Essays on Economic Policies over the Business Cycle
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A PhD thesis submitted to School of Business and Social Sciences, Aarhus University, in partial fulfilment of the requirements of the PhD degree in Economics and Business

May 2013
Preface

This PhD thesis was written in the period from February 2010 to January 2013 during my studies at the Department of Economics and Business, Aarhus University. I am grateful to the department for providing an excellent research environment and for generous financial support, which has allowed me to attend numerous courses, summer schools, workshops and conferences.

There are a number of people I wish to thank. First and foremost, I thank my supervisor Torben M. Andersen for encouraging me to enroll as a PhD student, for always taking the time for me, for very constructive and helpful comments along the way, and for always radiating contagious calmness and surplus energy. I feel truly privileged to have worked with and learned from such an inspiring knowledge bank. My co-supervisor Michael Svarer also deserves special thanks, most of all for his good spirits and for introducing me to the exciting topic of labor market policies across the business cycle, which in essence has defined my academic career until now. Furthermore, the third chapter of this thesis is joint work with my two supervisors, and the process of writing this chapter (along with the research assistance I have had the pleasure of doing for them during the last almost five years) has without a doubt learned me more than any course I have ever followed.

From September 2011 to December 2011 I had the great pleasure of being a visiting research scholar at Tepper School of Business at Carnegie Mellon University. My stay in this very strong research environment has inspired me tremendously, and the hospitality of the department and my host Dennis Epple in particular is gratefully acknowledged. During my stay I also benefitted greatly from discussing my work with Nicolas Petrosky-Nadeau and Laurence Ales.

Several colleagues at the Department of Economics and Business deserve a special thank you. Importantly, Mikkel and I have discussed each other’s work with great value, as well as multiple other subjects of both economic and non-economic nature. Also, his help in the final editorial phase has improved the layout of this thesis enormously. Many great coffee breaks with various more or less intellectual discussions have been spent with Mikkel, Kenneth S, Rune L, Jonas P, Lasse, Niels S, Maria H, and Rune V. Furthermore, the latter has challenged me with many fundamental questions of economics, from which I have benefitted greatly, as well as provided me with invaluable technical support when Fortran programming (and compiling!) did not go as planned. Henning also deserves many thanks in this respect.
Special thanks go to my office mates Niels H D-H, Firew, Juan Carlos, Rasmus L, Morten K (and apparently also Jonas P), and especially with the former I have spent many fun (and sometimes quite disturbing) hours discussing almost everything ranging from the NFL to advanced macroeconomic theory. I have also had the pleasure of spending several evenings playing either hockey or soccer with fellow PhD students and sometimes even senior faculty.

My deepest gratitude goes to my friends and family, especially Mikkel and Morten HR who have followed me closely on this journey into (the darkness of) economics, the guys from back home, my wife and in-laws, my siblings and their truly better halves, and my parents who have supported me and believed in me all the way as well as provided me with an outstanding home base. Finally, I would like to thank my wife and colleague Jannie for her never-ending encouragement, love and support, as well as for reminding me whenever necessary that there is more to life than macroeconomics (e.g. microeconometrics).

Mark Strøm Kristoffersen
Aarhus, January 2013

Updated preface

The predeffence took place on April 16, 2013. I am grateful to the members of the assessment committee, Birthe Larsen, Morten O. Ravn, and Allan Sørensen (chair), for their careful reading of the dissertation and their many constructive comments and suggestions. Some of the suggestions have already been incorporated, while others remain for future work.

Mark Strøm Kristoffersen
Copenhagen, May 2013
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Summary

This thesis comprises three self-contained chapters on economic policies over the business cycle. Social insurance and optimal stabilization policies have been studied for several years. However, during the so-called Great Moderation (see e.g. Bernanke (2004)), the focus in (most of) the economics literature shifted away from stabilization policies. In this period, output volatility declined dramatically, and therefore these policies were viewed to be of minor importance. In recent years, the literature has re-blossomed, in the wake of the financial and economic crisis, now referred to as the Great Recession. In this discussion, business cycle dependent labor market policies have been suggested as a means to improve the trade-off between incentives and insurance, and at the same time strengthening the automatic stabilizers. The three papers in this thesis, and in particular the last two, can be read in this context, although the methods used and the results derived are by no means specific to the Great Recession.

