \) be the lowest level of productivity at which a firm undertakes at least level \( x_i \) of activity \( i \), given \( \gamma \). Bounding this threshold from below by \( \theta_a \), it is given by

\[
\theta_i(x_i, \gamma; A, \beta, S) \equiv \max\{\theta_a, \inf\{\theta : x_i^*(\theta, \gamma; A, \beta, S) \geq x_i\}\}.
\]

Although the sorting pattern we just described is convenient for characterising an equilibrium, it has some undesirable features if productivity is the only source of firm heterogeneity. First, the strict relationship between productivity and the level of any activity seems unrealistically stark. Moreover, this very relationship

\(^{28}\)This pattern of firm sorting into the various activities can be seen as a straightforward extension of an insight of Mrazova and Neary (2013) to our case of multidimensional firm decisions.
introduces a gap between the range of available decisions in $S$ and the range of decisions observed in equilibrium. Given the wide variety of firm decisions seen in reality, such a limitation on the observable decisions is undesirable. Allowing for additional sources of firm heterogeneity through $\Gamma$ alleviates these issues. While the strict sorting pattern based on productivity holds for a given realisation of characteristics, $\gamma$, it does not necessarily hold across firms with different characteristics. If we consider all firms at once, this can break the strict relationship between the level of a given activity and productivity and thereby increase the number of observable decisions.

With $\Theta$ and $\Gamma$ being independently distributed, we can identify restrictions on the distribution of productivities, $F$, that give us monotone comparative statistics for the industry composition without restricting neither the distribution of characteristics, $G$, nor how $\gamma$ affects profits. These features are comforting to the extent that many potential sources of firm heterogeneity are not easily observable.

### 5.2 The Level and Selection Effects

On the basis of the sorting pattern described above, we now characterise the equilibrium distributions of the activities and highlight the effects at play. In the following, we focus on a particular level, $x_i$, of activity $i$ which could be any of the $n$ activities. Applying the law of large numbers, let $s_a \equiv 1 - F(\theta_a)$ be the share of firms with characteristics $\gamma$ that are active and $\bar{s}_a = \int s_a dG(\gamma)$ be the overall share of active firms. Similarly, let $s_i \equiv 1 - F(\theta_i)$ denote the share of firms with characteristics $\gamma$ undertaking at least level $x_i$ of activity $i$ and let $\bar{s}_i = \int s_i dG(\gamma)$ be the corresponding share across characteristics. Using these shares, the c.d.f. of the equilibrium distribution of activity $i$ can be expressed as

$$ H_i(x_i; \beta, S) = 1 - \frac{\bar{s}_i(x_i; A(\beta, S), \beta, S)}{\bar{s}_a(A(\beta, S), \beta, S)}. \quad (7) $$

Denote by $\Delta H_i$ the change in $H_i$ induced by an increase from $(\beta', S')$ to $(\beta'', S'')$. Using $t' = (\beta', S')$ and $t'' = (\beta'', S'')$ for shorthand notation, this change can be decomposed into the level and selection effects mentioned above.

---

29 For example, if we have two binary activities, then we can have four possible decisions, $S = \{(0, 0), (1, 0), (0, 1), (1, 1)\}$. However, if $x' = (1, 0)$ is the optimal decision for one firm, then $x'' = (0, 1)$ cannot be the optimal decision for some other firm since this would contradict Lemma 1.

30 For examples where multidimensional firm heterogeneity has this purpose, see Eaton et al. (2011), Amiti and Davis (2011), and Kasahara and Lapham (2013).

31 Note that $s_a \geq s_i$ and therefore $s_a \geq \bar{s}_i$. 

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\[
\Delta H_i = \frac{\bar{s}_i(x_i; A', t')}{\bar{s}_a(A', t')} - \frac{\bar{s}_i(x_i; A', t'')}{\bar{s}_a(A', t')} + \frac{\bar{s}_i(x_i; A', t'')}{\bar{s}_a(A', t'')} - \frac{\bar{s}_i(x_i; A', t'')}{\bar{s}_a(A', t'')}
\]

Direct Level Effect

\[
+ \frac{\bar{s}_i(x_i; A', t'')}{\bar{s}_a(A', t'')} - \frac{\bar{s}_a(A', t'')}{\bar{s}_a(A', t'')}
\]

Indirect Level Effect

\[
+ \frac{\bar{s}_i(x_i; A', t'')}{\bar{s}_a(A', t'')} - \frac{\bar{s}_i(x_i; A', t'')}{\bar{s}_a(A', t'')}
\]

Indirect Selection Effect

where again, \( A' = A(\beta', S') \) and \( A'' = A(\beta'', S'') \). Remember that the (total) level effect is due to changes in the levels of activity \( i \) undertaken by firms conditional on being active. In (8), this is represented by changes in the share of firms undertaking at least a given level of activity \( i \), \( \bar{s}_i \). The (total) selection effect is due to changes in the range of active firms, which is represented by changes in the share of active firms, \( \bar{s}_a \). Each of these two effects have a direct component induced by changes in \( (\beta, S) \) for a given \( A \) and an indirect component induced by changes in \( A \). Together, the direct and indirect level effects correspond to the firm-level responses analysed in Section 4. The additional effect of an increase in \( (\beta, S) \) on \( H_i \) through the direct and indirect selection effects is clearly a source of discrepancy between firm- and industry-level comparative statics.

5.3 Monotone Comparative Statics for the Industry Composition

The present section provides conditions that imply monotone comparative statics for the industry composition \( \Delta H_i \leq 0 \) in the general case where the demand level may either rise or fall. These conditions are relaxed in the next section by conditioning on use of either the carrot or the stick.

Let us start by considering the direct level effect on \( H_i \). We know from Section 4 that the direct effect of an increase in \( (\beta, S) \) on firms’ decisions is nonnegative. This tends to increase the share of firms that undertake at least a given level of activity \( i \), \( \bar{s}_i \).\(^{32}\) The direct level effect in (8) is therefore nonpositive which works in favour of an FOSD shift in \( H_i \). Since this effect may actually be zero for some levels of activity \( i \), we must ensure that the sum of the remaining effects in (8) is also nonpositive. First off, because the demand level could be constant such that both the indirect level and selection effects are zero, this requires that the direct selection effect is itself nonpositive.

For the direct selection effect to be nonpositive, we need that the direct effect on the share of active firms, \( \bar{s}_a \), is nonpositive. Intuitively, the marginal active

\[^{32}\]To see this formally, note that by Lemma 1, \( x^* \) is nondecreasing in \((\theta, \beta, S)\). Thus it follows from (6) that \( \theta_i \) is nonincreasing in \((\beta, S)\) given \((\gamma, A)\). Therefore, \( s_i \) is nondecreasing for all \( \gamma \) which means that \( \bar{s}_i \) is nondecreasing.
firms have low productivities and therefore undertake low levels of the activities conditional on being active. An increase in the share of active firms therefore tends to work against an FOSD shift in $H_i$. To ensure that the direct effect of a change in $(\beta, S)$ does not work in this direction, we impose the following assumption which applies in the remainder of the paper.

**Assumption 2.** The direct selection effect in (8) is nonpositive.

This assumption implies that the direct effect of increases in $(\beta, S)$ on the profits of the least productive active firms must be nonpositive. Assumption 2 is often satisfied in models of international trade since the least productive active firms are often not directly affected by the comparative statics considered. Note that under Assumption 2, $(\beta, S)$ can still be increased by both the carrot or the stick.\(^{33}\) Hence, Assumption 2 does not limit the firm-level responses to be either ambiguous or unambiguous. Further, Assumption 2 only implies that the direct effect on $\bar{s}_a$ is nonpositive. The total effect on $\bar{s}_a$ could be positive due to $\bar{s}_a$ being affected by the demand level as well.

Since the direct selection effect may also be zero, we need to ensure that the sum of the indirect effects on $H_i$ is nonpositive. An increase in $A$ tends to make all firms weakly increase their levels of activity $i$. This makes the indirect level effect nonpositive. At the same time, an increase in $A$ will allow some previously inactive low-productivity firms to produce profitably. The indirect selection effect is therefore nonnegative. These two effects are reversed when $A$ decreases but are obviously still opposing. Therefore, in order to ensure that the sum of these indirect effects on $H_i$ is nonpositive in both cases, we must have that the indirect level and selection effects are exactly offsetting. In other words, the percentage changes in $\bar{s}_a$ and $\bar{s}_i$ induced by the change in $A$ have to be equal.

To formulate a condition that ensure this, we utilise that productivity is only identified up to a positive monotone transformation in our setup. Given a "primitive" productivity variable, $\phi$, with c.d.f. $F_\Phi$, one can instead work with any positive monotone transformation, $\theta = V(\phi)$.\(^{34}\) Without loss of generality we can therefore let the distribution of (transformed) productivities, $F$, have constant hazard rate. This merely amounts to choosing the transformation, $V$, such that $F(\theta) = F_\Phi(V^{-1}(\theta))$ has constant hazard rate, $\lambda(\theta) = \lambda$.

\(^{33}\)One the one hand, Assumption 2 is clearly satisfied when the direct effect on the profits of all firms is nonpositive (stick). On the other hand, Assumption 2 is satisfied for a carrot approach where the direct effect on the profits of the least productive active firms is zero and the effect on the profits of all other firms is positive while being strictly positive for some firms. For an example of the latter, see Section 6.1.

\(^{34}\)This of course requires proper reformulation of the profit function. If profits in terms of $\phi$ are given by $\tilde{\pi}(x, \phi, \gamma; A, \beta)$, profits in terms of $\theta$ are $\pi(x, \theta, \gamma; A, \beta) = \tilde{\pi}(x, V^{-1}(\theta), \gamma; A, \beta)$. Assumption 1 being satisfied for primitive productivity $\phi$ and profits $\tilde{\pi}$ then implies that Assumption 1 is also satisfied for $\theta$ and $\pi$ and vice versa.
Proposition 2. W.l.o.g. let $F$ have constant hazard rate. For all $G$, increases in $(\beta, S)$ induce FOSD shifts in the equilibrium distributions of all activities if the optimal decision, $x^*$, and the choice to exit depend on $\theta$ and $A$ only through $\theta + \tilde{A}$ where $\tilde{A}$ is a positive monotone transformation of $A$.

The constant hazard rate implies by construction that the density at any productivity level is constant relative to the probability mass above it. This means that the percentage changes (induced by a change in $A$) in the share of active firms and the share of firms undertaking at least a given level of activity $i$ are equal for a given $\gamma$ if the changes in the thresholds $\theta_a$ and $\theta_i$ are equal. The conditions in Proposition 2 ensure exactly this. Further, they ensure that the effect does not depend on $\gamma$, such that the conclusion holds when aggregating over $\gamma$. For details of the proof, see Appendix B. Applying Proposition 2 in a given context can be done in the following way. First check that firms’ decisions and choice to exit can be expressed such that they depend on the demand level and a transformation, $\theta$, of primitive productivity only through $\theta + \tilde{A}$. A sufficient condition is that profits, $\pi$, do so.$^{35}$ Next, monotone comparative statics for the industry composition follow if this transformation of productivity has constant hazard rate.

As mentioned above, Assumption 2 does not restrict $A$ to either rise or fall. Therefore, the industry-level monotone comparative statics in Proposition 2 are independent of whether the firm-level response is ambiguous or not. One reason is that the indirect effect through the demand level, which is the source of the possible ambiguity at the firm level, is made neutral at the industry level by the conditions in Proposition 2. Propositions 1 and 2 enable us to draw the somewhat surprising conclusion that firm-level complementarities can assert themselves more clearly in the composition at the industry level than at the firm level. This point is not only theoretically interesting but also important to take into account when testing models that conform to our assumptions.

5.4 Monotone Comparative Statics for the Industry Composition: Carrot or Stick

Once we consider increases in $(\beta, S)$ where we can predict that $A$ either falls (carrot) or rises (stick), we no longer need to be on the knife edge where the indirect level and selection effects on $H_i$ exactly balance. This leads us to the following proposition which is proved in Appendix C.

Proposition 3. Let $\theta_a$ be invariant across $\gamma$’s and let the optimal decision, $x^*$, and the choice to exit depend on $\theta$ and $A$ only through $\theta + \tilde{A}$. For all $G$, increases

$^{35}$However, it is not necessary; see Section 6.4.
in $(\beta, S)$ by the carrot (stick) induce FOSD shifts in the equilibrium distributions of all activities if the distribution of productivities has nonincreasing (nondecreasing) hazard rate, $\lambda(\theta)$.

To understand the intuition behind Proposition 3, remember that $A$ induces the indirect level and selection effects through its effects on $\theta_i$ and $\theta_a$, respectively. Relative to the case with constant hazard rate, where the two effects balance, a nonincreasing hazard rate puts relatively more probability density at $\theta_a$ (since $\theta_a \leq \theta_i$) which means that the indirect selection effect dominates the indirect level effect. This works in favour of an FOSD shift in the equilibrium distribution of activity $i$ when $A$ falls. Conversely, a nondecreasing hazard rate means that the indirect level effect dominates which works in favour of FOSD shifts when $A$ increases.

5.5 Models of International Trade

In many models of international trade, the decision of firms can be formulated in such a way that profits only depend on the primitive productivity variable, $\phi$, and a demand-level variable, $A$, through their product $A\phi$.\footnote{This is e.g. the case in the list of nested trade models provided in the introduction.} This arises naturally when consumers have CES preferences, in which case $A$ is a function of total expenditure and a price index, but can also arise in other cases; see Sections 6.3 and 6.4. Since many of these models otherwise conform to our basic setup and feature complementarities as described in Assumption 1, it is interesting to see when the conditions of Propositions 2 and 3 are satisfied.

That profits only depend on $A$ and $\phi$ through $A\phi$ is equivalent to profits only depending on them through $\theta + \log A$ with $\theta = \log \phi$. Further, if profits only depend on the log of primitive productivity, $\theta$, and the demand level through $\theta + \log A$, then so do firms’ decisions and their choice to exit. Finally, the distribution of $\theta$ having constant hazard rate is equivalent with $\phi$ being Pareto distributed.\footnote{$\theta = \log \phi$ being distributed with constant hazard rate, $\lambda$, implies $F(\theta) = 1 - e^{-\lambda(\theta - \theta_0)}$. Substituting for $\theta$ gives $F_{\phi}(\phi) = 1 - (\phi_0/\phi)^\lambda$ where $\phi_0 = e^{\theta_0}$ is the lower bound on $\phi$. Thus $F_{\phi}$ is given by the Pareto.} Corollary 1 follows.

Corollary 1. Let profits depend on primitive productivity, $\phi$, and a demand-level variable, $A$, only through their product, $A\phi$. The conditions of Proposition 2 are satisfied if and only if $\log \phi$ has constant hazard rate, i.e., if and only if $\phi$ is Pareto distributed.

While the Pareto distribution is often used in these models of international trade due to other attractive features, our result points out a novel knife-edge
property with strong implications for industry-level comparative statics in the presence of complementarities. Further, even though this literature considers many diverse activities, many of these models can be formulated such that labour input is one of these. Therefore, the comparative statics considered, such as trade liberalisations, very often lead to the prediction of FOSD shifts in the size distribution of firms (as measured by labour input).  

Next, it is quite easy to come up with productivity distributions that satisfy the weaker distributional requirement in Proposition 3. If one wants monotone comparative statics for the industry composition when employing the carrot (stick), one just has to pick a distribution of log-productivities, $F$, with a nonincreasing (nondecreasing) hazard rate. The corresponding distribution of primitive productivities is then obtained as $F_\Phi(\phi) = F(\log \phi)$.

### 5.6 Comparative Statics without Direct Effects

Some parameter changes have no direct effects on either firms’ decisions or their choice to exit. While this may be true for some elements of $\beta$; it is obviously the case for the sunk entry cost, $f_e$, which we will use to illustrate the implications. First off, the firm-level comparative statics are determined solely by the indirect effect on $\tilde{x}^*$. From (4) it is clear that an increase in $f_e$ must increase $A$ and thereby the equilibrium decisions of all firms. At the industry level however, things may be different. Since $f_e$ does not have direct level or selection effects, only the indirect effect matters for the industry level. We can therefore conclude that the equilibrium distributions of the activities are completely unaffected if the conditions in Proposition 2 hold. Further, on the one hand, if $\lambda(\theta)$ is nondecreasing such that the indirect level effect dominates the indirect selection effect, then the equilibrium distributions of activities shift toward higher values when $f_e$ increases. On the other hand, if $\lambda(\theta)$ is nonincreasing such that the indirect selection effect dominates, then all of these distributions shift towards lower values regardless of the unambiguous increase in the equilibrium decisions of all firms.

### 6 Applications

The present section shows how our firm- and industry-level results can be applied to some existing models and to a case with more generally-formulated preferences.  

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38To see how this works in the basic Melitz (2003) model, see Section 6.1.

39See Section 6.4 for an example.

40Our results can also be applied to a number of other models from the trade literature covering a broad range of topics which we do not treat here. Among these are Antràs and Helpman (2004), Helpman et al. (2004), Arkolakis (2010), Amiti and Davis (2011), Bustos
6.1 Exporting and Labour Input

Consider a two-country Melitz (2003) model. We let the activities of the firms be export status, given by the indicator $1_{ex}$ for exporting, and total labour input for variable production, $l$. Requiring that the resulting output is optimally distributed across markets in case of exporting, we obtain the profit function $^{41}$

$$\pi(x, \theta; A, \beta) = (1 + 1_{ex} (1-\sigma) l^{\frac{\sigma-1}{\sigma}} e^{\frac{\sigma-1}{\sigma}(\theta+\log A)} - l - f - 1_{ex} f_{ex}), \quad (9)$$

where $A$ is a demand shifter, $\sigma > 1$ is the elasticity of substitution from the CES preferences, $\tau > 1$ is an iceberg trade cost, $f$ is a fixed cost of production, and $f_{ex}$ is a fixed cost of exporting. Here, we define $\theta = \log \phi$, with $\phi$ being the primitive productivity variable of Melitz (2003).

Now, let $x = (l, 1_{ex})$, $X = \mathbb{R}_{+} \times \{0, 1\}$, and $\beta = (-\tau, -f_{ex})$. Then it is easy to verify that the profit function, (9), is supermodular in $x$ and has increasing differences in $(x, \theta)$, $(x, A)$, and $(x, \beta)$. Hence, Assumption 1 regarding complementarities is satisfied. Since the model otherwise conforms to our setup, we can apply our propositions to analyse the effects of different types of trade liberalisation. Starting with the firm level, we consider an opening to trade and incremental liberalisations of trade through decreases in $\tau$ and $f_{ex}$. Opening to trade implies introducing a new activity (exporting) by moving from $S' = \mathbb{R}_{+} \times \{0\}$ to $S'' = \mathbb{R}_{+} \times \{0, 1\}$ which constitutes an increase in $S$. Both incremental liberalisations involve an increase in the attractiveness of exporting (an increase in $\beta$). All of these increases in $(\beta, S)$ are by means of the carrot since they have a nonnegative direct effect on the profits of any given firm. Therefore we know that upon either trade liberalisation, the demand level must fall. The indirect effects on the equilibrium decisions are thus negative. This implies that the total effect on the amount of labour employed in variable production is negative for some firms and positive for others. For example, when the iceberg trade cost is reduced, all exporters increase their use of labour while (ex-post) nonexporters reduce their use of labour. While the positive direct effect dominates the negative indirect effect for exporters, nonexporters are only affected by the negative indirect effect.$^{43}$


$^{41}$We disregard $\gamma$ as productivity is the only source of firm heterogeneity.