In the first chapter, 'Hand-to-Mouth Consumers and Fiscal Stabilization Policy in an Open Economy,' I investigate how the presence of so-called hand-to-mouth consumers, i.e., people who neither save nor borrow, affects the need and role for fiscal stabilization policies in a small open economy. Since the intertemporal mobility is smaller in economies with many hand-to-mouth consumers, it is often argued that the need for fiscal stabilization policies is larger, and that fiscal policy is more effective, cf. IMF (2009). My results reveal that the presence of hand-to-mouth consumers may have more complex interactions with fiscal stabilization policies. The optimal stabilization policy in case of productivity shocks is independent of the fraction of hand-to-mouth consumers, and the presence of these agents actually tends to reduce the need for an active policy stabilizing productivity shocks.

In the second chapter, 'Business Cycle Dependent Unemployment Benefits with Wealth Heterogeneity and Precautionary Savings,' I study business cycle dependent unemployment benefits in a model with labor market matching, wealth heterogeneity, precautionary savings, and aggregate fluctuations in productivity. In recent years the literature on business cycle dependent labor market policies has been fast-growing, but almost all existing studies ignore savings, and thus an important determinant of the welfare gains from economic policies. Therefore, I use the model of Krusell, Mukoyama, & Şahin (2010). The results are ambiguous: both procyclical and countercyclical unemployment benefits can increase welfare relative to business cycle invariant
benefits. Procyclical benefits are beneficial due to countercyclicality of the distortionary effect (on job creation) from providing unemployment insurance, whereas countercyclical benefits facilitate consumption smoothing. The calibration strategy and the responsiveness of wages to temporary changes in the level of unemployment benefits turn out to be crucial for the results, both qualitatively and quantitatively.

In the third chapter, 'Benefit Reentitlement Conditions in Unemployment Insurance Schemes,' co-authored with Torben M. Andersen and Michael Svarer, we study the interaction between the duration of unemployment benefits and the employment requirements to qualify for unemployment benefits when designing the optimal unemployment insurance scheme. We show that the reentitlement requirement can work as a substitute to the duration of unemployment benefits. The economic structure and preferences, captured by productivity and risk aversion, respectively, are found to have important consequences for the optimal design of the unemployment insurance scheme, and this may in part explain the variation in unemployment insurance schemes across OECD countries, as documented by Venn (2012). Finally, we consider a business cycle version of our model in which the optimal unemployment insurance scheme turns out to exhibit countercyclical generosity.

Bibliography


I det andet kapitel, ’Konjunkturafhængige arbejdsløshedsdagpenge med formueheterogenitet samt opsparing,’ undersøges effekterne af konjunkturafhængige arbejdsløshedsdagpenge i en model med matching på arbejdsmarkedet, formueheterogenitet, opsparing samt aggregerede produktivitetsfluktuationer. I de senere år er litteraturen om konjunkturafhængig arbejdsmarkedspolitik vokset hastigt, men langt de fleste studier ignorerer muligheden for delvis selvforsik-


Litteratur


Chapter 1

Hand-to-Mouth Consumers and Fiscal Stabilization Policy in an Open Economy
Hand-to-Mouth Consumers and Fiscal Stabilization Policy in an Open Economy*

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May 2013

Abstract

It is often claimed that the presence of hand-to-mouth consumers enhances the need for and the effects of fiscal stabilization policies. This paper studies this in a model of a small open economy with hand-to-mouth consumers, i.e., some households are liquidity constrained in the sense that they can neither save nor borrow. It is shown that the consequences of liquidity constraints are more complex than previously thought: The optimal stabilization policy in case of productivity shocks is independent of the fraction of hand-to-mouth consumers. Furthermore, the presence of these agents actually reduces the need for an active policy stabilizing productivity shocks.

JEL Classification: E32, E63, F41
Keywords: Liquidity constraints; Stabilization policy; Fiscal policy; Small open economy

*This paper was previously titled *Liquidity Constraints and Fiscal Stabilization Policy*. Comments from Torben M. Andersen, Laurence Ales, Mikkel N. Hermansen, Birthe Larsen, Morten O. Ravn, Allan Sørensen, as well as participants in seminars at Aarhus University and Tepper School of Business, Carnegie Mellon University, are gratefully acknowledged. All remaining errors are, of course, my own.