$^{42}$Note how proper ordering is crucial.

$^{43}$Referring to footnote 23, this shows that the direction of firm-level responses may vary across firms for a given activity. To see that responses can vary across activities within a given firm, one can split total labour for variable production into that used for production to the domestic market and the export market. Doing so, the model still conforms to our assumptions and it can easily be verified that upon a decrease in $\tau$, exporters increase their use of labour for production to the export market while decreasing the use for labour for production to the domestic market.

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Moving on to the industry level, we can invoke Proposition 2 since the trade liberalisations described above have no direct effects on the profits of the least productive firms (which do not export) wherefore Assumption 2 holds. Thus, we know that if log-productivities, $\theta$, have a distribution with constant hazard rate, then trade liberalisations give rise to monotone comparative statics for the industry composition. As mentioned in Section 5.5, this corresponds to primitive productivity, $\phi$, being Pareto distributed. In this case, either of the three types of trade liberalisation induces industry-level shifts towards exporting and more labour in variable production.\(^{44}\) The effects on the equilibrium distribution of labour input are illustrated in Figure 1.

Figure 1: Equilibrium distribution of labour input before (dotted) and after (solid) three trade liberalisations.

The flat parts, which occur at the share of active firms that do not export, are due to the discrete increase in optimal labour input when exporting. These flat parts shift downwards since the share of active firms which do not export decreases. It is also possible to obtain monotone comparative statics for the industry composition without the Pareto distribution. Since the demand level is decreasing in all cases, Proposition 3 tells us that whenever log-productivities, $\theta$, are distributed with nonincreasing hazard rate, the equilibrium distributions shift towards exporting and more labour input in variable production.

The observation that improved export opportunities are beneficial to some firms and detrimental to others is a well-known result of the Melitz (2003) model. However, our industry-level results illustrate conditions under which the industry level exhibits monotone comparative statics regardless of firm-level responses being ambiguous. Even though a range of firms may downsize, the equilibrium

\(^{44}\)Relating to the discussion in Section 5.3, this conclusion would not be affected by extending the model to include additional complementary activities.
distribution of labour input unambiguously shifts towards firms becoming larger under the distributional assumptions discussed above.

6.2 Exporting and Labour-Market Frictions

Helpman et al. (2010) consider the effects of trade liberalisation on wage inequality and unemployment. Workers are heterogeneous with respect to ability and the productivity of employed workers is determined by their average ability. Firms are able to screen workers to ensure that only workers with abilities above a chosen threshold, \( a_c \), are hired. Firms choose this threshold along with their export status, \( 1_{ex} \), and the number of workers to sample for screening, \( \nu \). As the Helpman et al. (2010) model otherwise conforms to our setup, we only need to obtain a profit function of the proper form for our results to apply in this context.

Assuming that countries are symmetric, the profit function can be expressed as

\[
\pi(x, \phi; \hat{A}, \beta) = (1 + 1_{ex} \tau^{1-\sigma})^{\frac{1}{\sigma}} \nu^\chi a_c^\kappa (A\phi)^{\frac{\sigma-1}{\sigma}} - b\nu - ca_c^\zeta - f - 1_{ex} f_{ex},
\]  

(10)

where \( \hat{A} \) is a demand shifter, \( \chi, \kappa, c, \zeta \) are positive parameters, and \( b \) is the cost of sampling workers. The remaining parameters have the same interpretations as in the previous example. The profit function, (10), with the decision \( x = (\nu, a_c, 1_{ex}) \) and \( \beta = (-\tau, -f_{ex}) \) would satisfy our assumptions if it was not for the fact that the marginal cost of sampling workers, \( b \), is an endogenous variable which depends on the tightness of the labour market. However, if we let firms choose sampling expenditures, \( b\nu \), instead of the number of sampled workers and rewrite the profit function, then we get

\[
\pi(x, \phi; A, \beta) = (1 + 1_{ex} \tau^{1-\sigma})^{\frac{1}{\sigma}} (b\nu)^\chi a_c^\kappa (A\phi)^{\frac{\sigma-1}{\sigma}} - b\nu - ca_c^\zeta - f - 1_{ex} f_{ex},
\]  

(11)

where the demand level, \( A = \hat{A}b^{\frac{\chi}{\chi-1}} \), now contains all endogenous variables outside the control of firms. Letting \( x = (b\nu, a_c, 1_{ex}) \) and \( \beta = (-\tau, -f_{ex}) \), the Helpman et al. (2010) model now conforms to our setup and assumptions. The authors let productivities, \( \phi \), be Pareto distributed. We can thus apply our industry-level results to trade liberalisations as these satisfy Assumption 2.

We find that opening to trade and incremental trade liberalisations through reductions in \( \tau \) or \( f_{ex} \) lead to unambiguous FOSD shifts in the industry composition towards more intensive screening of workers, i.e., higher levels of \( a_c \), higher sampling expenditures, \( b\nu \), and more exporting. Further, allowing firms to undertake higher levels of worker screening leads to FOSD shifts in the industry composition towards more exporting, higher sampling expenditures, and more intensive screening. Since all these comparative static exercises are by means of the carrot, we have a negative effect on the demand level, \( A \). All these industry-level results will therefore also hold when \( \theta = \log \phi \) has nonincreasing hazard.
rate. In Helpman et al. (2010), nonexporters are unable to profit from the trade liberalisations and thus cut down on both sampling expenditures and screening intensity.

6.3 Additively Separable Utility and Quality

Quite general preference structures can give rise to a profit function that satisfies the conditions in Propositions 2 and 3. With a slight reinterpretation of $\theta$ as (a transformation of) quality, we show that our industry-level results are more widely applicable than in just the case of CES preferences.

Let consumers have preferences $U = \int \phi_j u(c_j) \, dj$ with $j$ indexing varieties of a differentiated good and $c_j$ being consumption of variety $j$. In this case, $\phi_j$ should be interpreted as quality of variety $j$ instead of the productivity of the firm producing it. With identical consumers maximising utility subject to the budget constraint, $\int p_j c_j \, dj = I$, where $p_j$ is the price of variety $j$ and $I$ is income, the inverse demand function reads $p_j = A\phi_j u'(c_j)$, with $A$ being the inverse marginal utility of income. In this case, the revenue of a firm reads $r(q_j) = A\phi_j u'(q_j/L)q_j$, where $q_j$ is total output and $L$ is market size (the number of consumers). Now, we could go on to make assumptions on the cost structure and then write up a profit function which depends on both the choice of $q$, a number of additional activities, and some parameters such that our assumptions of complementarities are met. However, we confine the discussion to noting that with this revenue function, the profit function is well on its way to depend on $A$ and $\phi$ only through their product, $A\phi$. Drawing upon Section 5.5, one can then easily identify distributions of quality that can result in monotone comparative statics for the industry composition in case the profit function exhibits complementarities.

6.4 Variable Markups

This example is based on the Melitz and Ottaviano (2008) model but abstracts from trade. Consumers have quadratic preferences leading to the linear demand function, $q = L(p_{\text{max}} - p)$, where $q$ is quantity, $p$ is the price, $p_{\text{max}}$ is the choke price, and $L$ is market size. Firms have constant marginal costs given by the inverse of primitive productivity, $\phi^{-1}$, and no fixed costs. In this closed-economy version of the model, the profit function can be written as

$$\pi(x, \phi; A, \beta) = L\phi^{-2}(A\phi - x)(x - 1),$$

(12)

Note that in this case, $\phi_j$ represents quality in the sense that it scales up marginal utility, $u'(c_j)$, or marginal willingness to pay, for a given level of consumption.
where \( x = p/\phi^{-1} \) is the relative markup of the firm and \( A = p_{\text{max}} \). Let \( X = [1, \infty) \).\(^{46}\) Note that \( \phi^{-1} > A \) implies that a firm exits.

The profit function is nondecreasing in \( A \) and has increasing differences in \( (x, A) \). Further, optimised profits are monotone nondecreasing in \( \phi \) even though profits are not.\(^{47}\) While (12) does not have increasing differences in \( (x, L) \), (12) does exhibit the single crossing property. Thus, we can still apply our results with \( \beta = L \).\(^{48}\) Note that \( L \) does not affect the optimal decision, \( x^* \), directly. Therefore, an increase in market size only has an indirect effect. As all firms can earn weakly higher profits the higher is \( L \) given \( A \), an increase in \( L \) has a negative indirect effect on the equilibrium decision. In total, a larger market makes all firms strictly reduce their markups.

What about the industry level? First, note that (12) exhibits the single crossing property in \( (x, \phi) \) and that this is actually sufficient for Lemma 1. We can therefore proceed with our analysis. The model conforms to our setup in all other respects as well. Further, Assumption 2 is satisfied for an increase in \( L \) since this increase does not directly affect the optimal profits of the least productive firms. Finally, since \( A \) and \( \phi \) only matter for a firm’s decision on markups and exit through \( A\phi \) and Melitz and Ottaviano (2008) assume productivities, \( \phi \), to be distributed Pareto, the conditions of Corollary 1 are satisfied. We can therefore conclude that an increase in \( L \) implies that the equilibrium distribution of markups shifts towards higher values. However, this only holds in the weakest possible sense. Let \( \theta = \log \phi \). As the increase in \( L \) has zero direct effects on \( \theta_a \) and \( \theta_1 \), the equilibrium distribution of markups is constant when primitive productivities, \( \phi \), are distributed Pareto (\( \theta \) has constant hazard rate). Even though the model features pro-competitive effects of a larger market size, in that all firms strictly reduce their markups, the equilibrium distribution of markups is constant in \( L \) due to selection effects (low-productivity firms with low markups exit).

Since \( L \) only affects the equilibrium distribution of markups through the indirect effects and \( A \) is decreasing, the equilibrium distribution of markups shifts towards lower values—in line with the intuition of the pro-competitive effect at the firm level—if log-productivities are distributed with increasing hazard rate. However, if the distribution of log-productivities has a decreasing hazard rate, then the equilibrium distribution of markups shifts towards higher values following an increase in \( L \). In this case, sufficiently many low-productivity firms are driven out of business to ensure that the consumers are going to face higher

\(^{46}\) No firm would want to choose a price lower than its marginal cost wherefore \( x \geq 1 \) is an uncontroversial restriction.

\(^{47}\) Solving for \( x^* = \frac{1}{2}(A\phi + 1) \) and plugging back into (12) yields \( \pi^* = \frac{L}{4}(A - \phi^{-1})^2 \).

\(^{48}\) As (12) is trivially supermodular in \( x \), increasing differences in \( (x, \beta) \) is a sufficient condition for the partial comparative statics in Lemma 1. The single crossing property is both necessary and sufficient; see Appendix A.
markups on average when the market size has increased. The pro-competitive ef-
facts at the firm level therefore in no way ensure that the equilibrium distribution
of markups shifts toward lower values.\footnote{This relates to a remark made by Arkolakis et al. (2012). However, as our criterion is first-order stochastic dominance instead of monotone likelihood ratio dominance, our condition of a decreasing/nondecreasing hazard rate of the distribution of log-productivities is different from theirs (log-concavity/convexity of this distribution).}

\section*{6.5 Multi-Product Firms}

Bernard et al. (2011), Arkolakis and Muendler (2011), and Mayer et al. (2011) consider heterogeneous-firms models where firms are allowed to sell multiple prod-
ucts on each of the markets they face. The basic differences between the three
models are the consumer preferences and whether the number of products on a
given market is continuous or discrete. If we let markets be symmetric and the
activities be the number of products to sell on each market, our assumptions are
satisfied in these models.\footnote{For Mayer et al. (2011) one needs to rely on the single crossing property instead of increasing differences.}

Productivities are assumed to be Pareto distributed such the total indirect effect on the industry composition is zero. For reasons of tractability, Bernard et al. (2011), Arkolakis and Muendler (2011), and Mayer et al. (2011) assume that the choice of the number of products to sell on a given market has no effect on the choice concerning other markets. One may thus ar-
agree that the profit function is only supermodular in the activities in the weakest
possible sense as it is simply modular. In this case, allowing firms to export, or
conducting an incremental trade liberalisation, leaves the share of firms selling
at least a given number of products domestically completely unaffected.\footnote{Both exercises satisfy Assumption 2 when the least productive firms do not export.}

Why then is this an interesting application of our results? We believe for two reasons.

First, because it tells us that, even though individual firms may scale down
the number of products sold domestically in the wake of trade liberalisation,
the endogenous distribution of domestic products across (domestic) firms is not
shifted towards fewer domestic products. Considering a given firm, Bernard et al.
(2011) note: "...trade liberalization reduces the range of products supplied to
the domestic market." But this should be taken as the firm-level result that it
is. From the consumers’ point of view, supplied variety from domestic firms is
affected only through the mass of domestic firms, as the distribution of domestic
products across (domestic) firms is completely unaffected. Further, one would
observe an FOSD shift if log-productivities had nonincreasing instead of constant
hazard rate.

Second, maybe the activities ought not to be independent. Although it
reduces the tractability of the models, it seems more reasonable a priori that

\footnote{Both exercises satisfy Assumption 2 when the least productive firms do not export.}
the activities are complementary. If you are already selling a given product on one market, then it should be easier to sell it on another market as well since production—and to some extent (international) distribution—of the product is already up and running. In this case of nontrivial complementarity, our results of Section 5 tell us that trade liberalisation induces FOSD shifts in the industry composition towards broader domestic product ranges. Interestingly, this result may also arise even if you let the number of domestic and export products be directly independent. The numbers of domestic and export products could become indirectly complementary by including a third activity, e.g. technology upgrading as in Bustos (2011), to which they are both complementary.

7 Concluding Remarks

If one considers a given firm in isolation, then complementarities like the ones we analyse imply monotone comparative statics in the parameters of the firm’s profit maximisation problem. This has been known at least since Milgrom and Roberts (1990a). However, when placing firms in a context of monopolistic competition, this may no longer be the case due to indirect effects through changes in the demand level. New or better opportunities that become available to all firms may make a given firm scale down existing activities even when these existing activities are complementary to the activities affected. Thus, for some firms, such opportunities may turn out to be a threat detrimental to many dimensions of the firms’ operations.

Our main finding is that firm-level complementarities may manifest themselves much more clearly in the industry composition than in the behaviour of individual firms. Despite the ambiguities at the firm level, we show that one may very well observe that the equilibrium distributions of the activities unambiguously shift towards higher values. Key to this result is the observation that these distributions depend not only on the activity levels undertaken by firms conditional on being active but also on the selection of active firms.

These results are important for several reasons. First, they provide general insights on the implications of firms-level complementarities in models of monopolistic competition; a workhorse market structure in many strands of the economics literature. Our industry-level results may prove particularly useful for the new macro literature featuring heterogeneous firms; see e.g. Ghironi and Melitz (2005). Second, our results provide strong, novel, and testable predictions—especially at the industry level—for a large number of recent trade models. We believe that it will be both useful and interesting to confront these predictions with data. On the one hand, such empirical investigations can shed light on the appropriateness of commonly-used functional form assumptions. On the other hand, this approach
Further, we have defined the weights where we have used the definitions of increasing differences in increase in Theorem 2. Let \( h \) implies that \( h(x', x'') \in X \) if for all \( x', x'' \in X, h(x', t) \geq h(x' \land x'', t) \) \( h(x' \lor x'', t) \geq h(x', t) \) and \( h(x', t) > h(x' \land x'', t) \) \( h(x' \lor x'', t) > h(x'', t) \). Hence, if an increase in a subset of the elements of \( x \) raises \( h \) at a given level of the remaining elements, exactly the same increase in the same subset of the elements of \( x \) will increase \( h \) when the remaining elements also increase. In the language of Milgrom and Shannon (1994), quasisupermodularity expresses a weak kind of complementarity between the elements of \( x \). The function \( h(x, t) \) satisfies the single crossing property in \( (x, t) \) if for \( x'' > x' \) and \( t'' > t' \), \( h(x'', t') > h(x', t') \) implies that \( h(x'', t'') > h(x', t'') \) and \( h(x'', t') \geq h(x', t') \) implies that \( h(x'', t'') \geq h(x', t'') \). Hence, if an increase in \( x \) raises \( h \) when \( t \) is low, exactly the same increase in \( x \) will raise \( h \) when \( t \) is high. One can verify by the relevant definitions that any supermodular function is also quasisupermodular and any function with increasing differences in \( x, t \) also satisfies the single crossing property in \( (x, t) \). Let \( S \subseteq X \). The following monotonicity theorem is due to Milgrom and Shannon (1994).

**Theorem 2.** \( \arg \max_{x \in S} h(x, t) \) is monotone nondecreasing in \( (t, S) \) if and only if \( h \) is quasisupermodular in \( x \) on \( X \) for each \( t \in T \) and satisfies the single crossing property in \( (x, t) \) on \( X \times T \).

## A Quasisupermodularity and Single Crossing

Let \( X \) be a lattice and \( T \) be a partially ordered set. The real-valued function \( h(x, t) \) is quasisupermodular in \( x \) on \( X \) if for all \( x', x'' \in X, h(x', t) \geq h(x' \land x'', t) \) \( h(x' \lor x'', t) \geq h(x', t) \) and \( h(x', t) > h(x' \land x'', t) \) \( h(x' \lor x'', t) > h(x'', t) \). Hence, if an increase in a subset of the elements of \( x \) raises \( h \) at a given level of the remaining elements, exactly the same increase in the same subset of the elements of \( x \) will increase \( h \) when the remaining elements also increase. In the language of Milgrom and Shannon (1994), quasisupermodularity expresses a weak kind of complementarity between the elements of \( x \). The function \( h(x, t) \) satisfies the single crossing property in \( (x, t) \) if for \( x'' > x' \) and \( t'' > t' \), \( h(x'', t') > h(x', t') \) implies that \( h(x'', t'') > h(x', t'') \) and \( h(x'', t') \geq h(x', t') \) implies that \( h(x'', t'') \geq h(x', t'') \). Hence, if an increase in \( x \) raises \( h \) when \( t \) is low, exactly the same increase in \( x \) will raise \( h \) when \( t \) is high. One can verify by the relevant definitions that any supermodular function is also quasisupermodular and any function with increasing differences in \( x, t \) also satisfies the single crossing property in \( (x, t) \). Let \( S \subseteq X \). The following monotonicity theorem is due to Milgrom and Shannon (1994).

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## B Proof of Proposition 2

Note that a zero (total) indirect effect in (8) is equivalent with

\[
\int \omega_a e^{-f_{\theta_a}(\gamma; A'; A''; S'')} \lambda(\theta) d\theta = \int \omega_i e^{-f_{\theta_i}(x_i; \gamma; A'; A''; S'')} \lambda(\theta) d\theta \tag{13}
\]

where we have used the definitions of \( \bar{s}_a \) and \( \bar{s}_i \) along with \( 1 - F(\theta) = e^{-f(\theta) \lambda(s) ds} \). Further, we have defined the weights \( \omega_a = s_a(\gamma; A', A''; S'')/\bar{s}_a(\gamma; A', A''; S'') \) and \( \omega_i = s_i(x_i; \gamma; A', A''; S'')/\bar{s}_i(x_i; A', A''; S'') \) which both integrate to one.
If $\lambda(\theta) = \lambda$ and the changes in $\theta_a$ and $\theta_i$ induced by the change in $A$ are the same, then the condition (13) surely holds. Further, when firms’ decisions and their choice to exit only depend on $A$ and $\theta$ through $\theta + \tilde{A}$, then $\theta_a + \tilde{A}$ and $\theta_i + \tilde{A}$ must be constant in $A$. This means that

$$\theta_a(\gamma; A'', \beta, S) - \theta_a(\gamma; A', \beta, S) = \tilde{A}' - \tilde{A}''$$  \hspace{1cm} (14)

and

$$\theta_i(x_i, \gamma; A'', \beta, S) - \theta_i(x_i, \gamma; A', \beta, S) = \tilde{A}' - \tilde{A}''$$  \hspace{1cm} (15)

which together with the constant hazard rate means that (13) is satisfied.

C  Proof of Proposition 3

Note that the (total) indirect effect on $H_i$ is nonpositive when

$$\int \omega_i e^{-\int_{\theta_a(\gamma; \tilde{A}'', \beta', S'')}^0 \lambda(\theta) d\theta} dG(\gamma) \leq \int \omega_i e^{-\int_{\theta_i(x_i, \gamma; \tilde{A}'', \beta', S'')}^0 \lambda(\theta) d\theta} dG(\gamma).$$

When $\theta_a$ is independent of $\gamma$, this reduces to

$$e^{-\int_{\theta_a(\tilde{A}'', \beta', S'')}^0 \lambda(\theta) d\theta} \leq \int \omega_i e^{-\int_{\theta_i(x_i, \gamma; \tilde{A}'', \beta', S'')}^0 \lambda(\theta) d\theta} dG(\gamma).$$  \hspace{1cm} (16)

Because (14) and (15) hold, the length of the interval over which $\lambda(\theta)$ is integrated is the same on both sides of the inequality in (16). Remember that $\theta_i(x_i, \gamma; A', \beta'', S'') \geq \theta_a(A', \beta'', S'')$. Since $\theta_a(A'', \beta, S') \geq \theta_a(A', \beta, S')$ and $\theta_i(x_i, \gamma; A'', \beta, S') \geq \theta_i(x_i, \gamma; A', \beta, S')$ hold when $A$ falls, (16) holds for all $A' \geq A''$ when $\lambda(\theta)$ is nonincreasing. Conversely, $\theta_a(A'', \beta, S') \leq \theta_a(A', \beta, S')$ and $\theta_i(x_i, \gamma; A'', \beta, S') \leq \theta_i(x_i, \gamma; A', \beta, S')$ hold when $A$ rises. Hence, (16) holds for all $A' \leq A''$ when $\lambda(\theta)$ is nondecreasing. Since using the carrot (stick) corresponds to the former (latter) case, the proposition follows.

References


Firms and International Trade


Firms and International Trade


Chapter Three

Trade Liberalisation and Vertical Integration*

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Abstract

We build a three-country model of international trade in final and intermediate goods and study the relation between different types of trade liberalisation and vertical integration. Firms are heterogeneous with respect to both productivity and factor intensity as observed in data. Final-good producers face decisions on exporting, vertical integration of intermediate-input production, and whether the intermediate-input production should be offshored to a low-wage country. We find that due to firm-level complementarities, the shares of final-good producers that pursue either vertical integration, offshoring, or exporting are all increasing when intermediate- or final-goods trade is liberalised and when the cost of vertical integration is reduced. At the same time, one will observe individual firms that shift away from either vertical integration, offshoring, or exporting. All these results hold for a class of productivity distributions to which the Pareto distribution belongs.

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1 Introduction

We develop a three-country model of trade in final and intermediate goods in order to investigate the relation between vertical integration and liberalisations of international trade in final and intermediate goods. The relationship between final-good producers and intermediate-good suppliers is characterised by incomplete contracts. Final-good producers face a joint decision on the three activities vertical integration, offshoring, and exporting. Importantly, the model is characterised by firm-level complementarities between these three activities, in the sense that undertaking one of the activities raises the gains from undertaking the others. Our main contribution is to derive a series of strong and testable results that illustrate how these complementarities have clear implications for the industry composition. In particular, we find that the shares of final-good producers that pursue either vertical integration, offshoring, or exporting, i.e., the prevalences of these activities, are all increasing when intermediate- or final-goods trade is liberalised and when the cost of vertical integration falls. Meanwhile, one observes individual firms shifting away from either vertical integration, offshoring, or exporting under the comparative statics mentioned above. This observation is compatible with rising prevalences of the three activities because some low-productivity firms, which do not undertake any of these activities, endogenously shut down due to fiercer competition.

These main findings relate to the ongoing discussion about the relationship between trade liberalisation, or more generally competition, and vertical integration. While elements of the popular press and the seminal studies by McLaren (2000) and Antràs and Helpman (2004) (henceforth AH) have accentuated a negative relation between trade liberalisation and vertical integration, other theoretical contributions like Grossman and Helpman (2004), Ornelas and Turner (2008), and Conconi et al. (2012) have shown that the relationship between trade liberalisation and vertical integration is often ambiguous. This paper relates to all these studies by unveiling a clear positive relationship between different types of trade liberalisation and the prevalence of vertical integration. Further, we show that this finding is compatible with ambiguities at lower levels of aggregation. First, some firms shift away from vertical integration in the wake of trade

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1 We define vertical integration (outsourcing) as the acquisition of an intermediate input from an affiliated (unaffiliated) supplier. Offshoring refers to the phenomenon that production of this intermediate input takes place in the low-wage South.

2 In relation to this, Grossman and Helpman (2002) and Acemoglu et al. (2010) show that the relation between competition and vertical integration can be ambiguous. These theoretical results have been corroborated by the empirical evidence in Aghion et al. (2006) and Acemoglu et al. (2010). Recent work by Alfaro et al. (2012) finds a negative relation between trade liberalisation and vertical integration in firm-level data.
liberalisation. Second, the share of firms undertaking vertical integration domestically may decrease or increase, partly depending on whether intermediate- or final-goods trade is being liberalised.

Our model builds on the two prominent models of international trade by Melitz (2003) and AH. In an important departure from AH, we accentuate a natural complementarity between the activities vertical integration and offshoring. In addition, our model serves as a natural extension of AH for several reasons. First, the tradeoffs governing the integration and offshoring decisions in the AH model, which does not include a possibility of exporting, can reasonably be expected to depend on the export status of the firm. One reason is a complementarity between the activities offshoring and exporting for which Amiti and Davis (2011), Bas (2012), and Kasahara and Lapham (2013) provide tentative evidence. Another reason is that the export decision partly determines the scale of the firm, which is likely to affect the decision to vertically integrate. Consistent with these speculations, Kohler and Smolka (2011) note that Spanish exporters are more likely than non-exporters to pursue vertical integration and offshoring.

Second, we extend the AH model to allow firms within the same industry to be heterogeneous with respect to headquarter intensity as well as productivity. Headquarter intensity refers to the elasticity of output with respect to headquarter services, which are one input into production. Heterogeneity in headquarter intensity serves two main purposes. Primarily, allowing for heterogeneity in headquarter intensity is a direct theoretical modelling response to the empirical findings of for instance Corcos et al. (forthcoming). These authors reveal that factor intensities like capital and skill intensity—commonly used empirical proxies for headquarter intensity, cf. Antràs (forthcoming)—exhibit considerable variation across firms within narrowly defined industries. Corcos et al. (forthcoming) also argue that firm-level capital and skill intensities are important determinants of the decision to vertically integrate since the probability of intra-firm importing increases in these firm-level intensities conditional on firm productivity and

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3The underlying forces behind this complementarity are present in the AH model but are obscured due to the desire to force the model to generate a rich equilibrium sorting pattern based on productivity alone.

4Further, Bernard et al. (2012) note that across U.S. industries, there is a positive correlation (0.87 and significant) between the share of exporters and the share of importers of intermediate inputs.

5Acemoglu et al. (2010) note that vertically integrated firms are larger than non-integrated firms, which indicates that size may play a role in the decision to integrate.

6In the French firm-level data used by Corcos et al. (forthcoming), the capital and skill intensities of production exhibit much more variation within than across industries. Further, within-industry heterogeneity in factor intensity is even more pronounced than in its productivity counterpart. Bernard et al. (2003) also argue that industry is a poor indicator of firm factor intensity among U.S. manufacturers.
importing. These new empirical observations indicate a need for extending the influential open-economy property-rights theory of the firm, pioneered by Antràs (2003) and AH and building on the work of Grossman and Hart (1986), to include firm-specific headquarter intensities, and our contribution is a first attempt in this regard.\footnote{As mentioned by Antràs (forthcoming) and Corcos et al. (forthcoming), this extension should make the open-economy property rights model more suitable for future analyses based on the firm-level sourcing data which are starting to appear. Empirical tests of the open-economy property-rights theory of the firm have broadly provided empirical support to the model but mostly been based on industry-level data on intra-firm trade; see e.g. Nunn and Trefler (forthcoming) and the survey by Antràs (forthcoming).} Furthermore, allowing for heterogeneity in headquarter intensity as well as in productivity is a natural way to obtain an industry equilibrium with the wide variety of organisational decisions observed by e.g. Tomiura (2007) and Kohler and Smolka (2011). An interesting and reassuring aspect of our model is that the sorting of firms into activities based on their productivity and headquarter intensity is broadly consistent with the empirical findings of Corcos et al. (forthcoming). However, we also illustrate that one has to be careful when applying a conventional intuition about the relation between headquarter intensity and vertical integration to a firm-level analysis of vertical integration and intra-firm importing in an open-economy context.

Third, our model departs from its AH backbone in dispensing with the common assumption of Pareto-distributed firm-specific productivity parameters in favour of a broader class of distributions that includes the Pareto.

Despite the apparent complexity of our model, it remains surprisingly tractable. The main reason is the firm-level complementarities inherent in our model. Drawing on the analysis of Bache and Laugesen (2013), these complementarities allow us to keep track of both the sorting of firms into activities and comparative statics. Compared to Bache and Laugesen (2013) who analyse a framework that is much more generally formulated, the specific setup we analyse in this paper allows us to go a step further and show that the prevalences of the three activities we consider are strictly increasing as opposed to nondecreasing. That is, to arrive at strictly monotone comparative statics for the industry composition. Another innovation relative to Bache and Laugesen (2013) is to analyse the prevalence of firm strategies that combine various activities such as vertical FDI. Finally, in the present paper, we are able to derive results on the sorting of firms in more than just the productivity dimension.

2 Model

We build a three-country heterogeneous-firms trade model with two symmetric northern (\(N\)) countries that interact through intra-industry trade in differentiated

\[ \text{...} \]
final goods. In addition to the differentiated-goods industry, which is monopolistically competitive, each northern country contains a perfectly-competitive homogeneous-good industry. Our analysis shall focus on the former. The third country is South (S) which does neither demand nor produce differentiated goods. While South also has a perfectly-competitive industry producing the homogeneous good, South basically serves as a possible production site for intermediate inputs to production. Offshoring denotes the phenomenon that a northern final-good producer decides to let its production of intermediate inputs take place in South. The attraction of producing in South is its perfectly elastic supply of labour at the relatively low wage, $w_S$. The northern wage is normalised to unity, $w_N = 1$, such that $w_S < 1$ is also the relative wage. In general equilibrium, this wage difference is justified by a labour productivity difference in the production of the freely-traded homogeneous good, $q_0$, which is produced and consumed in all three countries.

The preferences of the representative consumer in each $N$ country are represented by the utility function,

$$U = q_0 + \log \left( \int_{i \in \omega} q(i)^\alpha \, di \right)^{1/\alpha}, \quad 0 < \alpha < 1,$$

where $q(i)$ denotes the quantity consumed of variety $i$ of the differentiated goods. Each final-good producer produces a single unique variety and $\omega$ denotes the endogenous measure of available varieties. Demand for variety $i$ is given by the demand function,

$$q(i) = Ap(i)^{-\sigma},$$

where $p(i)$ is the price, $\sigma = 1/(1 - \alpha)$, and the demand shifter $A$ is taken as given by firms while being endogenous in the aggregate.

### 2.1 Firm Entry

Prospective final-good firms in $N$ pay $f_E$ units of local labour in order to enter the monopolistically-competitive industry and to draw a productivity, $\theta$, from a known distribution. This distribution, $F(\theta)$, is $C^1$ on the interior of its domain which is unbounded from above. We let the distribution of $\log \theta$ have nonincreasing hazard rate. Note that this is the case under the common assumption that $\theta$ is Pareto distributed since this implies that $\log \theta$ is distributed with constant hazard rate. Simultaneously with the realisation of $\theta$, firms also realise their idiosyncratic characteristic $\eta$ which also affects their technology of

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8As mentioned in the introduction, our model builds upon Melitz (2003) and AH.

9Note that $\log \theta$ being distributed with nonincreasing hazard rate implies that $\log(\theta^\varepsilon)$ is distributed with nonincreasing hazard rate for $\varepsilon > 0$. 

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production. For reasons which will become clear below, we refer to \( \eta \) by the term headquarter intensity. \( \eta \) is independently distributed from \( \theta \) according to the distribution \( G(\eta) \) which is strictly increasing on its domain, \( \eta \in (0,1) \). Upon the realisation of \( (\theta,\eta) \), firms make their optimal decisions, \( klx \), where \( k \in \{O,V\}, l \in \{N,S\}, x \in \{D,X\} \), or exit the industry. The decision comprises three sub-decisions. First, a make-or-buy decision concerning procurement of an intermediate input, \( m \). This determines the ownership structure, \( k \), which can be either vertical integration (henceforth integration), \( V \), or outsourcing, \( O \). Second, an offshoring decision concerning the location of the production of the intermediate input. Through offshoring, this input may be produced in \( S \) and used in final-good production in \( N \). This determines \( l \in \{N,S\} \) where \( l = S \) under offshoring and \( l = N \) otherwise. The intermediate input will never be shipped between northern countries in equilibrium. Third, we have a decision about exporting. The export status, \( x \), can either be \( X \) for exporter or \( D \) for nonexporter or domestic firm. If no choice of \( klx \) entails positive profits, the firm exits the industry and forfeits its cost of entry.

### 2.2 Production

Production of final-good variety \( i \) is given by

\[
q(i) = \theta(i)\zeta(i) h(i)^{\eta(i)} m(i)^{1-\eta(i)},
\]

where \( \zeta(i) \equiv \eta(i)^{-\eta(i)}(1-\eta(i))^{-(1-\eta(i))} \). \( h(i) \) and \( m(i) \) denote investments in headquarter services and a manufacturing intermediate input, respectively.\(^{10}\) In the following, we focus attention to a given final-good variety and drop the index \( i \). Investments in headquarter services, \( h \), are undertaken by the particular final-good producer, \( H \), itself. Investments in intermediate inputs, \( m \), are undertaken by an intermediate-input supplier, \( M \). Due to a perfectly elastic supply of homogeneous \( M \) in all countries, matching is always unproblematic for \( H \). One unit of either input, \( h \) or \( m \), is produced from one unit of local labour in the country of its production. In contrast to \( m \), \( h \) can only be produced in the \( N \) country where \( H \) entered.

Trade in final and intermediate goods is costly. Iceberg costs of international final-goods trade are \( \tau > 1 \). We include the intermediate-goods iceberg trade costs, \( \tau_m > 1 \), in \( w_S \) such that \( w_S = \tilde{w}_S \tau_m \) where \( \tilde{w}_S \) is the relative southern wage net of trade costs that we pin down by the homogeneous good. Finally,

\(^{10}\)We use the term headquarter intensity for \( \eta(i) \) since \( \frac{\partial q(i)}{\partial h(i)} \frac{h(i)}{q(i)} = \eta(i) \). Skill, R&D, and advertisement intensities have earlier been used as empirical proxies for \( \eta(i) \) since \( h(i) \) is often thought of as a white-collar input to production. Antràs (2003) uses capital intensity as an empirical proxy for \( \eta(i) \) since \( h(i) \) is intensive in capital in this model.
production implies fixed costs, $f_{klx}$, where

$$f_{klx} = f_k + \mathbb{1}_S(l)f_S + \mathbb{1}_X(x)f_X.$$  \hfill  (2)

The assumption $0 < f_O < f_V$ has been used extensively in the previous literature ranging from Grossman and Helpman (2002) and AH to Antràs and Chor (forthcoming) and we simply follow suit. $\mathbb{1}_S(l)$ and $\mathbb{1}_X(x)$ are indicator functions for offshoring and exporting, respectively.\(^{11}\) As in Amiti and Davis (2011), offshoring and exporting both imply discrete increases in fixed costs since $f_S, f_X > 0$. All fixed costs are denominated in northern labour.

3 The Decision

Under exporting, the final-good production is distributed across the two northern markets. The final-good producer, $H$, faces the following problem of maximising total revenue for a given level of production,

$$R(\theta, \eta, h, m, x; A) = \max_{q_D, q_X} \left[ A^{1/\sigma} \left[ q_D^\alpha + \mathbb{1}_X(x) \left( q_X/\tau \right)^\alpha \right] \right]$$

s.t. $q_D + q_X \leq \theta \zeta h^\eta m^{1-\eta}$,

where $q_D$ and $q_X$ denote the quantities produced for, respectively, the domestic and the export markets when only $q_X/\tau$ units of final goods arrive for sale on the export market. The optimal allocation entails a revenue of

$$R(\theta, \eta, h, m, x; A) = A^{1/\sigma} \theta^\alpha h^{\alpha \eta} m^{\alpha (1-\eta)} \zeta^\alpha (1 + \tau^{1-\sigma}) \mathbb{1}_X(x)^{(1-\alpha)}. \hfill  (3)$$

The next step is to analyse the equilibrium investments in $h$ and $m$. To this end, we impose an assumption of complete asset specificity meaning that the inputs, $h$ and $m$, are completely tailored to the production of the particular variety under scrutiny and useless elsewhere. Importantly, we assume that only the decision, $klx$, and not the subsequent production of $h$ and $m$, is contractible.\(^{12,13}\) After $klx$ is chosen, $H$ and $M$ simultaneously and noncooperatively determine

\(^{11}\)That is, $\mathbb{1}_S(S) = \mathbb{1}_X(X) = 1$ and $\mathbb{1}_S(N) = \mathbb{1}_X(D) = 0$.

\(^{12}\)Since Coase (1937), it has been known that firm boundaries are indeterminate in a world of complete contracts. Because we want to determine firm boundaries, we resort to an assumption of incomplete contracting where input investments are ex-post observable to the transacting parties but not verifiable by third parties. For an interesting extension into partial contractibility, see Antràs and Helpman (2008).

\(^{13}\)AH assume that $k$ and $l$ are contractible sub-decisions but their model does not include the possibility of exporting. We assume that parties can contract on export status as exports usually leave a paper trail. This trail is e.g. created from dealings with customs and shipping agencies.
their respective relationship specific investments, \( h \) and \( m \), while foreseeing future Nash bargaining over final-good revenue. Through a process of Nash bargaining, \( H \) reaps the fraction \( \beta_k \) of equilibrium revenue while \( M \) reaps the complementary fraction. Following Grossman and Hart (1986) and Antràs (2005), we impose the following assumption about relative bargaining power.\(^{14}\)

\[
\beta_V > \beta_O = \frac{1}{2}.
\] (4)

The inequality in (4) and the implied input-investment incentives are key for the tradeoff between integration and outsourcing. More on this below. The well-known intuition behind (4) is that integration entails more extensive property rights for \( H \) and thus improved bargaining power. When choosing their investments in \( h \) and \( m \), \( H \) and \( M \) foresee the share of the revenue, (3), they will receive as a consequence of the Nash bargaining. Hence, by backwards induction, the optimal decision, \( klx \), solves the programme,

\[
\max_{k \in \{O,V\}, l \in \{N,S\}, x \in \{D,X\}} \ R(\theta, \eta, h_{klx}, m_{klx}, x; A) - h_{klx} - w_l m_{klx} - f_{klx}
\] (5)

s.t. \( h_{klx} = \arg \max_h \beta_k R(\theta, \eta, h, m_{klx}, x; A) - h \),

\( m_{klx} = \arg \max_m (1 - \beta_k) R(\theta, \eta, h_{klx}, m, x; A) - w_l m \).