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1.1 Introduction

It is often claimed that the presence of hand-to-mouth consumers enhances both the need for and the effects of fiscal stabilization policies. According to IMF (2009) fiscal policy is more effective when economic agents face tighter liquidity constraints, a conclusion partly based on the findings in Tagkalakis (2008). The basic intuition is that intertemporal mobility is lower, when liquidity constraints are tight, and therefore the response to both shocks and policy stimulus is larger. There is a large literature on liquidity constraints originating from Jappelli & Pagano (1989) and Campbell & Mankiw (1991), see e.g. Mankiw (2000) and Galí, López-Salido, & Vallés (2007), but despite this very few studies (empirical as well as theoretical) support these presumptions.

The existing literature on the interaction between the presence of hand-to-mouth consumers and fiscal policy is scarce. One exception is Galí et al. (2007) who find that the presence of hand-to-mouth consumers causes government spending to crowd in private consumption. However, other studies find little evidence in support of this crowding-in effect, see e.g. Horvath (2009).

Furthermore, most of the existing literature only considers closed economies and thus abstracts from potentially important open economy effects. One exception is Heathcote (2005) who considers both a closed economy and a small open economy setting in his study of the short-run effects of changing the timing of income taxes when consumers face a borrowing constraint. He finds that temporary tax cuts boost aggregate consumption, and this increase in aggregate consumption is larger in the small open economy setting (exogenous factor prices) than in the closed economy (endogenous factor prices). Another exception is Beetsma & Giuliodori (2011) who find that the stimulating effect of government purchases is weaker for the more open EU countries.

The financial and economic crisis has emphasized that access to credit is an important determinant of economic fluctuations and aggregate demand. Also, Sarantis & Stewart (2003) find that the average proportion of current income consumers is 70.6% across the 20 OECD countries considered, and it varies from 33.1% (in the UK) to 99.3% (in the Netherlands).

In a more recent study, Kreiner, Lassen, & Leth-Petersen (2012) use a Danish unanticipated stimulus reform (where households had the possibility to get the so-called Special Pension, SP, paid out) to test whether households respond more strongly to a fiscal stimulus if they face credit market imperfections. They find a strong positive relationship between the response in household consumption and the tightness of credit constraints.

Nowadays, hand-to-mouth consumers are included in most models used for evaluation of fiscal policies. As an example consider FiMod, the DSGE model of Banco de España and

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1 There are several possible microfoundations for hand-to-mouth consumption, e.g. myopia or borrowing constraints. In the literature, hand-to-mouth consumers are sometimes referred to as rule-of-thumb, non-Ricardian or liquidity constrained consumers in the sense that they can neither save nor borrow. This paper uses these terms interchangeably, even though we do not take a stand on the microfoundation.
Deutsche Bundesbank used for fiscal policy simulations, cf. Stähler & Thomas (2012).

In light of this, it is surprising that the economic literature is largely silent regarding the interactions between liquidity constraints and stabilization policies. The novelty of the current paper is to consider the relationship between hand-to-mouth consumption and fiscal stabilization in a relatively simple model of a small open economy (with both tradeable and non-tradeable goods). The aim of this paper is to provide new insights on the effects of introducing hand-to-mouth consumers in intertemporal models of small open economies, and (perhaps surprising) results are derived for the need and role of an active fiscal stabilization policy.

In case of productivity shocks the optimal fiscal stabilization policy is derived, and this policy turns out to be independent of the fraction of hand-to-mouth consumers. Furthermore, the results indicate that the need for stabilization of productivity shocks is actually decreasing in the fraction being liquidity constrained. The intuition is that demand (of non-tradeables) has to increase following a positive productivity shock in order to counteract the increase in supply (of tradeables). This increase in demand is larger when more households consume in a hand-to-mouth fashion, since these households do not smooth out the windfall gain. Hence, the consequences of liquidity constraints are not as straightforward as usually argued.

The rest of the paper is organized as follows: The model is set up in Section 1.2, while Section 1.3 considers the steady state, and Section 1.4 analyzes a supply shock. Finally, Section 1.5 offers some concluding remarks.

1.2 Model

This paper considers a two-sector model for a small open economy, where one sector produces a tradeable good and the other sector produces a non-tradeable good. The model set-up largely follows Andersen & Holden (2002), except for the liquidity constraints. The price of tradeables (in domestic currency) is given exogenously from the world market, whereas the price of non-tradeables is determined endogenously. The population is normalized to unity. A fraction $\lambda$ of the households is hand-to-mouth consumers (liquidity constrained) since they simply spend their current income, whereas a fraction $1-\lambda$ has full access to saving and borrowing. Agents are risk averse, and the households own the firms. Furthermore, capital markets are incomplete, i.e., there exists an internationally traded bond but equities are not traded internationally. The public sector collects taxes and there is a public demand for non-tradeable goods. Finally, business cycle fluctuations are generated by supply shocks.