In (5), we have implicitly used that \( H \) extracts all rents from \( M \) through a participation fee which assures that \( M \) is left at its outside option of zero. Consequently, the decision simply maximises joint bilateral profits. By combining (3) with the Nash equilibrium input investments from the constraints in (5) and the fixed costs of production in (2), we arrive at the profits of \( H \) as a function of the decision \( klx \),

\[
\pi_{klx}(\Theta, \eta; A) = A \Theta \psi_k(\eta) \gamma_l(\eta)(1 + \tau^{1-\sigma})^{l(x)} - f_{klx},
\] (6)

where \( \Theta = \theta^{\sigma - 1}, \gamma_l(\eta) = w_l^{(1-\eta)(1-\sigma)}, \) and

\[
\psi_k(\eta) = \frac{1 - \alpha[\beta_k \eta + (1 - \beta_k)(1 - \eta)]}{[\frac{1}{\alpha}]^\beta_k (1 - \eta)^{(1-\eta)\sigma - 1}}.
\]

Note that \( \gamma_N = 1 \) and \( \psi_O = (1 - \alpha/2)(\alpha/2)^{\sigma - 1} \) are independent of \( \eta \). Immediately upon entry and the realisation of \( (\theta, \eta) \), \( H \) chooses the decision, \( klx \), that maximises (6) given that these profits are positive. In case optimal profits,

\[
\pi^*(\Theta, \eta; A) = \max_{klx} \pi_{klx}(\Theta, \eta; A),
\] (7)

\(^{14}\)AH provide microfoundations for the inequality in (4) which is based on differences in outside options. In their notation, we assume that \( \delta^N = \delta^S \), i.e., an equally large fraction of final-good production is lost under bargaining break-down under integration whether or not offshoring occurs. This simplifying assumption is shared with Antràs (2003). AH assume that \( \delta^N \geq \delta^S \), i.e., break-down is no less costly under offshoring.
are negative, \( H \) exits the industry and forfeits the fixed cost of entry. We assume that, for all \( \eta \), at least some low-productivity firms choose not to produce. In equilibrium, the following free-entry condition holds since the pool of potential entrants is unbounded.

\[
f_E = \int \int \max\{0, \pi^*(\Theta, \eta; A)\} \, dF(\theta) \, dG(\eta). \tag{8}
\]

The industry’s demand level, \( A \), is implicitly determined by (8) as a function of all parameters.\(^{15}\)

### 3.1 Input Sourcing and Headquarter Intensity

Before discussing the sorting pattern and the results of our model, we want to make clear how headquarter intensity affects the attractiveness of offshoring and integration. First, we notice that variable profits, \( A\Theta \psi_k(\eta)\gamma_k(\eta)(1 + \tau^{1-\sigma})^1 \times(x) \), are always increasing in offshoring because \( \gamma_S(\eta) > \gamma_N = 1 \) for \( \eta \in (0, 1) \). The reason is that offshoring lowers the marginal cost of the intermediate-input production. All else equal, the gains from offshoring are higher the smaller is \( \eta \) since a lower headquarter intensity implies that the intermediate good is more important in the production of the final good. Formally, \( \gamma_S(\eta) \) is continuous and strictly decreasing in \( \eta \) with \( \gamma_S(\eta) \to 1 \) as \( \eta \to 1 \).

Next, we take a careful look at the decision to integrate. The equilibrium input investments in \( h \) and \( m \) are always suboptimally low compared to the first best, perfect-contracting, input investments. This holds because \( H \) and \( M \) each cover the full marginal costs of their investments while they reap only a fraction of the marginal gains from these investments; see (5). The result is that either ownership structure obtains only a fraction of the variable profits that would arise under perfect contracting. The factor \( \psi_k(\eta) \) in variable profits represents the efficiency of the relationship between \( H \) and \( M \). This depends on the choice of \( k \) since integration assigns \( H \) a larger share of revenue and \( M \) a smaller share compared to outsourcing. Thus, integration improves \( H \)'s incentive to invest in \( h \) but worsens \( M \)'s incentives to invest in \( m \). Consequently, integration is a more efficient relationship than outsourcing when headquarter intensity is high but a less efficient relationship when headquarter intensity is low. This is the intuition behind the following lemma which is based on Proposition 3 and its proof in Antràs and Helpman (2008).

\(^{15}\)Given the implicit restriction on the distribution of productivities that expected profits are finite, the existence and uniqueness of an equilibrium and \( A \) follow from the continuity and strict monotonicity of (7) in \( A, \pi^* < 0 \) when \( A = 0, \pi^* \to \infty \) when \( A \to \infty \), and the intermediate value theorem.
Lemma 1. $\psi_V(\eta)$ is continuous and strictly increasing in $\eta$ and there exists a unique headquarter intensity, $\eta^* \in (0, 1)$, where $\psi_O = \psi_V(\eta^*)$. Thus, $\eta > \eta^* \Leftrightarrow \psi_V(\eta) > \psi_O$ and $\eta < \eta^* \Leftrightarrow \psi_V(\eta) < \psi_O$.

Note that offshoring involves a tradeoff between higher fixed costs and lower marginal costs for all values of $\eta$. Integration on the other hand, only involves a tradeoff between higher relationship efficiency and higher fixed costs for $\eta \in (\eta^*, 1)$. For lower headquarter intensities, outsourcing achieves higher relationship efficiency and lower fixed costs than integration. Thus, no firms with $\eta \in (0, \eta^*]$ choose to integrate.

3.2 Complementarities and Sorting

Complementarities between the activities faced by firms are central to our analysis of firms’ decisions. First off, offshoring and exporting are complementary for all firms. This is because exporting involves additional sales and offshoring effectively reduces the marginal cost. Higher sales are worth more when the goods are produced more cheaply and vice versa. Next, consider the interaction between integration and the two other activities. Remember that whenever integration is considered by firms ($\eta > \eta^*$), it is because integration lets the firms obtain a larger share of the variable profits that would be possible under perfect contracting. Reducing the share of variable profits lost due to incomplete contracting is obviously worth more when these variable profits are higher. This is the case both when serving an additional market through exporting and when the marginal cost is lowered through offshoring. In total, whatever the $\eta \in (0, 1)$, the activities that firms actually consider are complementary (fixed costs are additively separable).\footnote{Formally, imposing the partial ordering, $V > O$, $S > N$, and $X > D$, the complementarities discussed follow from the profit function being supermodular in $lx$ for $\eta \in (0, \eta^*]$ and in $klx$ for $\eta \in (\eta^*, 1)$.}

Undertaking one activity increases the gains from undertaking others. It should be noted that due to sorting considerations, integration and offshoring are not modelled to be complementary in the AH model. However, such a complementarity arises quite naturally when focusing on the essence of these activities, cf. the discussion above.\footnote{Compared to AH, we obtain a supermodular profit function through the combination of additively separable fixed costs and the parameter restriction $\delta_N = \delta_S$ mentioned in footnote 14. When $\delta_N = \delta_S$, the sorting pattern presented by AH can only be achieved if the fixed costs of the firm are strictly supermodular in the offshoring and integration decision.}

The activities integration, offshoring, and exporting are not only complementary to each other, they are also complementary to productivity. Integration (when considered) and exporting are complementary to productivity for the same reason that these activities are complementary to offshoring; lower marginal costs
increase the gains from these activities. Offshoring is complementary to productivity since the isoelastic demand function means that scaling down marginal cost by a given factor, as implied by offshoring, is worth more when productivity is high.\textsuperscript{18,19}

These properties of the profit function imply that given the headquarter intensity, $\eta$, the sorting of firms into activities obeys a cutoff rule. For all $\eta \in (0, 1)$, there exist productivity cutoffs for offshoring and exporting such that all firms with higher (lower) productivities do (do not) undertake the activity in question. In addition, there exists a similar productivity cutoff for integration for $\eta \in (\eta^*, 1)$. Conditioning on $\eta$, the three activities will therefore each be associated with a productivity premium consistent with empirical studies.\textsuperscript{20} We assume that, for all $\eta$, not all firms undertake either integration, offshoring, or exporting. Hence, as in AH, the least productive active firms choose the decision $klx = OND$. Further, in order to observe a wide variety of organisational forms, we assume that for $\eta$ close to 0 and 1, the productivity cutoff for offshoring and integration, respectively, is lower than that for exporting.\textsuperscript{21}

### 3.3 Intra-Firm Importing

Interestingly, our model delivers a pattern of firm sorting into activities that is broadly consistent with two empirical findings of Corcos et al. (forthcoming) who use firm-level import data from France to investigate the determinants of the choice between intra-firm and arms’-length importing. First, they show that, conditional on headquarter intensity (proxied by capital and skill intensity) and offshoring, higher firm-level productivity makes intra-firm importing more likely relative to arms’-length importing. Second, they find that conditional on firm-

\textsuperscript{18}See Mrazova and Neary (2013) for a detailed discussion of the complementarity between offshoring and productivity in the AH model.

\textsuperscript{19}Formally, the complementarity between productivity and either offshoring or exporting is seen from the profit function having increasing differences in $(l, x; \theta)$ when we use the partial ordering of activities mentioned in footnote 16. The complementarity between productivity and integration on $(\eta^*, 1)$ follows from the profit function having increasing differences in $(k; \theta)$ on this interval.

\textsuperscript{20}There is a productivity premium associated with integration in data; see Federico (2010) and Kohler and Smolka (2011). For evidence about a size and age premium for integration, see Acemoglu et al. (2010). Importers of intermediate goods are more productive than nonimporters; see Bernard et al. (2012) who also discuss the well-established exporter productivity premium.

\textsuperscript{21}Loosely speaking, this means that when integration and offshoring provide the greatest gains for firms, these activities are more attractive than exporting. Thus, exporting is sufficiently expensive not to be the most attractive activity for all headquarter intensities. This guarantees that we will observe nonexporters which offshore and nonexporters which integrate in equilibrium. This assumption is not central for our results; see Section 4.
level productivity and offshoring, higher firm-level headquarter intensity makes intra-firm importing more likely relative to arms'-length importing. Corcos et al. (forthcoming) interpret these empirical results as firm-level support for the key predictions of the open-economy property rights theory of the firm concerning intra-firm importing.

The first empirical finding of Corcos et al. (forthcoming) squares perfectly with the cutoff rule described above and the implied sorting pattern in the productivity dimension. The compliance of our model with the second empirical finding mentioned above is a bit more subtle.

**Proposition 1.** Consider the sorting of firms into decisions in a given equilibrium. Among the offshoring firms with a given headquarter intensity, the share that also integrates is higher the higher is the headquarter intensity. However, considering firms with a given productivity, we may observe that one firm undertakes arms'-length importing while having higher headquarter intensity than another firm that undertakes intra-firm importing.

*Proof.* See Appendix B.

The first part of Proposition 1 is reminiscent of an industry-level result obtained by AH. In contrast to this, our result covers the full spectrum of headquarter intensities within a single industry. To understand the second part of Proposition 1, consider the choice of integration conditional on offshoring. A higher $\eta$ increases the relationship-efficiency of integration relative to outsourcing, which works in the favour of intra-firm importing. However, a higher $\eta$ also decreases the attractiveness of offshoring, which reduces the incentive to integrate due to the complementarity. If this latter effect is sufficiently strong, it is indeed possible that a higher $\eta$ means that firms shift from intra-firm importing to arms'-length importing.\footnote{The adverse complementarity effect is smaller when the North-South wage gap decreases. Hence, the sorting pattern of firms in our model is more likely to resonate perfectly with the second finding of Corcos et al. (forthcoming) when the wage gap becomes smaller.} Proposition 1 illustrates that one has to be careful when applying the conventional intuition about the relation between $\eta$ and integration, which was discussed in Section 3.1, to a firm-level analysis of intra-firm importing in an open-economy context.\footnote{We note that Proposition 1 is not dependent on the inclusion of exporting into the model. This follows from Appendix B.}

### 4 Comparative Statics

Apart from generating a two-dimensional pattern of firm sorting that is broadly consistent with empirical evidence, our model gives rise to strong predictions based on comparative static analysis of the industry composition.
**Definition 1.** The prevalence of a given activity is the share of active final-good firms that pursue this activity. The prevalence of vertical FDI is the share of active final-good firms that both integrate and offshore.

**Proposition 2.** Reductions in \( (f_V, w_S, f_S, \tau, f_X) \) imply that the prevalences of integration, offshoring, and exporting strictly increase. Reductions in \( (f_V, w_S, f_S, \tau) \) imply that the prevalence of vertical FDI strictly increases.

**Proof.** See Appendix A.

Proposition 2 illustrates a strong industry-level interdependence among the activities arising from the firm-level complementarities. In general, increasing the attractiveness of any of the three activities makes the industry composition of firms shift towards all three activities becoming more prevalent. These results may seem to be an obvious consequence of the complementarities of the model. That the results are by no means trivial is illustrated by the following proposition.

**Proposition 3.** Reducing the costs associated with any one of the activities integration, offshoring, or exporting induces some individual firms to shift away from the other two activities.

**Proof.** See Appendix C.

Our main findings in Propositions 2 and 3 relate to the ongoing discussion about the relation between trade liberalisation and integration. In this strand of literature, the results vary almost as much as the modelling strategies. Our contribution in this regard is twofold. First, we unveil a clear positive relation between trade liberalisations and integration at the industry level. In particular, reductions in fixed or variable costs of trade in final or intermediate goods induce a shift in the industry composition towards integration (Proposition 2). Second, this is not incompatible with shifts towards outsourcing at the firm level (Proposition 3).

To see how Propositions 2 and 3 are compatible, consider trade liberalisation. Apart from their direct effects on firms’ decisions (for a given \( A \)), trade liberalisations result in fiercer competition (a lower equilibrium demand level, \( A \)). The reason is that, given the demand level, liberalisation of either kind increases the profits for all firms weakly and for some firms strictly. Thus, for the free-entry condition, (8), to hold, competition must toughen and the demand level, \( A \), falls. This tends to reduce the size of firms and thereby discourage integration. However, it also forces the least productive firms, which do not integrate, to shut

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\[ \text{Reduction the variable cost of intermediate-goods trade corresponds to lowering } w_S = \tau_m \tilde{w}_S. \]
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down. With a nonincreasing hazard rate of log-productivities, the net result is an increase in the prevalence of integration.\footnote{For further discussion, we refer to Bache and Laugesen (2013).}

Even though Proposition 2 implies a clear positive relation between all four types of trade liberalisation and integration, it should be noted that the particular type of trade liberalisation can matter for the effect on integration once we address the share of firms which undertake northern integration, i.e., the share of active final-good firms that choose the strategy $VNx$, $x \in \{D, X\}$. The formal result, which is stated below, again illustrates the point that firm-level complementarities may manifest themselves more clearly at higher levels of aggregation.

**Proposition 4.** Let productivities be Pareto distributed. Then liberalisations of intermediate-goods trade reduce the prevalence of northern integration while liberalisations of final-goods trade may increase the prevalence of northern integration.

**Proof.** See Appendix D.

While the first part of Proposition 4 is in accordance with a finding of AH, it is important to note that our main result in Proposition 2 is not. AH find that the prevalence of outsourcing rises when intermediate-goods trade is liberalised ($w_S$ decreases). This incongruity does not depend on our inclusion of exporting, so we abstract from this activity for the moment. In order to accord with what is observed in data, a model has to generate an equilibrium where all possible combinations of the outsourcing and offshoring decisions, $kl \in \{ON, OS, VN, VS\}$, are observed. Since AH want to achieve this (in headquarter-intensive industries) with productivity as the only source of heterogeneity, they cannot allow integration and offshoring to be complementary at the firm level. Letting firms be heterogeneous with respect to both productivity and headquarter intensity means that we are able to analyse the implications of a clear-cut complementarity between the sourcing decisions faced by firms while generating the desired diversity of observed organisational forms. We contribute by showing that this plausible complementarity implies an unambiguously positive relationship between trade liberalisations and the prevalence of integration.

Finally, we analyse the effects of a change in the barriers to entry through a change in $f_E$. Like the previously analysed trade liberalisations, a decrease in $f_E$ implies a fall in the demand level, $A$, due to free entry. As a consequence, the productivity cutoffs for integration, offshoring, exporting, and vertical FDI increase meaning that firms shift away from these activities. However, as before, low-productivity firms are being pushed out and the nonincreasing hazard rate of log-productivities imply that the net effects on the prevalences of the activities are positive. Further, if the distribution of log-productivity has strictly decreasing
hazard rate, the prevalences mentioned above strictly increase; see Bache and Laugesen (2013). These results relate to an existing literature on the relation between competition and integration mentioned in footnote 2 and illustrate again the discrepancy between firm- and industry-level comparative statics.

Although our assumptions are not necessarily controversial, let us briefly discuss how certain assumptions can be relaxed without affecting Proposition 2 crucially. First off, assume that outsourcing does not result in symmetric but rather generalised Nash bargaining, i.e., $\beta_O$ is not necessarily one half. Proposition 2 still holds in this case if one assumes that productivities are Pareto distributed.\footnote{The reason is that, in this case, $\psi_O$ depends on $\eta$ which means that the exit productivity cutoff does so as well. Then we need a constant hazard rate of the log-productivity distribution to use the results of Bache and Laugesen (2013) and argue that the prevalence of any given activity is nondecreasing in $\nu \equiv (-f_V, -w_S, -f_S, -\tau, -f_X)$ which is the first step of the proof in Appendix A. From here, the second step proceeds as before.} Next, relaxing the assumption that $G(\eta)$ is strictly increasing simply means that we cannot be sure that the effects in Proposition 2 are strictly positive. Nevertheless, we still know that they are nonnegative, cf. Appendix A. Dispensing with the assumption that some firms with high and low $\eta$ integrate and offshore, respectively, without exporting simply means that we cannot be sure that the effects of reducing $f_X$ on the prevalences of integration and offshoring are strictly positive, nor that the effect on the prevalence of exporting of reducing $(f_V, w_S, f_S)$ is strictly positive. These effects will however still be nonnegative while all other strict results still hold. Finally, it should also be noted that, if one removes one of the activities integration, offshoring, or exporting from the choice set of firms, Proposition 2 is still valid for the remaining activities and the relevant cost reductions.\footnote{This can be shown by repeating the steps of the proof in Appendix A ignoring one of the activities.} Thus, the strong industry-level interdependence of e.g. exporting and integration implied by Proposition 2 is not lost by not allowing firms to offshore.

\section{Concluding Remarks}

Our main contribution is to obtain strong and testable results about the interdependencies among integration, offshoring, exporting, and vertical FDI at the industry level of analysis. Of particular interest in light of the existing literature is the clear positive relationship between trade liberalisations and the prevalence of vertical integration. Notably, these results are compatible with ambiguities at lower levels of aggregation. Central to our analysis are the firm-level complementarities we identify. Apart from the introduction of exporting, our model is a natural extension of Antràs and Helpman (2004) for the following reasons. First, we allow firms within the same industry to be heterogeneous with respect
to both productivity and headquarter intensity. This is a response to recent empirical evidence which reveals that industry is a poor indicator for headquarter intensity and that firm-level headquarter intensity is an important determinant of the decision to vertically integrate. Allowing for heterogeneity in headquarter intensity is shown to be relatively uncomplicated even for a general distribution of headquarter intensities. We believe that the inclusion of firm-specific headquarter intensities makes the open-economy property rights theory of the firm more suitable for future empirical tests based on the firm-level data sets with input-sourcing information which are starting to appear. Reassuringly, we obtain a sorting pattern broadly in line with Corcos et al. (forthcoming). Further, we emphasise that one has to be careful when applying the conventional intuition about the relationship between headquarter intensity and vertical integration at the firm level in an open-economy context. Second, we dispense with the common assumption of Pareto-distributed firm productivities and utilise instead a broader class of distribution functions to which the Pareto distribution belongs.