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2Distinguishing between tradeables and non-tradeables is a very simple way to model a small open economy since the impacts of foreign shocks are collected into the price of tradeables. Tinbergen (1965) was the first to use this method (in a Keynesian model). He referred to these goods as international and national goods, respectively.
1.2.1 Households

It is assumed that the households inelastically supply a given amount of labor \((L)\). Each household has an infinite horizon, and their objective is to maximize expected lifetime utility given by

\[
U_t = \mathbb{E}_t \left[ \sum_{j=0}^{\infty} (1 + \rho)^{-j} u (b_{t+j}) \right] \tag{1.1}
\]

where \(\mathbb{E}_t\) denotes the expectation operator given information at time \(t\), the subjective rate of time preference is \(\rho > 0\), and \(u (\cdot)\) is the instantaneous utility function given by

\[
u (b_{t+j}) = b_{t+j} - \frac{k}{2} (b_{t+j})^2, \quad k > 0 \tag{1.2}
\]

where \(b_{t+j}\) is a composite index of consumption of non-tradeables \((c_{t+j}^{NT})\) and tradeables \((c_{t+j}^{T})\) defined as

\[
b_{t+j} = \frac{1}{\Omega} \left( c_{t+j}^{NT} \right)^{\alpha} \left( c_{t+j}^{T} \right)^{1-\alpha}, \quad 0 < \alpha < 1 \tag{1.3}
\]

with \(\Omega \equiv \alpha^\alpha (1 - \alpha)^{1-\alpha}\). Hence, there is risk aversion with respect to the composition of the consumption bundle.

Since preferences are homothetic, the optimal consumption decision can be split into two, i.e., first the household maximizes the value of the composite consumption bundle for a given level of nominal expenditures \(M_{t+j}\) in period \(t + j\), and secondly the household chooses how much to spend each period. Denoting the price of non-tradeables (tradeables) by \(P_{t+j}^{NT} \ (P_{t+j}^{T})\) nominal expenditures in period \(t + j\) are defined as

\[
M_{t+j} = P_{t+j}^{NT} c_{t+j}^{NT} + P_{t+j}^{T} c_{t+j}^{T}. \tag{1.4}
\]

Now, consider the maximization of the value of the consumption bundle for \(M_{t+j}\) given. Since the composite index is of the Cobb-Douglas type, optimal consumption implies

\[
c_{t+j}^{NT} = \alpha \frac{M_{t+j}}{P_{t+j}^{NT}} \tag{1.5}
\]

\[
c_{t+j}^{T} = (1 - \alpha) \frac{M_{t+j}}{P_{t+j}^{T}} \tag{1.6}
\]

and therefore the optimal value of the consumption bundle can be written

\[
b_{t+j} = \frac{M_{t+j}}{Q_{t+j}} \tag{1.7}
\]

where the consumer price index \(Q_{t+j}\) is defined as

\[
Q_{t+j} \equiv \left( P_{t+j}^{NT} \right)^{\alpha} \left( P_{t+j}^{T} \right)^{1-\alpha}. \tag{1.8}
\]

Next, we consider the choice of nominal expenditures \(M_{t+j}\).
1.2.1.1 Liquidity constrained households

An exogenous fraction of the households ($\lambda$) is assumed to be liquidity constrained (denoted $c$) in the sense that they simply spend their current income. As these households do not have access to saving and borrowing, they maximize (1.1) by letting

$$M^c_{t+j} = I_{t+j}$$

where $I_{t+j}$ denotes the after-tax nominal income in period $t+j$, i.e., the level of nominal expenditures is given by the nominal income. To focus on the pure effects from introducing hand-to-mouth consumers, nominal income is assumed to be independent of the liquidity status and is determined as

$$I_{t+j} = P^{NT}_{t+j} y^{NT}_{t+j} + P^T_{t+j} y^T_{t+j} - T_{t+j}$$

where $y^{NT}_{t+j}$ ($y^T_{t+j}$) denotes output from the non-tradeables (tradeables) sector, and $T_{t+j}$ is a lump-sum tax paid by all households.