A Proof of Proposition 2

As an intermediate step, we establish that increases in \( \nu \equiv (-f_V, -w_S, -f_S, -\tau, -f_X) \) lead to nondecreasing prevalences of integration, offshoring, and exporting. This is done by drawing upon the results, in particular Proposition 3, of Bache and Laugesen (2013) (henceforth BL). To do so, we adopt the following ranking of the values of the three choice variables in \( klx \): \( V > O \), \( S > N \), and \( X > D \). Remember that the profit function reads

\[
\pi_{klx}(\Theta, \eta; A, \nu) = A\Theta\psi_k(\eta)\gamma_l(\eta)(1 + \tau^{1-\sigma})^{1-x(x)} - f_{klx}.
\]  

Note that any increase in \( \nu \) must imply that \( A \) falls since the profits of all firms weakly increase given the level of \( A \), the profits of some firms strictly increase given \( A \), and we have free entry. In the terminology of BL, an increase in \( \nu \) leads to comparative statics using the carrot. Our assumption that the least productive active firms with any \( \eta \) choose \( klx = OND \) means that the productivity cutoff for being active,

\[
\Theta_{exit}(\eta; A, \nu) \equiv \inf\{ \Theta : \pi_{OND}(\Theta, \eta; A, \nu) > 0 \} = \frac{f_O}{A\psi_O},
\]

is invariant across \( \eta \) and importantly, is not affected directly by changes in \( \nu \) but only indirectly through changes in \( A \).

Let \( s_j^{\eta \leq \eta^*} \) and \( s_j^{\eta > \eta^*} \) be the shares of active final-good firms with \( \eta \) less than or greater than \( \eta^* \), respectively, that end up undertaking activity \( j \in \{V, S, X\} \).
The overall share of active firms undertaking activity \( j \) is then given by
\[
s_j = G(\eta^*) s_j^{\eta \leq \eta^*} + [1 - G(\eta^*)] s_j^{\eta > \eta^*}.
\]
Since \( \eta^* \) is not affected by changes in \( \nu \), we have shown that \( s_j \) is nondecreasing in \( \nu \) if we can establish that \( s_j^{\eta \leq \eta^*} \) and \( s_j^{\eta > \eta^*} \) are nondecreasing in \( \nu \). Consider first the latter. For \( \eta > \eta^* \), the profit function, (9), is supermodular in \((k, l, x)\) and has increasing differences in \((k, l, x; \Theta)\), \((k, l, x; A)\), and \((k, l, x; \nu)\). Since our setup conforms to all the other conditions for invoking Proposition 3 of BL for \( \eta > \eta^* \), we conclude that \( s_j^{\eta > \eta^*} \) is nondecreasing in \( \nu \) with \( j \in \{V, S, X\} \). Next, consider \( s_j^{\eta \leq \eta^*} \). For these values of \( \eta \), we know that no firms choose integration and we can treat \( k \) as exogenously fixed at \( k = O \). Thus \( s_j^{\eta \leq \eta^*} = 0 \). Further, for \( \eta \leq \eta^* \), the profit function with \( k = O \) is supermodular in \((l, x)\) and has increasing differences in \((l, x; \Theta)\), \((l, x; A)\), and \((l, x; \nu)\). We can therefore use Proposition 3 of BL to conclude that \( s_j^{\eta \leq \eta^*} \) with \( j \in \{V, S, X\} \) is indeed nondecreasing in \( \nu \) as well. It follows that \( s_j \) is nondecreasing in \( \nu \) for \( j \in \{V, S, X\} \).\(^{28}\)

The second step in this proof is to show that \( s_j, j \in \{V, S, X\} \), is strictly increasing in \( \nu \). For this purpose we note that following BL, the effect of \( \nu \) on \( s_j \) can be split into two parts: the total direct effect of changes in \( \nu \) (for a given \( A \)) and the total indirect effect due to the change in \( A \). Both of these are nonnegative. This follows from BL and was implicitly used in the first step of this proof.\(^{29}\) We will show that the effect of \( \nu \) on \( s_j \) is strictly positive by arguing that the total direct effect is strictly positive. Since the direct effect of \( \nu \) on the productivity cutoff for being active is zero as argued above,\(^{30}\) the total direct effect on \( s_j \) is determined by the direct effect on the share of all firms (not just active) that undertake activity \( j \), i.e., the direct level effect in the language of BL. This share is determined by a cutoff, \( \Theta_j(\eta; A, \nu) \), giving the threshold productivity for undertaking activity \( j \) depending on the headquarter intensity, \( \eta \).\(^{31}\) The direct effects of increases in \( \nu \) on the cutoffs \( \Theta_V(\eta; A, \nu) \), \( \Theta_S(\eta; A, \nu) \), and \( \Theta_X(\eta; A, \nu) \) are nonpositive for all \( \eta \); see BL. Hence, if we can establish that these effects are strictly negative, each for an interval of \( \eta \), we are done.\(^{32}\)

First, we establish that \( \Theta_V, \Theta_S, \) and \( \Theta_X \) are continuous functions of \( \eta \) on the intervals \((\eta^*, 1), (0, 1)\), and \((0, 1)\), respectively. We will only provide details

\(^{28}\)The assumption that \( G(\eta) \) is strictly increasing is redundant for this intermediate result.

\(^{29}\)Following BL, it is easy to show that \( s_j, j \in \{V, S, X\} \), is strictly increasing in \( \nu \) when the distribution of \( \log \Theta \) (\( \log \theta \)) has a strictly decreasing hazard rate. We would however like to allow for Pareto-distributed productivities, \( \theta \), i.e., a distribution of \( \log \Theta \) with constant hazard rate.

\(^{30}\)In the language of BL, the direct selection effect is zero.

\(^{31}\)Note that \( \Theta_V \) is infinite for \( \eta \in (0, \eta^*) \). The following discussion of \( \Theta_V \) will therefore only concern its behaviour on \((\eta^*, 1)\).

\(^{32}\)Remember that \( G(\eta) \) is strictly increasing on \( \eta \in (0, 1) \).
on how to show this for $\Theta_s$ as showing it for $\Theta_s$ and $\Theta_x$ will be completely analogous. Let us define $\Theta_s$ properly. To do so, let

$$\pi_k(\Theta, \eta; A, \nu) = \max_{lx} \pi_{klx}(\Theta, \eta; A, \nu).$$

Note that $\pi_k$ is continuous in $(\Theta, \eta)$ since $\pi_{klx}$ is. Now, on $(\eta^*, 1)$, $\Theta_V(\eta; A, \nu)$ is given by

$$\pi_V(\Theta, \eta; A, \nu) - \pi_O(\Theta, \eta; A, \nu) = 0. \quad (10)$$

Since the LHS of (10) is continuous in $(\Theta, \eta)$ and strictly increasing in $\Theta$, it follows from the implicit function theorem that $\Theta_V$, as determined by (10), is continuous in $\eta$.

Next, we establish that $\Theta_V \rightarrow \infty$ as $\eta \rightarrow \eta^*$ from above and $\Theta_V$ is bounded from above as $\eta \rightarrow 1$. Let $\Theta_V|lx(\eta; A, \nu)$ be implicitly defined by the equation

$$\pi_V|lx(\Theta_V|lx, \eta; A, \nu) - \pi_{Olx}(\Theta_V|lx, \eta; A, \nu) = 0. \quad (12)$$

Let $\Theta_V(\eta; A, \nu) = \min_{lx} \Theta_V|lx$. Then for firms with productivities below $\Theta_V$, outsourcing is the optimal ownership structure since regardless of the firms’ choices of $lx$, profits are higher under $O$ than under $V$. This is due to the LHS of (12) being strictly increasing in $\Theta$. Thus, $\Theta_V$ represents a lower bound on $\Theta_V$ for all $\eta$. In a similar fashion, $\Theta_V = \max_{lx} \Theta_V|lx(\eta; A, \nu)$ is an upper bound on $\Theta_V$. Now, since the LHS of (12) is increasing in $lx$ and $\Theta$ by the supermodularity and increasing differences properties of $\pi_{klx}$, respectively, we have $\underline{\Theta}_V = \Theta_V|sX$ and $\bar{\Theta}_V = \Theta_V|ND$. Since the lower bound on $\Theta_V$, $\Theta_V|sX \rightarrow \infty$ as $\eta \rightarrow \eta^*$ from above, the same holds for $\Theta_V$. Further, as the upper bound on $\Theta_V$, $\Theta_V|ND$ is bounded from above as $\eta \rightarrow 1$, so is $\Theta_V$. By similar lines of arguments, one can establish that $\Theta_S \rightarrow \infty$ as $\eta \rightarrow 1$, $\Theta_S$ is bounded from above as $\eta \rightarrow 0$, and that $\Theta_X$ is bounded from above both as $\eta \rightarrow 0$ and as $\eta \rightarrow 1$.

---

33That the LHS of (10) is weakly increasing in $\Theta$ follows from $\pi_{klx}$ having increasing differences in $(k, l, x; \Theta)$ since this implies that $\pi_k$ has increasing differences in $(k; \Theta)$. That it is in fact strictly increasing can be seen by writing out (10) to get

$$A\Theta[\psi_V(\eta)\gamma_{l_1}(\eta)(1 + \tau^{1-\sigma})^{l_1x_1} - \psi_O\gamma_{l_2}(\eta)(1 + \tau^{1-\sigma})^{l_2x_2}] - (f_{Vl_1x_1} - f_{Ol_2x_2}) = 0, \quad (11)$$

where $l_1x_1$ and $l_2x_2$ are the optimal choices of $lx$ under $k = V$ and $k = O$, respectively. Since $V > O$, we have $l_1 \geq l_2$ and $x_1 \geq x_2$. Further, since $\psi_V(\eta) > \psi_O$ for $\eta \in (\eta^*, 1)$, the square bracket on the LHS of (11) is strictly positive on $(\eta^*, 1)$ and thus the LHS of (10) is strictly increasing in $\Theta$ on $(\eta^*, 1)$.

34The intuition for $\min_{lx} \Theta_V|lx = \Theta_V|sX$ (max_{lx} $\Theta_V|lx = \Theta_V|ND$) is that integration is promoted as much (little) as possible by other complementary activities in this case.

35Since $\Theta_V|sX = \frac{f_V - f_O}{(\psi_V(\eta) - \psi_O)}$ and $|\psi_V(\eta) - \psi_O| \rightarrow 0$ as $\eta \rightarrow \eta^*$, it follows that $\Theta_V|sX \rightarrow \infty$ as $\eta \rightarrow \eta^*$ from above. Further, since $\Theta_V|ND = \frac{f_V - f_O}{(\psi_V(\eta) - \psi_O)}$, $\psi_V(\eta) > \psi_O$ for $\eta > \eta^*$, and $\psi_V(\eta)$ is increasing in $\eta$, $\Theta_V|ND$ is bounded from above as $\eta \rightarrow 1$. 

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Our assumption that some firms with \( \eta \) close to 0 choose to offshore without exporting implies that \( \Theta_S < \Theta_X \) for \( \eta \) sufficiently close to 0. Further, the assumption that some firms with \( \eta \) close to 1 choose to integrate without exporting implies that \( \Theta_V < \Theta_X \) for \( \eta \) sufficiently close to 1. Combined with the properties derived above an application of the intermediate value theorem implies that \( \Theta_V \) and \( \Theta_X \), \( \Theta_V \) and \( \Theta_S \), and \( \Theta_S \) and \( \Theta_X \) intersect in the interior of \((\eta^*, 1), (\eta^*, 1)\), and \((0, 1)\), respectively. This means that \( V \) and \( S \) share the same cutoff productivity for at least one value of \( \eta \), as does \( V \) and \( X \) and \( S \) and \( X \). The final steps of the proof show that this implies that these joint cutoffs must each be relevant for an interval of \( \eta \), and that for each of the activities, \( j \in \{V, S, X\} \), at least one of its joint cutoffs is strictly decreasing conditional on \( A \) when \( \nu \) increases.

Consider the joint cutoffs for activity \( V \). First off, we have established that for \( \eta \) sufficiently close to \( (\text{but above}) \) \( \eta^* \), \( \Theta_V > \Theta_S, \Theta_X \) and that for \( \eta \) sufficiently close to 1, \( \Theta_V < \Theta_S, \Theta_X \). That is, \( \Theta_V \) must be equal to respectively \( \Theta_S \) and \( \Theta_X \) for at least one value of \( \eta \). Suppose that \( \Theta_V \) and \( \Theta_S \) not equal on an interval of \( \eta \). Then there must exist an \( \eta = \eta' \) for which \( \Theta_V(\eta'; A, \nu) = \Theta_S(\eta'; A, \nu) = \Theta' \) with \( \Theta_V > \Theta_S \) for \( \eta \) just below \( \eta' \) and \( \Theta_V < \Theta_S \) for \( \eta \) just above \( \eta' \). First, suppose that \( \Theta_X(\eta'; A, \nu) \neq \Theta' \). That is, for \( (\Theta, \eta) \) sufficiently close to \( (\Theta', \eta') \) all firms choose the same export status, \( x \). Then we must have that \( \pi_{V_{Sx}}(\Theta', \eta'; A, \nu) = \pi_{O_{Sx}}(\Theta', \eta'; A, \nu) = \pi_{V_{N_{x}}}(\Theta', \eta'; A, \nu) \). However, this cannot be the case since \( \pi_{O_{Sx}} - \pi_{O_{N_{x}}} = 0 \) implies that \( \pi_{V_{Sx}} - \pi_{V_{N_{x}}} > 0 \). Second, suppose \( \Theta_X(\eta'; A, \nu) = \Theta' \). Then we must have that

\[
\pi_{V_{Sx}}(\Theta', \eta'; A, \nu) = \pi_{O_{N_{x}}}(\Theta', \eta'; A, \nu) = \pi_{V_{N_{x}}}(\Theta', \eta'; A, \nu) = \pi_{O_{S_{x}}}(\Theta', \eta'; A, \nu)
\]

for some \( x_1, x_2 \). But as before, this cannot be true. To see why, suppose (13) holds for \( x_1 = D \) and \( x_2 = X \). Then \( \pi_{O_{S_{x}}} - \pi_{O_{N_{x}}} = 0 \) implies that \( \pi_{V_{S_{x}}} - \pi_{V_{N_{x}}} > 0 \) and we have a contradiction. If (13) holds for \( x_1 = X \) and \( x_2 = D \), then \( \pi_{V_{N_{x}}} - \pi_{O_{N_{x}}} = 0 \) implies \( \pi_{V_{S_{x}}} - \pi_{O_{S_{x}}} > 0 \) and again we have a contradiction. If \( x_1 = x_2 = D \), \( \pi_{V_{N_{x}}} - \pi_{O_{N_{x}}} = 0 \) implies \( \pi_{V_{S_{x}}} - \pi_{O_{S_{x}}} > 0 \). But this means that \( \pi_{V_{S_{x}}}(\Theta', \eta'; A, \nu) \) is strictly higher than \( \pi_{O_{N_{x}}}(\Theta', \eta'; A, \nu) \), and due to continuity, firms with \( \eta = \eta' \) and \( \Theta \) just below \( \Theta' \) find \( kx = VSD \) more profitable than \( kx = O_{N_{x}} \). But these firms should choose \( O_{N_{x}} \) if \( \Theta_V(\Theta', \eta'; A, \nu) = \Theta_S(\Theta', \eta'; A, \nu) = \Theta_X(\Theta', \eta'; A, \nu) = \Theta' \) and we have a contradiction. Finally, if \( x_1 = x_2 = X \), then \( \pi_{V_{S_{x}}} - \pi_{V_{N_{x}}} = 0 \) implies \( \pi_{O_{S_{x}}} - \pi_{O_{N_{x}}} < 0 \) but by \( \pi_{O_{S_{x}}} = \pi_{O_{N_{x}}} \), we thus have \( \pi_{O_{N_{x}}} > \pi_{O_{N_{x}}} \), which, by a similar ar-

\[\text{Right at } (\Theta', \eta') \text{ firms are indifferent between } V_{S_{x}} \text{ and } O_{N_{x}} \text{ which gives the first of the equalities. For } \eta \text{ just below } \eta', \Theta_S \text{ gives indifference between } O_{N_{x}} \text{ and } O_{S_{x}}. \text{ As } \Theta_S \text{ is continuous, this indifference at } \Theta_S \text{ extends to } \eta = \eta' \text{ which gives the second equality. The last equality follows from a similar argument using } \Theta_V \text{ for } \eta \text{ just above } \eta'.\]

\[\text{The argument is completely analogous to before.}\]
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gument as before, is incompatible with $\Theta_V(\Theta', \eta'; A, \nu) = \Theta_S(\Theta', \eta'; A, \nu) = \Theta_X(\Theta', \eta'; A, \nu) = \Theta'$. Thus, we must have that $\Theta_V = \Theta_S$ holds for an interval of $\eta$ since otherwise we arrive at a contradiction. Arguments similar to those used above imply that the same must hold for $\Theta_V$ and $\Theta_X$ and for $\Theta_S$ and $\Theta_X$.

To conclude the proof of the first part of the proposition, note that when $\Theta_S = \Theta_V$, this joint cutoff is given by

$$\pi_{V; x_1}(\Theta_V, \eta; A, \nu) - \pi_{ON; x_2}(\Theta_V, \eta; A, \nu) = 0 \quad (14)$$

for some $x_1 \geq x_2$. But the LHS of (14) is strictly increasing in both $\Theta$ and $(-f_V, -w_S, -f_S)$ which means that, given $A$, $\Theta_V$ is strictly decreasing in $(-f_V, -w_S, -f_S)$. This holds for an interval of $\eta$ as argued above. As $\Theta_V = \Theta_S$ on this interval of $\eta$, the same can be said about $\Theta_S$. Further, when $\Theta_V = \Theta_X$, it is given by

$$\pi_{V; l_1, \chi}(\Theta_V, \eta; A, \nu) - \pi_{O; l_2}(\Theta_V, \eta; A, \nu) = 0 \quad (15)$$

for some $l_1 \geq l_2$. As the LHS of (15) is strictly increasing in $\Theta$ and $(-f_V, -\tau, -f_X)$, $\Theta_V$ and $\Theta_X$ are strictly decreasing in $(-f_V, -\tau, -f_X)$, given $A$, on an interval of $\eta$. Finally, when $\Theta_S = \Theta_X$, it is given by

$$\pi_{k_1, S \chi}(\Theta_S, \eta; A, \nu) - \pi_{k_2, ND}(\Theta_S, \eta; A, \nu) = 0 \quad (16)$$

for some $k_1 \geq k_2$. As the LHS of (16) is strictly increasing in $\Theta$ and $(-w_S, -f_S, -\tau, -f_X)$, $\Theta_S$ and $\Theta_X$ are strictly decreasing in $(-w_S, -f_S, -\tau, -f_X)$, given $A$, on an interval of $\eta$. Combining these results leads you to conclude that whenever $\nu \equiv (-f_V, -w_S, -f_S, -\tau, -f_X)$ increases, $\Theta_V, \Theta_S$, and $\Theta_X$ each strictly decreases on some interval of $\eta$ given $A$. As argued above, this gives us the first part of the proposition.

That the prevalence of vertical FDI (the share of all active final-good firms that undertake both $V$ and $S$) is strictly increasing in $(-f_V, -w_S, -f_S, -\tau)$ is quite simple to show at this point. Note that vertical FDI only occurs for $\eta > \eta^*$. First, let us define the productivity cutoff for vertical FDI,

$$\Theta_{VS}(\eta; A, \nu) = \max\{\Theta_V(\eta; A, \nu), \Theta_S(\eta; A, \nu)\}.$$

This productivity cutoff is clearly nonincreasing in $(-f_V, -w_S, -f_S, -\tau)$ given $A$. By the same arguments as above, we need to show that, given $A$, $\Theta_{VS}$ is strictly decreasing in $(-f_V, -w_S, -f_S, -\tau)$ on an interval of $\eta$ in order to prove the last part of the proposition. We have already established that $\Theta_V = \Theta_S$ on an interval of $\eta$ for which they are strictly decreasing in $(-f_V, -w_S, -f_S)$ for a given $A$. This means that the same holds for $\Theta_{VS}$ on this interval of $\eta$. Finally, since $\Theta_V > \Theta_S, \Theta_X$ for all $\eta$ sufficiently close to, but above, $\eta^*$, $\Theta_{VS} = \Theta_{V|S_X}$ for these $\eta$. It is easy to verify that $\Theta_{V|S_X}$ is strictly decreasing in $-\tau$ given $A$, and we are therefore done.

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B Proof of Proposition 1

This proof draws on results and definitions from Appendix A and should be read in extension of this. Denote by \( s_{V|S}(\eta) \) the share of final-good firms with headquarter intensity \( \eta \) which integrate given that they are offshoring. To prove that \( s_{V|S} \) is increasing in \( \eta \), we first show that if \( s_{V|S} \) is one for some \( \eta \), then it must be one for all higher values of \( \eta \). If this is not the case, there must exist an \( \eta' \in (\eta^*, 1) \) such that \( \Theta_S(\eta') = \Theta_V(\eta') = \Theta' \) and for \( \eta \) just above \( \eta' \), \( \Theta_S < \Theta_V \). For \( \eta \) just above \( \eta' \), \( \Theta_S \) must thus be given by

\[
\pi_{OSX_1}(\Theta_S, \eta; A, \nu) - \pi_{ONX_2}(\Theta_S, \eta; A, \nu) = 0
\]

for some \( x_1 \geq x_2 \). This gives us

\[
\Theta_S(\eta; A, \nu) = \frac{f_{OSX_1} - f_{ONX_2}}{A[\gamma_S(\eta)(1 + \tau^{1-\sigma})\mathbf{1}_X(x_1) - (1 + \tau^{1-\sigma})\mathbf{1}_X(x_2)]} \tag{17}
\]

for \( \eta \) at and just above \( \eta' \). Further, we have that \( \pi_{VSX_3}(\Theta', \eta'; A, \nu) = \pi_{OSX_1}(\Theta', \eta'; A, \nu) = \pi_{ONX_2}(\Theta', \eta'; A, \nu) \) where \( VSX_3 \) is undertaken by firms with \( \eta = \eta' \) and productivity just above \( \Theta' \). It must hold that \( x_3 \geq x_1 \). Now, define \( \tilde{\Theta}_V \) as

\[
\pi_{VSX_3}(\tilde{\Theta}_V, \eta; A, \nu) - \pi_{ONX_2}(\tilde{\Theta}_V, \eta; A, \nu) = 0. \tag{18}
\]

This gives us

\[
\tilde{\Theta}_V(\eta; A, \nu) = \frac{f_{VSX_3} - f_{ONX_2}}{A[\gamma_S(\eta)\psi_V(\eta)(1 + \tau^{1-\sigma})\mathbf{1}_X(x_3) - \psi_O(1 + \tau^{1-\sigma})\mathbf{1}_X(x_2)]} \tag{19}
\]

Differentiating (17) and (19) with respect to \( \eta \) gives

\[
\frac{\partial \Theta_S}{\partial \eta} = -\Theta_S \frac{1}{1 - \gamma^{-1}(1 + \tau^{1-\sigma})\mathbf{1}_X(x_2) - \mathbf{1}_X(x_1)} \frac{\partial \gamma}{\partial \eta} \frac{1}{\gamma_S} \tag{20}
\]

and

\[
\frac{\partial \tilde{\Theta}_V}{\partial \eta} = -\tilde{\Theta}_V \frac{1}{1 - \gamma^{-1}(1 + \tau^{1-\sigma})\mathbf{1}_X(x_2) - \mathbf{1}_X(x_3)} \left( \frac{\partial \gamma}{\partial \eta} \frac{1}{\gamma_S} + \frac{\partial \psi_V}{\partial \eta} \frac{1}{\psi_V} \right), \tag{21}
\]

where \( \xi \equiv \psi_V(\eta)/\psi_O > 1 \) since we consider \( \eta > \eta^* \). From (20) and (21) expressions one can see that \( \frac{\partial \Theta_V}{\partial \eta}(\eta'; A, \nu) < \frac{\partial \Theta_S}{\partial \eta}(\eta'; A, \nu) \) when using that \( \gamma_S(\eta) \) and \( \psi_V(\eta) \) are strictly decreasing and strictly increasing in \( \eta \), respectively.\(^{38}\) But this

\(^{38}\)First, this follows directly if \( \frac{\partial \gamma}{\partial \eta} \frac{1}{\gamma_S} + \frac{\partial \psi_V}{\partial \eta} \frac{1}{\psi_V} \geq 0 \). If \( \frac{\partial \gamma}{\partial \eta} \frac{1}{\gamma_S} + \frac{\partial \psi_V}{\partial \eta} \frac{1}{\psi_V} < 0 \), it follows from noting that \( 0 > \frac{\partial \gamma}{\partial \eta} \frac{1}{\gamma_S} + \frac{\partial \psi_V}{\partial \eta} \frac{1}{\psi_V} > \frac{\partial \gamma}{\partial \eta} \frac{1}{\gamma_S} \), \( 0 < \gamma_S^{-1}(1 + \tau^{1-\sigma})\mathbf{1}_X(x_2) - \mathbf{1}_X(x_3) < \gamma_S^{-1}(1 + \tau^{1-\sigma})\mathbf{1}_X(x_2) - \mathbf{1}_X(x_1) < 1 \), and that for \( \eta = \eta' \), \( \Theta_S = \tilde{\Theta}_V \).
means that for \( \eta \) just above \( \eta' \), we have \( \Theta_S > \Theta_V \). However, from (18), this implies that for firms with \( \eta \) just above \( \eta' \) and \( \Theta \) just below \( \Theta_S \),

\[
\pi_{Vsx_3}(\Theta_S, \eta; A, \nu) - \pi_{ONx_2}(\Theta_S, \eta; A, \nu) > 0,
\]

which is contradictory to these firms choosing \( klx = ONx_2 \). Thus, there must exist an \( \eta'' > \eta' \) such that for a given \( \eta \) greater than \( \eta'' \) all firms that offshore also integrate and for a given \( \eta \) less than \( \eta'' \) not all firms that offshore also integrate. We now know that \( s_{V|S}(\eta) = 0 \) for \( \eta \in (0, \eta^* \) and \( s_{V|S}(\eta) = 1 \) for \( \eta \in [\eta'', 1) \). What remains to be shown is that \( s_{V|S}(\eta) \) is monotone increasing in \( \eta \) on \( (\eta^*, \eta'') \). To do so, note that on this interval, \( \Theta_V > \Theta_S \). That is, \( \Theta_V \) is given by

\[
\pi_{Vsx_4}(\Theta_V, \eta; A, \nu) - \pi_{OSx_5}(\Theta_V, \eta; A, \nu) = 0
\]

for some \( x_4 \geq x_5 \). This gives us

\[
\Theta_V(\eta; A, \nu) = \frac{f_{Vsx_4} - f_{OSx_5}}{A\gamma_S(\eta)[\psi_V(\eta)(1 + \tau^{1-\sigma})^x(x_4) - \psi_O(1 + \tau^{1-\sigma})^x(x_3)]}.
\]  (22)

Further, \( \Theta_S \) is given by,

\[
\pi_{OSx_6}(\Theta_S, \eta; A, \nu) - \pi_{ONx_7}(\Theta_S, \eta; A, \nu)
\]

for some \( x_6 \geq x_7 \). This gives us

\[
\Theta_S(\eta; A, \nu) = \frac{f_{OSx_6} - f_{ONx_7}}{A\psi_O[\gamma_S(\eta)(1 + \tau^{1-\sigma})^x(x_6) - (1 + \tau^{1-\sigma})^x(x_7)]}.
\]  (23)

Note that \( x_5 \geq x_6 \). Introducing \( B(\eta) = [\gamma_S(\eta)(1 + \tau^{1-\sigma})^x(x_6) - (1 + \tau^{1-\sigma})^x(x_7)] \), the cutoffs (22) and (23) can be written as

\[
\Theta_V(\eta; A, \nu) = \frac{(f_{Vsx_4} - f_{OSx_5})[(1 + \tau^{1-\sigma})^x(x_4) - (1 + \tau^{1-\sigma})^x(x_5)]\gamma_S(\eta)^{-1}}{AB(\eta)[\psi_V(\eta)(1 + \tau^{1-\sigma})^x(x_4) - \psi_O(1 + \tau^{1-\sigma})^x(x_5)]} \]  (24)

and

\[
\Theta_S(\eta; A, \nu) = \frac{f_{OSx_6} - f_{ONx_7}}{A\psi_O B(\eta)},
\]  (25)

respectively. First note that if one keeps \( B(\eta) \) constant, then (24) is decreasing in \( \eta \) for constant \((x_4, x_5, x_6, x_7)\). Next, for \( \eta \in (\eta^*, \eta'') \), we can express \( s_{V|S}(\eta) \) as

\[
s_{V|S}(\eta) = \frac{1 - \bar{F}(\log \Theta_V)}{1 - \bar{F}(\log \Theta_S)},
\]  (26)

where \( \bar{F}(\log \Theta) = F(\Theta^{1/(\sigma-1)}) \) is the distribution of \( \log \Theta = (\sigma - 1) \log \theta \). Note first that for given \((x_4, x_5, x_6, x_7)\), any effect of an increase in \( \eta \) that does not go
through $B(\eta)$ will increase $s_{V|S}(\eta)$ since it reduces $\Theta_V$. Next, we show that the effect of $\eta$ through $B$ on $s_{V|S}$ is positive as well. To see that this is true, rewrite (26) to get
\[
s_{V|S}(\eta) = \exp \left\{ - \int_{\log \Theta_S}^{\log \Theta_V} \lambda(\Theta) \, d\Theta \right\},
\]
where $\lambda(\Theta)$ is the hazard rate of $\tilde{F}$. Now, differentiating (27) with respect to $B(\eta)$ gives us
\[
\frac{\partial s_{V|S}}{\partial B} = s_{V|S} \left( \lambda(\log \Theta_S) \frac{\partial \log \Theta_S}{\partial B} - \lambda(\log \Theta_V) \frac{\partial \log \Theta_V}{\partial B} \right).
\]
It is clear from (24) and (25) that $\frac{\partial \log \Theta_V}{\partial B} = \frac{\partial \log \Theta_S}{\partial B} < 0$. Further, since the distribution of $\log \Theta$ has nonincreasing hazard rate, we know that $\lambda(\log \Theta_V) \leq \lambda(\log \Theta_S)$ as we consider an interval where $\Theta_V > \Theta_S$.\(^{39}\) Thus, $s_{V|S}$ is decreasing in $B$ which means that the effect of $\eta$ through $B$ on $s_{V|S}$ is positive for given $(x_4, x_5, x_6, x_7)$. Thus, in total, we have shown that on the interval $(\eta^*, \eta^{**})$, $s_{V|S}$ is increasing in $\eta$ whenever $(x_4, x_5, x_6, x_7)$ does not change. However, we know that if $(x_4, x_5, x_6, x_7)$ changes at some point as $\eta$ increases, then at this point, the value of $s_{V|S}$ is unaffected since $\Theta_V$ and $\Theta_S$ are continuous in $\eta$. Further, just above and below such points, $s_{V|S}$ is increasing in $\eta$. Thus, we can conclude that $s_{V|S}$ is increasing in $\eta$ on $(\eta^*, \eta^{**})$, and we are done.

The reason we cannot say that individual firms will unambiguously find intra-firm importing more attractive as $\eta$ increases conditional on productivity is that $\eta$ has two opposing effects on the RHS of (22). One is that $\psi_V$ increases and the other is that $\gamma_S$ decreases. If the latter effect dominates for some $\eta \in (\eta^*, \eta^{**})$, then we see that $\Theta_V$ is above $\Theta_S$ and increasing in $\eta$. But then, for some level of productivity, a higher $\eta$ will cause firms to shift from operating with $kl = VS$ to $kl = OS$, i.e., higher headquarter intensity causes a shift from intra-firm importing to arms’-length importing at the firm level.

### C Proof of Proposition 3

For $\eta$ sufficiently close to 0, the cutoffs for offshoring and exporting are given by
\[
\pi_{OSD}(\Theta_S, \eta; A, \nu) - \pi_{OND}(\Theta_S, \eta; A, \nu) = 0 \quad (28)
\]
and
\[
\pi_{OSX}(\Theta_X, \eta; A, \nu) - \pi_{OSD}(\Theta_X, \eta; A, \nu) = 0, \quad (29)
\]

\(^{39}\)As $\log \theta$ has nonincreasing hazard rate and $\sigma > 1$, $\log \Theta = \log \theta^{\sigma-1} = (\sigma - 1) \log \theta$ has nonincreasing hazard rate as well.
respectively. Next, for $\eta$ sufficiently close to 1, the cutoffs for integration and exporting are given by

$$\pi_{VND}(\Theta_V, \eta; A, \nu) - \pi_{OND}(\Theta_V, \eta; A, \nu) = 0$$

(30)

and

$$\pi_{VNX}(\Theta_X, \eta; A, \nu) - \pi_{VND}(\Theta_X, \eta; A, \nu) = 0,$$

(31)

respectively. Now, any increase in $(-f_V, -w_S, -f_S, -\tau, -f_X)$ results in a reduction in $A$. Consider a reduction in the fixed cost associated with integration, $f_V$. This does not affect the cutoff conditions (28) and (29) directly but does so indirectly through $A$. Since LHS of both equations are strictly increasing in $(\Theta, A)$, the result of the decline in $A$ is a strict increase in the cutoffs given by (28) and (29) for the relevant $\eta$’s. That is, some firms shift away from offshoring and exporting. Next, consider a reduction in the costs associated with offshoring, $(w_S, f_S)$. Since the LHS of (30) and (31) are not directly affected by these changes and strictly increase in $(\Theta, A)$, the result is a strict increase in the cutoffs given by these two equations for the relevant $\eta$’s. That is, some firms shift away from integration and exporting. Finally, consider a reduction in $(\tau, f_X)$. Using the same line of arguments again implies that the cutoffs given by (28) and (30) are strictly increasing for the relevant $\eta$’s. That is, some firms shift away from integration and offshoring.

D Proof of Proposition 4

First we prove that increasing $(-w_S, -f_S)$ always reduces the prevalence of northern integration. To do so, consider for the moment the initial equilibrium and note that it follows from Appendix B that if $\Theta_V \leq \Theta_S$ for some $\eta = \eta'$, then $\Theta_V \leq \Theta_S$ for all $\eta \geq \eta'$. Next, suppose that $\Theta_V < \Theta_S$, then

$$\Theta_S(\eta; A, \nu) = \frac{f_{VSx_1} - f_{VNX_1}}{A\psi_V(\eta)[\gamma_S(\eta)(1 + \tau^{1-\alpha})^{1}(x_1) - (1 + \tau^{1-\alpha})^{1}x_2]},$$

for some $x_1 \geq x_2$, and

$$\Theta_V(\eta; A, \nu) = \frac{f_{VNX_3} - f_{ONX_4}}{A[\psi_V(\eta)(1 + \tau^{1-\alpha})^{1}x_4] - \psi_O(1 + \tau^{1-\alpha})^{1}x_4]},$$

for some $x_3 \geq x_4$ where $x_2 \geq x_3$. Thus, when $\Theta_V < \Theta_S$, the ratio is given by

$$\frac{\Theta_S}{\Theta_V} = \frac{(f_{VSx_1} - f_{VNX_2})[(1 + \tau^{1-\alpha})^{1}(x_1) - \psi_O\psi_V(\eta)^{-1}(1 + \tau^{1-\alpha})^{1}x_2]}{(f_{VNX_3} - f_{ONX_4})[\gamma_S(\eta)(1 + \tau^{1-\alpha})^{1}(x_1) - (1 + \tau^{1-\alpha})^{1}x_2]}.$$  

(32)
Since $\gamma_S(\eta)$ and $\psi_V(\eta)$ are decreasing and increasing in $\eta$, respectively, it is obvious that the ratio in (32) is increasing in $\eta$ for given $(x_1, x_2, x_3, x_4)$. Further, as (32) is continuous at points where $(x_1, x_2, x_3, x_4)$ jumps, (32) is increasing in $\eta$ whenever $\Theta_V < \Theta_S$. Thus, if $\Theta_V < \Theta_S$ for some $\eta = \eta''$, we have that $\Theta_V < \Theta_S$ for all $\eta > \eta''$. Let $\eta^{**}$ be given by

$$\eta^{**} \equiv \inf\{\eta : \Theta_V < \Theta_S\} = \inf\{\eta : 1 < \frac{\Theta_S}{\Theta_V}\}.$$  

Then we can express the prevalence of northern integration, $s_{VN}$, as

$$s_{VN} = \frac{\int_{\eta^{**}}^1 [F(\Theta_S) - F(\Theta_V)]dG(\eta)}{1 - F(\Theta_{exit})}. $$

Let us show that $\eta^{**}$ is increasing in $(-w_S, -f_S)$. By (32), which is valid for $\eta \geq \eta^{**}$, it is obvious that changes in $A$ do not affect $\eta^{**}$. Further, given $A$, increases in $(-w_S, -f_S)$ reduce $\Theta_S$ and does not affect $\Theta_V$ whenever $\Theta_V < \Theta_S$. It follows that $\eta^{**}$ must rise in $(-w_S, -f_S)$. Next, using the Pareto distribution with shape parameter $z(\sigma - 1)$ for $F$, $s_{VN}$ can be expressed as

$$s_{VN} = \frac{\int_{\eta^{**}}^1 [(A\Theta_V)^{-z} - (A\Theta_S)^{-z}]dG(\eta)}{(A\Theta_{exit})^{-z}}. $$  \hspace{1cm} (33)

Now, note that $\eta^{**}$ increases in $(-w_S, -f_S)$, that $A\Theta_V$ and $A\Theta_{exit}$ are unaffected by changes in $(-w_S, -f_S)$ for $\eta > \eta^{**}$, and that $A\Theta_S$ is decreasing in $(-w_S, -f_S)$. Thus, $s_{VN}$ decreases in $(-w_s, -f_S)$.

Second, to show that $s_{VN}$ can be increasing when final-goods trade is liberalised, assume that $\Theta_X < \Theta_S$ for $\eta \geq \eta^{**}$. Note that this implies that $A\Theta_X = A\Theta_V$ for an interval of $\eta$ in $(\eta^{**}, 1)$. Under these assumptions, an increase in $-f_X$ has no effect on $\eta^{**}$, $A\Theta_S$, or $A\Theta_{exit}$ in (33). However, for the interval of $\eta$ in $(\eta^{**}, 1)$ where $\Theta_X = A\Theta_V$, $A\Theta_V$ is decreasing and $A\Theta_V$ is nonincreasing otherwise. This means that $s_{VN}$ increases in $-f_X$.

We can also illustrate that $s_{VN}$ can be unambiguously increasing in $-\tau$ if we relax the assumption that $\Theta_X > \Theta_V$ for $\eta$ sufficiently close to 1. Assume that $\Theta_X < \Theta_V$ for all $\eta$. Then $x_1 = x_2 = x_3 = x_4 = X$ in (32) which means that $\eta^{**}$ and $\Theta_S$ for $\eta \geq \eta^{**}$ are unaffected by $\tau$. Expressing $s_{VN}$ as

$$s_{VN} = \frac{\int_{\eta^{**}}^1 (A\Theta_V)^{-z}[1 - (\frac{\Theta_S}{\Theta_V})^{-z}]dG(\eta)}{(A\Theta_{exit})^{-z}}, $$

it now follows that $s_{VN}$ is increasing in $-\tau$ since $A\Theta_{exit}$ is unaffected and $A\Theta_V$ is decreasing.

\footnote{For $\eta = \eta^{**}$, $\Theta_S = \Theta_V$.}
References


Firms and International Trade


Chapter Four

An Industry-Equilibrium Analysis of the LeChatelier Principle*

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Abstract

By considering firms operating in a perfectly- or monopolistically-competitive industry with free entry, we show that well-established results on the celebrated LeChatelier principle (LCP) do not extend into an endogenous competitive environment. For instance, labour demand may be more elastic in the short run (where capital is fixed) than in the long run even if capital and labour are either complements or substitutes in profits. This may also be true locally at a point of long-run equilibrium. A novel insight is that industry-equilibrium effects introduce an asymmetry such that the LCP may hold for wage increases but not for wage decreases. These results are important for the interpretation of estimated labour-demand elasticities. Finally, we show that the LCP may hold for the total industry labour demand in situations where it does not hold for the labour demand of individual firms.

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1 Introduction

One cornerstone of comparative statics in economics is the LeChatelier principle (LCP) which arguably has its simplest expression when considering a profit-maximising firm choosing two inputs, say capital and labour. In this context, the LCP states that the elasticity of firms’ labour demand with respect to the wage is smaller in magnitude in the short run, where the level of capital is fixed, than in the long run where capital is free to adjust. The requirement for this outcome to hold true is most clearly expressed by Milgrom and Roberts (1996): the LCP requires capital and labour to be complements (substitutes) in the dual sense that a reduction in the wage leads to increased (decreased) demand for capital and that an increase (decrease) in the use of capital leads to increased demand for labour. If this is the case, we say that capital and labour are complements (substitutes) in demand. The LCP holds under these conditions as capital and labour form a positive feedback system where allowing capital to adjust magnifies the adjustment of labour demand (Milgrom, 2006). The question then is to figure out under which conditions the two inputs are indeed complements or substitutes in demand.

Since Samuelson (1947) introduced the LCP into the field of economics, it has been known that the LCP always holds locally (at an initial point of long-run equilibrium) for the labour demand of a firm considered in isolation. However, it quickly became clear that for noninfinitesimal changes in wages, the LCP does not always apply and examples that violate a global LCP now abound; see e.g. Samuelson (1960), de Meza (1981), Milgrom and Roberts (1994), and Milgrom and Roberts (1996). Ample attention has therefore been devoted to formulating conditions that ensure a global version of the LCP. Notably, Milgrom and Roberts (1996) argue that if capital and labour are global complements (substitutes) in the profit function of the firm under scrutiny, then they are global complements (substitutes) in demand as well and consequently, the LCP holds globally for labour demand. This analysis, like most other studies of the LCP, relies on the firm operating in an exogenous competitive environment where the only endogenous variables affecting the firm’s profits are its own choices of inputs. While insightful and providing a natural first step, such an approach is not fully satisfying. When considering exogenous shocks that affect all firms in an industry, such as a change in the wage, precluding the adjustments of competitors from influencing the choices of a given firm is dubious at best.

The present paper takes the next step in the analysis of the LCP by considering the labour demand of firms in industry equilibrium. In particular, we let
firms operate in a perfectly- or monopolistically-competitive industry with free entry. Central to our analysis is the fact that exogenous changes in the wage now induce endogenous adjustments in the fierceness of competition. Importantly, the LCP may now fail to hold locally and we describe the conditions under which this is the case. Assuming that capital and labour are either global complements or global substitutes in profits, we show that this assumption is no longer sufficient for these inputs to be global complements or global substitutes in demand. A global LCP is therefore no longer generally valid under these assumptions on the profit function. Further, our setup features cases where the LCP holds globally when the wage increases but not when the wage decreases. The possibilities of breakdowns and, perhaps especially, asymmetries in the LCP call for caution in both obtaining and interpreting estimates of labour-demand elasticities at different horizons. Finally, we note the possibility of a discrepancy between the LCP for firms’ labour demand (as considered so far) and an LCP for aggregate labour demand in the industry.

The possible discrepancy between LCPs at the firm and industry levels has also been noted by Koebel and Laisney (2010). These authors focus predominantly on an aggregate LCP for an industry characterised by Cournot competition. As described above, the present paper is primarily concerned with the implications of endogenising the competitive environment for established results regarding the LCP for firms’ input demand. While this is done in a setting general enough to encompass both perfect and monopolistic competition, our formulation of the industry equilibrium is admittedly rudimentary. This is intentional as a comprehensive analysis of the LCP in endogenous competitive environments is beyond the scope of this paper. Rather, our goal is to show that, even with our simple notion of industry equilibrium, the implications for the validity of the LCP are profound as evident from the results outlined above.

2 Setup

The industry under scrutiny is characterised by either perfect or monopolistic competition. Firm profits depend on one endogenous industry-wide variable outside the control of individual firms. We refer to this variable as the demand level, \( A \).\(^4\) Firms use two inputs in production namely capital and labour, \( K \) and \( L \), which are in perfectly elastic supply. That is, the industry is small enough not

\(^3\)This is not only a way to make the relation to the existing literature transparent. Milgrom and Roberts (1995) and Topkis (1995) argue that complementarities arise quite naturally between the various dimensions of firms’ choices.

\(^4\)Under perfect competition, \( A \) would simply be the price. Under monopolistic competition, \( A \) could e.g. be the demand shifter from the demand function arising from CES preferences; see Section 4.
to affect factor prices in the aggregate. Firm profits are given by
\[
\pi(K, L; A, \beta) = R(K, L; A) - rK + \beta L - f,
\]
where \( r \) is the interest rate, \(-\beta = w\) is the wage rate, and \( f \) is a fixed cost.\(^5\)
We use the notation \( \beta = -w \) for expositional convenience in the following. Since \( r \) and \( f \) will be kept constant throughout, we do not write profits as explicitly depending on these parameters. We consider symmetric equilibria and focus on a representative firm. We assume that the revenue function, \( R \), is increasing in \((K, L; A)\) and that a higher value of \( A \) makes it more attractive to increase \( K \) and \( L \), all else equal. Formally, the latter property means that \( R \) has increasing differences in \((K; A)\) and \((L; A)\) which, in turn, implies that \( R_{LA} \) and \( R_{KA} \) are nonnegative if \( R \) is smooth. Subscripts denote partial derivatives. We let \( K \) and \( L \) be either global complements or global substitutes in the profit function. That is, \( R \), and thus also \( \pi \), is either super- or submodular in \( K \) and \( L \). Supermodularity (submodularity) implies that \( R_{KL} \geq 0 \) (\( R_{KL} \leq 0 \)) holds globally if \( R \) is smooth.

In the following, we focus on the effects of changes in the wage, \(-\beta\), on labour demand in the short run, where capital is fixed, and in the long run where capital can adjust. Profit maximisation gives us the optimal levels of \( K \) and \( L \),\(^6\)
\[
L^*(K; A, \beta) = \arg \max_L \pi(K, L; A, \beta)
\]
and
\[
K^*(A, \beta) = \arg \max_K \pi(K, L^*(K; A, \beta); A, \beta).
\]
It follows from our assumptions and the monotonicity theorem of Topkis (1978) that \( L^* \) is increasing in \((A, \beta)\). In the case where \( K \) and \( L \) are complements (substitutes) in profits, \( K^* \) is increasing (decreasing) in \( \beta \) and \( L^* \) is increasing (decreasing) in \( K \). Our analysis will make extensive use of these properties.

2.1 Industry Equilibrium

To assess the consequences of an endogenous competitive environment, we need to impose an equilibrium condition. We let this be free entry which requires that
\[
\pi(K, L^*(K; A, \beta); A, \beta) = 0. \tag{1}
\]
\(^5\)The fixed cost can e.g. be thought of as a fixed amount of capital, \( \tilde{f} \), that must be rented for production to take place, i.e., \( f = r\tilde{f} \). The results below will be qualitatively similar if \( f \) also comprises a fixed amount of labour.

\(^6\)We implicitly assume that the maximisers exist and treat them as unique. Existence is ensured e.g. if \( K \) and \( L \) are both chosen from compact choice sets and \( \pi \) is upper semi-continuous in \( K \) and \( L \) (Milgrom and Roberts, 1996).
Assuming the existence of an equilibrium, this free-entry condition gives us the demand level, \( A = A(K; \beta) \). Note that \( A(K; \beta) \) is decreasing in \( \beta \) as the left-hand side of (1) is increasing in \((A, \beta)\). The free-entry condition, (1), is the natural choice for an equilibrium condition in the long run due to our focus on perfect or monopolistic competition. In addition, it is convenient as it only involves the already introduced profit function. We assume that the free-entry condition also holds in the short run. If new firms enter in the short run, they do so with the same (fixed) level of capital as incumbents. Abstracting from the possible complication of different equilibrium conditions at different horizons means that all short-to-long-run effects arising in our setup will solely be the consequence of capital becoming free to adjust. This makes our analysis more directly comparable to the existing literature on the LCP.

3 Comparative Statics

In the following, we restrict attention to cases where short-run labour demand is well-behaved in the sense that it decreases in the wage. That is, cases where \( \hat{L}^*(K; \beta) \equiv L^*(K; A(K; \beta), \beta) \) is increasing in \( \beta \). For the LCP to hold, the total long-run adjustment in \( L \) must exceed the short-run adjustment. As mentioned in the introduction, this requires \( K \) and \( L \) to be either: (i) complements in the dual sense that increasing \( \beta \) leads to an increase in \( K \) in the long run and \( L^* \) is increasing in \( K \); or (ii) substitutes in the dual sense that increasing \( \beta \) leads to a reduction in \( K \) in the long run and \( L^* \) is decreasing in \( K \). Importantly, these properties have to hold when we take the endogeneity of \( A \) into account. We refer to case (i) as \( K \) and \( L \) being complements in demand and to case (ii) as \( K \) and \( L \) being substitutes in demand. In either of these cases, \( K \) and \( L \) are said to be part of a positive feedback system where the adjustment in \( K \) magnifies the adjustment in \( L \) (Milgrom, 2006).

Consider briefly the case of an exogenous demand level, \( A \). Treating \( A \) as fixed, it follows immediately from the properties of \( K^* \) and \( L^* \) that capital and labour being complements (substitutes) in profits implies that they are complements (substitutes) in demand and the LCP holds globally. This is the main result.

---

7 The existence and uniqueness of an equilibrium are guaranteed by the intermediate value theorem if we e.g. let \( R(K, L; 0) = 0 \) and \( R(K, L; A) \) be continuous and strictly increasing in \( A \) with \( R \to \infty \) as \( A \to \infty \).

8 This does not follow immediately as \( A(K; \beta) \) is decreasing in \( \beta \) and \( L^* \) is increasing in \((A, \beta)\). It is however always satisfied in the notable special case where \( R(K, L; A) = A\hat{R}(K, L) \) with \( \hat{R} \) being smooth. Such a revenue function arises under perfect competition where \( A \) is the price. Further, under monopolistic competition, it can arise from the combination of a production function and the demand function obtained from consumers having additively separable preferences over varieties in the industry.
of Milgrom and Roberts (1996); a benchmark result from the related literature in which we take offset. As will become evident below, $A$ being endogenous in our setup implies that the conditions for the LCP to hold (complements or substitutes in demand) do not translate as easily into requirements on primitives (complements or substitutes in profits).

### 3.1 Local LCP

To show that the endogeneity of $A$ implies that $K$ and $L$ being complements in profits is not sufficient to ensure that they are also complements in demand, let us consider the conditions under which the LCP holds locally at an initial point of long-run equilibrium, assuming $R$ is smooth. Since the results about increases and decreases in $\beta$ are symmetric in this context, we focus on an increase in $\beta$, i.e., a reduction in the wage.

Consider the response in $L$ when $K$ is kept fixed. By the first-order condition for $L$, $R_L = -\beta$, this is given by

$$\frac{\partial \hat{L}^*}{\partial \beta} = \frac{1}{-R_{LL}} (1 - \varepsilon_{R_L A} \varepsilon_{A, \beta}),$$

where $\varepsilon_{x,y} \equiv \frac{\partial x}{\partial y} y$ is used to denote the elasticity of $x$ with respect to $y$. Let the second-order conditions be satisfied, wherefore $R_{LL}$ is negative. By our focus on well-behaved demand functions, we know that (2) is positive. Note that the short-run increase in $L$ is smaller in magnitude than if $A$ had been exogenous ($\varepsilon_{A, \beta} = 0$).

Next, consider the total response in $L$ when $K$ is allowed to adjust,

$$\frac{d \hat{L}^*}{d \beta} = \frac{\partial \hat{L}^*}{\partial \beta} + \frac{\partial \hat{L}^*}{\partial K} dK.$$

The LCP holds locally if $\frac{\partial \hat{L}^*}{\partial K} dK$ is positive when evaluated at the initial equilibrium which corresponds to $K$ and $L$ being either (local) complements or (local) substitutes in demand. By the first-order condition for $L$,

$$\frac{\partial \hat{L}^*}{\partial K} = \frac{R_{KL}}{-R_{LL}}.$$

This derivative is positive if $K$ and $L$ are complements in profits and negative if they are substitutes. Hence, if $K$ and $L$ are complements (substitutes) in profits,

9See Appendix A for derivation.

10Thus, $\varepsilon_{R_L A} = \frac{R_{L A}}{R_L} \geq 0$ and $\varepsilon_{A, \beta} = \frac{\partial A}{\partial \beta} A \geq 0$. Using the first-order condition for $L$ and (1), the latter can be expressed in terms of primitives as $\varepsilon_{A, \beta} = \frac{R_{LL}}{R_{A A}}$.

11Here, we have used that $\frac{\partial \pi}{\partial K} = 0$ holds locally, wherefore $\frac{\partial A}{\partial K} = 0$. 

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then the LCP holds locally only if they are also complements (substitutes) in the sense that an increase in $\beta$ increases (decreases) $K$ when the endogeneity of $A$ is taken into account.\footnote{In line with the existing literature, the LCP always holds trivially when $\frac{dK}{d\beta} = 0$.} To determine whether this is the case, we consider the derivative,\footnote{For a derivation of this expression, see Appendix A.}

$$\frac{dK}{d\beta} = \frac{R_{KL}(1 - \varepsilon_{R_L A \varepsilon_A \beta})}{R_{KK}R_{LL} - R_{KL}^2} \left(1 + \frac{\varepsilon_{R_K A \varepsilon_A \beta}}{\varepsilon_{R_K L \varepsilon_L \beta}}\right), \quad (4)$$

where the second-order conditions imply that $R_{KK}R_{LL} > R_{KL}^2$. Take first the case of complements in profits ($R_{KL} > 0$). In this case, (4) must be positive for the LCP to hold locally. This is only the case if $\varepsilon_{R_K A \varepsilon_A \beta} \leq \varepsilon_{R_K L \varepsilon_L \beta}$ where all terms are positive (after taking the absolute value of $\varepsilon_{L,\beta}$). This condition states that the positive effect of $\beta$ through $L$ on the marginal revenue product of $K$ must be larger in magnitude than the negative effect through the fall in $A$ on the marginal revenue product of $K$. When this is the case, an increase in $\beta$ makes firms increase $K$ when allowed to adjust and (4) is positive. The LCP therefore holds locally. On the other hand, if the negative effect of $\beta$ through $A$ on the marginal revenue product of $K$ dominates the positive effect through $L$, then $K$ falls when allowed to adjust. Thus, for the local LCP to hold under complements in profits, $K$ and $L$ must be sufficiently complementary such that the indirect effect of a falling demand level is dominated by the direct complementarity effect through $L$.

Next, consider the case of substitutes in profits ($R_{KL} < 0$). In this case, $K$ always tends to fall when allowed to adjust after a wage decrease.\footnote{When $R_{KL} < 0$, both $\varepsilon_{R_K A \varepsilon_A \beta}$ and $\varepsilon_{R_K L \varepsilon_L \beta}$ are positive.} The reason is that both the decrease in $A$ and the increase in $L$ make the firm want to reduce $K$. Further, as established above, $K$ and $L$ being substitutes in profits implies that this reduction in $K$ makes a firm want to increase $L$ compared to the case where $K$ is fixed. Locally, $K$ and $L$ are therefore always substitutes in demand when they are substitutes in profits. Hence, contrary to the case of complements in profits, the LCP always holds locally when $K$ and $L$ are substitutes in profits. Finally, by (3), it is clear that the LCP always holds locally in a weak form when $K$ and $L$ are independent in profits ($R_{KL} = 0$). The proposition below summarises these findings.\footnote{Our focus on well-behaved short-run demand functions is not crucial for obtaining a breakdown of the local LCP. In case $\frac{dL}{d\beta}$ is negative, one gets the following results. The local LCP always holds (in an inverted edition) when $K$ and $L$ are complements or independent in profits. When $K$ and $L$ are substitutes in profits, the local LCP holds (in an inverted edition) if and only if $\varepsilon_{R_K A \varepsilon_A \beta} \leq |\varepsilon_{R_K L \varepsilon_L \beta}|$.}
Proposition 1. The LCP always holds locally when $K$ and $L$ are either substitutes or independent in profits. The LCP holds locally when $K$ and $L$ are complements in profits if and only if $\varepsilon_{R_K,A} \leq \varepsilon_{R_K,L} |\varepsilon_{L,\beta}|$.

3.2 Global Considerations

As we have just seen, the general decline in $A$ following an increase in $\beta$ may mean that $K$ does not increase in the long run when $K$ and $L$ are complements in profits. Failure in this regard is the only reason for the LCP to break down locally when short-run demand functions are well-behaved. Considering a global version of the LCP, another consideration becomes relevant. Since the purpose of adjusting $K$ is to increase profits, the result is a short-to-long-run decline in $A$ regardless of whether $K$ increases or decreases. Locally, the change in $K$ has no effect on profits (first-order condition) and hence $A$. However, for noninfinitesimal changes in $\beta$, this second-order effect interferes with the LCP.

Let us begin by considering increases in $\beta$ which mean that the wage is reduced; a reduction in $\beta$ is treated separately in Section 3.3. Let $A', A^c$, and $A''$ represent the initial, short-run (constrained), and long-run values of $A$, respectively. It follows that when $\beta' < \beta''$,

$$A' \geq A^c \geq A''$$

Further, let $K'$ and $K''$ denote the initial and long-run equilibrium values of $K$, respectively. It follows that

$$L^*(K''; A'', \beta'') - L^*(K'; A'', \beta'') = L^*(K''; A', \beta'') - L^*(K''; A^c, \beta'')$$

$$+ L^*(K''; A^c, \beta'') - L^*(K'; A', \beta'').$$ (5)

The previous section already discussed why the direct effect of the change in $K$ could be negative in case of complements in profits. Whenever noninfinitesimal increases in $\beta$ are considered, the indirect effect through $A$ of changing $K$ must be taken into account as well. Importantly, this effect is always negative since $A^c \geq A''$ and $L^*$ is increasing in $A$. For the case of complements in profits, this is an additional reason the LCP may not hold globally besides $K$ moving in the wrong direction. For the case of substitutes in profits, this is a reason that the LCP may not hold globally even though it always does so locally.

To illustrate how the indirect effect of the adjustment in $K$ can result in a discrepancy between the local and global validity of the LCP, consider the case where $K$ and $L$ are independent in profits. That is, let $\pi$ be simultaneously super- and submodular in $K$ and $L$. When $\pi$ is smooth, this implies $\pi_{KL} = 0$. In this
case, the LCP holds locally; see Proposition 1. However, globally the LCP breaks down. To see why, simply note that the direct effect of the change in $K$ is zero such that (5) is given by the negative indirect effect.

**Proposition 2.** For noninfinitesimal increases in $\beta$, the LCP does not hold when $K$ and $L$ are independent in profits and the short-to-long-run adjustment in $L$ is nontrivial. 

One could easily imagine that the negative indirect effect of the change in $K$ in (5) dominates whenever $K$ and $L$ are sufficiently weak complements or substitutes in profits. That is, even though the LCP always holds locally for $K$ and $L$ being substitutes, it may not hold globally if they are only weakly so. The example of Section 4 confirms this possibility.

### 3.3 Decreasing $\beta$: Asymmetry in the LCP

In this section, we consider noninfinitesimal reductions in $\beta$, i.e., discrete increases in the wage. First note that a reduction in $\beta$ means that $A$ is larger than initially both in the short and the long run. However, the fact that $K$ can adjust in the long run still implies that $A$ must be lower in the long run relative to the short run. When $\beta' > \beta''$, we therefore get the following ranking of the initial, short-run, and long-run demand levels,

$$A' \leq A'' \leq A^c.$$ 

Note that a decrease in $\beta$ reduces $L$ in the short run due to our focus on well-behaved demand functions. Thus, in this case, the LCP holds if $L$ is reduced even further in the long run. But this means that the fact that adjusting $K$ reduces $A$ from the short to the long run works in favour of the LCP holding in contrast to the case where $\beta$ was increased. In certain cases, the LCP may therefore hold for reductions in $\beta$ but not for increases. The asymmetry between increases and decreases in $\beta$ with respect to the LCP can be clearly illustrated if we revisit the case where $K$ and $L$ are independent in profits. In this case, (5) is again given by the negative indirect effect. This means that the short-run reduction in $L$ is magnified in the long run and that the LCP holds. \(^{16}\)

**Proposition 3.** For noninfinitesimal reductions in $\beta$, the LCP holds when $K$ and $L$ are independent in profits.

\(^{16}\)Note that when $K$ and $L$ are very close to being independent, the reduction in $\beta$ can increase $K$ in the long run and this adjustment in $K$ can make $L$ fall further than in the short run. That is, $K$ and $L$ can behave as substitutes in demand, regardless of whether they are (very weak) complements or substitutes in profits. Section 4 confirms this possibility.
Propositions 2 and 3 give rise to the following corollary.

**Corollary 1.** *Whether the LCP holds or not may depend on the direction of change in β.*

Again, one could imagine that, for the same reasons leading to Proposition 3, the LCP holds for reductions in β whenever K and L are sufficiently weak complements or substitutes in profits. That is, asymmetry in the LCP may not be confined to the case of K and L being independent in profits. That there indeed can be asymmetries in the LCP for K and L being weak complements or weak substitutes is confirmed by the example of Section 4.

### 3.4 LCP at the Industry Level

The last point we want to make before considering an example is that the LCP may hold at the industry level even in cases where it does not hold for individual firms. In order to illustrate this possibility as simply as possible, we let expenditure in the industry be exogenously given by E. Then we get the number of firms as $M = E/R(K, L; A)$ and the total use of labour in the industry is $LM = EL/R(K, L; A)$. It should be clear that endogenous short- and long-run changes in $M$ can cause discrepancies between an LCP for L and one for $LM$. The following proposition makes this possibility clear.

**Proposition 4.** Let $R(K, L; A) = A\tilde{R}(K, L)$. Then the LCP always holds locally at the industry level (for $LM$) even though it may not do so at the firm level (for L).

*Proof.* See Appendix B.

To see the intuition for Proposition 4, recall the reason that the LCP may not hold locally at the firm level. When K and L are complements in profits, a breakdown happens when K fails to rise following an increase in β. In this case, L rises less when K is allowed to adjust since a decline in K tends to induce a decline in L (complements). However, the (short-to-long-run) declines in K and L also tend to reduce revenue of the individual firm which means that the number of firms must rise (total industry revenues are constant). Thus, whenever there is a force working against the LCP holding locally for individual firms, the same force tends to increase the number of firms. When $R(K, L; A) = A\tilde{R}(K, L)$, which is the case under perfect competition and under monopolistic competition when consumers have additively separable utility across varieties, it turns out that the local LCP at the industry level (for $LM$) always holds.
4 Example

We conclude our analysis of the LCP by considering an example that illustrates many of the points discussed above. Assume that the revenue function, \( R(K, L; A) \), is obtained by combining the isoelastic inverse demand function, \( p = Aq^{\rho-1} \), which could originate from consumers with CES preferences across varieties in the industry, with the production function,

\[
q = \left[ \alpha K^{\frac{\sigma-1}{\sigma}} + (1 - \alpha)L^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}},
\]

where \( \sigma > 0 \) is the elasticity of substitution in production and \( 0 < \alpha, \rho < 1 \). Profits are thus given by

\[
\pi = A \left[ \alpha K^{\frac{\sigma-1}{\sigma}} + (1 - \alpha)L^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} - rK + \beta L - f.
\]

Note that these assumptions give rise to a revenue function of the form \( R(K, L; A) = A\tilde{R}(K, L) \). Capital and labour are complementary in profits (\( \pi_{KL} > 0 \)) when \( \sigma \in (0, \frac{1}{1-\rho}) \) and substitutes (\( \pi_{KL} < 0 \)) for \( \sigma \in (\frac{1}{1-\rho}, \infty) \).\(^{17}\) When \( \sigma = \frac{1}{1-\rho} \), capital and labour are independent in profits (\( \pi_{KL} = 0 \)). First off, following an increase (reduction) in \( \beta \), \( L \) is larger (smaller) in both the long and the short run when compared to the initial value, regardless of whether \( K \) and \( L \) are complements, substitutes, or independent in profits.\(^{18}\) Appendix C shows that the LCP always holds globally if \( K \) and \( L \) are sufficiently strong complements, \( \sigma \in (0, 1) \), regardless of whether \( \beta \) increases or decreases. However, when \( \sigma \in (1, \infty) \), the LCP may or may not hold and there may be asymmetries between increases and decreases in \( \beta \).

The upper half of Figure 1 depicts the total long-run adjustment relative to the short-run adjustment in \( L \) for an increase and a reduction in the wage, \( w = -\beta \), of 20 percent for different values of \( \sigma \). In both cases, the LCP holds whenever the relative adjustment is above one. For the increase in \( \beta \), the LCP holds whenever \( K \) and \( L \) are sufficiently strong complements or sufficiently strong substitutes in production. When they are independent, the LCP fails to hold (Proposition 2). Further, the LCP fails to hold when \( K \) and \( L \) are weak complements or weak substitutes. For the reduction in \( \beta \), the LCP holds both if \( K \) and \( L \) are strong complements, \( \sigma < 1 \), and if they are very weak complements, independent (Proposition 3), or substitutes in profits. For the remaining intermediate values of \( \sigma \), the complementarity is not strong enough to make \( K \) fall when \( \beta \) is reduced.

\(^{17}\)The concavity introduced by \( \rho \) implies that \( K \) and \( L \) are not complements in the profit function for all \( \sigma > 0 \) even though they are always complements in the production function.

\(^{18}\)See Appendix C.
but still strong enough to make $L$ increase when $K$ increases.\textsuperscript{19} This means that $L$ increases from the short to the long run and thus exhibits undershooting. An asymmetry given by the fact that the LCP holds for reductions in $\beta$ but not for increases (Corollary 1) is clear in the figure for any value of $\sigma$ close to $\frac{1}{1-\rho}$. That is, when $K$ and $L$ are very weak complements or substitutes in profits.

The bottom half of Figure 1 shows the total long-run adjustment relative to the short-run adjustment in $LM$. Relating to the local result of Proposition 4, it is clearly seen that the LCP holds at the industry level, both for the increase and for the decrease in $\beta$ for all values of $\sigma$ considered, independently of whether the LCP holds at the firm level or not.

![Figure 1: Long-run changes in $L$ (top half) and $LM$ (bottom half) relative to their short-run changes for different values of $\sigma$. Whenever the graph is above 1, the LCP holds. The vertical dashed line indicates $\sigma = 1$ and the vertical solid line indicates independence between $K$ and $L$, $\sigma = \frac{1}{1-\rho}$.

5 Concluding Remarks

The present paper has shown that introduction of even very simple industry-equilibrium effects have important implications for the LCP. While research in-
terest in the LCP has somewhat waned in recent years, our contribution emphasises that there is more work to be done in understanding the conditions under which the LCP applies in more realistic environments. More rigorous inquiry into the implications of the specific nature of the competitive environment for the validity of the LCP (both locally and globally) seems like a promising area for future research.

A Derivation of (2) and (4)

To derive (2), we total differentiate the first-order condition \( R_L = -\beta \) with respect to \( \beta \), using that locally \( \frac{\partial A}{\partial L} = 0 \),

\[
R_{AL} \frac{\partial A}{\partial \beta} + R_{LL} \frac{\partial L^*}{\partial \beta} = -1.
\]

Rearranging, using the first-order condition \( R_L = -\beta \), gives (2).

To derive (4), we total differentiate the first-order conditions, \( R_K = r \) and \( R_L = -\beta \), with respect to \( \beta \),

\[
\begin{align*}
R_{LL} \frac{dL}{d\beta} + R_{KL} \frac{dK}{d\beta} &= -(1 - \varepsilon_{RL,A} \varepsilon_{A,\beta}), \\
R_{KL} \frac{dL}{d\beta} + R_{KK} \frac{dK}{d\beta} &= -R_{AK} \frac{\partial A}{\partial \beta}.
\end{align*}
\]

Using Cramer’s Rule,

\[
\frac{dK}{d\beta} = \frac{R_{KL}(1 - \varepsilon_{RL,A} \varepsilon_{A,\beta}) - R_{AK} R_{LL} \frac{\partial A}{\partial \beta}}{R_{KK} R_{LL} - R_{KL}^2}.
\]

Rearranging using the definitions of elasticities and (2) yields (4).

B Local LCP at the Industry Level

This appendix shows that the LCP always holds locally at the industry level, i.e., for the aggregate use of labour, \( LM \), in the industry when \( R(K, L; A) = A \tilde{R}(K, L) \). Due to symmetry across the homogeneous firms, we get \( A \tilde{R}(K, L)M = E \). From this, we get the aggregate use of labour expressed as

\[
LM(K; \beta) = \frac{E \tilde{L}^*(K; \beta)}{A(K; \beta) \tilde{R}(K, \tilde{L}^*(K; \beta))}.
\]
To see that the LCP holds locally for \( LM \), we first note that
\[
\frac{\partial LM}{\partial \beta} = \frac{AE\tilde{R}\frac{\partial L^*}{\partial \beta} - EL\tilde{R}\frac{\partial A}{\partial \beta} - AEL\tilde{R}_L\frac{\partial L^*}{\partial \beta}}{(AR)^2}
\]
\[
= \frac{E(AR + \beta L)\frac{\partial L^*}{\partial \beta} + EL^2}{(AR)^2} > 0,
\]
where we have used \( A\tilde{R}_L = -\beta, \frac{\partial A}{\partial \beta} = -\frac{L}{R} \), and the fact that \( \frac{\partial L^*}{\partial \beta} > 0 \) in the case we consider here. The LCP will hold locally for \( LM \) if
\[
\frac{dLM}{d\beta} - \frac{\partial LM}{\partial \beta} = \frac{\partial LM}{\partial K} \frac{dK}{d\beta} > 0.
\]
(7)

Now, consider \( \frac{dK}{d\beta} \), which can be obtained by simplifying (4) using \( R = A\tilde{R} \),
\[
\frac{dK}{d\beta} = \frac{\tilde{R}_KL}{A(R_KK_RLL - \tilde{R}^2_{KL})} \left( 1 - \frac{\tilde{R}_L L}{R} - \frac{-\tilde{R}_{LL} L\tilde{R}_K}{R_{KL}\tilde{R}} \right).
\]
(9)

It is obvious that (8) and (9) have the same sign such that (7) is positive and the LCP always holds locally at the industry level under the considered functional form of \( R(K, L; A) \).

C Comparative Statics in the CES Example

The first-order conditions for \( K \) and \( L \) are given by
\[
\pi_K = A\rho \alpha \left[ (1 - \alpha) L^{\frac{\sigma - 1}{\sigma}} + \alpha K^{\frac{\sigma - 1}{\sigma}} \right]^{\frac{\sigma - 1}{\sigma - 1}} K^{-\frac{1}{\sigma}} - r = 0,
\]
(10)
\[
\pi_L = A\rho (1 - \alpha) \left[ (1 - \alpha) L^{\frac{\sigma - 1}{\sigma}} + \alpha K^{\frac{\sigma - 1}{\sigma}} \right]^{\frac{\sigma - 1}{\sigma - 1}} L^{-\frac{1}{\sigma}} + \beta = 0.
\]
(11)
The second-order derivatives are given by

\[
\pi_{KK} = A\rho^2 \alpha^2(1 - \alpha)^2 \left[ (1 - \alpha)L^{\sigma - 1}_\sigma + \alpha K^{\sigma - 1}_\sigma \right]^{\sigma - 1}_\sigma K^{-\frac{\sigma - 1}{\sigma} - 2} L^{-\frac{\sigma - 1}{\sigma}} (L + \frac{1}{\alpha})^{-1} - \left( \rho - 1 + \frac{1}{\alpha} \right)^2
\]

\[
\pi_{LL} = A\rho^2 \alpha^2(1 - \alpha)^2 \left[ (1 - \alpha)L^{\sigma - 1}_\sigma + \alpha K^{\sigma - 1}_\sigma \right]^{\sigma - 1}_\sigma L^{-\frac{\sigma - 1}{\sigma} - 2} (L + \frac{1}{\alpha})^{-1} - \left( \rho - 1 + \frac{1}{\alpha} \right)^2
\]

\[
\pi_{KL} = A\rho \alpha(1 - \alpha) \left[ (1 - \alpha)L^{\sigma - 1}_\sigma + \alpha K^{\sigma - 1}_\sigma \right]^{\sigma - 1}_\sigma (L + \frac{1}{\alpha})^{-1} - \left( \rho - 1 + \frac{1}{\alpha} \right)^2
\]

such that the second-order conditions are satisfied. \( K \) and \( L \) are complements in profits, \( \pi_{KL} > 0 \), if \( \sigma < \frac{1}{1 - \rho} \) and substitutes in profits, \( \pi_{KL} < 0 \), if \( \sigma > \frac{1}{1 - \rho} \); see (12). If \( \sigma = \frac{1}{1 - \sigma} \), \( K \) and \( L \) are independent in profits. The second-order conditions hold regardless of whether \( K \) and \( L \) are substitutes, complements, or independent in profits. The free-entry condition equates optimal profits, \( \pi^* \), with zero, i.e.,

\[
\pi^* = 0.
\]

From (10), (11), and (13), we derive the following long-run input demands.

\[
K^{LR} = \frac{\beta}{(1 - \alpha)^\sigma (-\beta)^{1 - \sigma} + \alpha^\sigma r^{1 - \sigma}}
\]

\[
L^{LR} = \frac{\beta}{(1 - \alpha)^\sigma (-\beta)^{1 - \sigma} + \alpha^\sigma r^{1 - \sigma}}
\]

It follows that \( \frac{\partial L^{LR}}{\partial \beta} > 0 \). Further, \( \frac{\partial K^{LR}}{\partial \beta} > 0 \) if \( \sigma < 1 \), \( \frac{\partial K^{LR}}{\partial \beta} < 0 \) if \( \sigma > 1 \), and \( \frac{\partial K^{LR}}{\partial \beta} = 0 \) if \( \sigma = 1 \). Thus, \( K \) only rises in the long run following an increase in \( \beta \) if \( K \) and \( L \) are sufficiently strong complements (\( \sigma < 1 \)). If \( K \) and \( L \) are only weak complements, independent, or substitutes in profits (\( \sigma \in (1, \frac{1}{1 - \rho}) \), \( \sigma = \frac{1}{1 - \rho} \), and \( \sigma \in (\frac{1}{1 - \rho}, \infty) \), respectively), an increase in \( \beta \) ultimately leads to a reduction in \( K \).
Using (11) and (13), the short-run value of $L$ is implicitly determined by

$$
\frac{rK}{1-\rho} \left[ \frac{-\beta}{r} \frac{\alpha}{1-\alpha} \left( \frac{L^{SR}}{K} \right)^{1/\sigma} - \rho \right] - \beta L^{SR} = \frac{\rho}{1-\rho} f. \tag{14}
$$

It follows immediately that $\frac{\partial L^{SR}}{\partial \beta} > 0$. For later use, we note that when $\sigma > 1$, $(-\beta)(L^{SR})^{1/\sigma}$ falls when $\beta$ increases. Using (14), $\frac{\partial L^{SR}}{\partial K}$ shares sign with

$$
1 - \frac{\sigma - 1}{\rho \sigma} - \frac{\beta}{r} \frac{\alpha}{1-\alpha} \left( \frac{L^{SR}}{K} \right)^{1/\sigma}. \tag{15}
$$

Suppose that $\sigma < 1$. Then it is obvious that $\frac{\partial L^{SR}}{\partial \beta} > 0$. Thus, in this case, we have that an increase in $\beta$ implies $L^0 < L^{SR} < L^{LR}$ and a reduction in $\beta$ implies $L^0 > L^{SR} > L^{LR}$ where $L^0$ denotes the initial level of labour demand. The LCP therefore holds for both cases. Consider next the case where $\beta$ increases and $\sigma \in (1, \frac{1}{1-\rho})$ such that $K$ and $L$ are complements in profits, but only weakly so. Then we know that $\frac{\sigma - 1}{\rho \sigma} < 1$ and that $(-\beta)(L^{SR})^{1/\sigma}$ falls when $\beta$ increases. This means that (15) is positive at $K = K^0$. Further, (15) changes continuously as $K$ falls from $K^0$ to $K^{LR}$ and when it hits $1 - \frac{\sigma - 1}{\rho \sigma} > 0$, we are in the new long-run equilibrium. Thus, $\frac{\partial L^{SR}}{\partial K}$ is positive on $K \in (K^{LR}, K^0)$. But then, for $\sigma \in (1, \frac{1}{1-\rho})$, an increase in $\beta$ implies that $L^0 < L^{LR} < L^{SR}$ and the LCP does not hold in this case.

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