Thus, the constrained households’ consumption of non-tradeables and tradeables, respectively, is

$$c^{NT,c}_{t+j} = \alpha \frac{I_{t+j}}{P^{NT}_{t+j}}$$

$$c^{T,c}_{t+j} = (1 - \alpha) \frac{I_{t+j}}{P^T_{t+j}}.$$

Finally, using (1.9) in (1.7), the value of the optimal consumption bundle can be written

$$b^c_{t+j} = i_{t+j}$$

where $i_{t+j} = I_{t+j}/Q_{t+j}$.

1.2.1.2 Non-liquidity constrained households

The remaining households (fraction $1-\lambda$) have access to saving and borrowing (denoted $nc$), and therefore they maximize (1.1) subject to the intertemporal budget constraint

$$\sum_{j=0}^{\infty} \prod_{k=0}^{j} (1 + r_{t+k})^{-1} M^{nc}_{t+j} \leq \sum_{j=0}^{\infty} \prod_{k=0}^{j} (1 + r_{t+k})^{-1} I_{t+j} + F_t$$

where $F_t$ is nominal wealth at the beginning of period $t$, $r_{t+k}$ is the nominal interest rate, and nominal income $I_{t+j}$ is defined in (1.10). It is assumed that equities are not traded internationally, and therefore the risk associated with variations in domestic production (and thus income) cannot be fully diversified via the international capital market. Thus, households are

\footnote{In the rest of the paper it is assumed that marginal utility is always positive, i.e., $k b_{t+j} < 1 \forall j$.}
subjected to uninsurable risk due to incomplete capital markets, which leaves a role for an active stabilization policy, cf. Andersen (2001).

However, we still allow for some risk diversification since the non-liquidity constrained households have access to an internationally traded bond, which is assumed to offer a rate of return specified in terms of the consumption bundle, that is

$$
\frac{(1 + r_{t+1}) Q_t}{Q_{t+1}} = 1 + \delta_t. \tag{1.15}
$$

To prevent the country from accumulating or decumulating foreign debt forever, it is assumed that $\delta_t = \rho \forall t$, i.e., the objective and the subjective discount rates are equal, which implies that the real rate of return on the bond is riskless, and henceforth denoted $1 + \delta$.

These assumptions enable us to write the intertemporal budget constraint (1.14) as

$$
\sum_{j=0}^{\infty} (1 + \delta)^{-j} b_{t+j}^{nc} \leq \sum_{j=0}^{\infty} (1 + \delta)^{-j} i_{t+j} f_t \tag{1.16}
$$

where $i_{t+j} = I_{t+j}/Q_{t+j}$ and $f_t = F_t/Q_t$ are measured in real terms. To simplify notation it is useful to define

$$
A_t \equiv \sum_{j=0}^{\infty} (1 + \delta)^{-j} E_t [i_{t+j}] + f_t \tag{1.17}
$$

which is the expected present value of the household’s wealth – measured in real terms. Maximization of (1.1) subject to (1.16) yields

$$
b_{t}^{nc} = \frac{\delta}{1 + \delta} A_t \tag{1.18}
$$

and the associated no-Ponzi game condition is

$$
\lim_{T \to \infty} (1 + \delta)^{-T} f_{t+T} = 0. \tag{1.19}
$$

Furthermore, we have that

$$
E_t [i_{t+j}] = b_{t+j}^{nc}, \quad j \geq 0 \tag{1.20}
$$
$$
E_t [A_{t+j}] = A_t, \quad j \geq 0 \tag{1.21}
$$

and hence, consumption and wealth follow random walks.

Finally, for the non-constrained households the consumption of non-tradeables and tradeables is

$$
\begin{align*}
c_{t,NT,ne} &= \alpha \frac{\delta}{1 + \delta} A_t Q_t P_{t}^{NT} \tag{1.22} \\
c_{t,ne} &= (1 - \alpha) \frac{\delta}{1 + \delta} A_t Q_t P_{t}^{T}. \tag{1.23}
\end{align*}
$$

\footnote{Andersen & Holden (2002) discuss this assumption in more detail.}
1.2.1.3 Aggregation

Aggregate demand for the two goods is then

\[ c_t^{NT,Agg} = \lambda c_t^{NT,c} + (1 - \lambda) c_t^{NT,nc} \] (1.24)
\[ c_t^{T,Agg} = \lambda c_t^{T,c} + (1 - \lambda) c_t^{T,nc} \] (1.25)

with \( \lambda \in [0, 1] \).

1.2.2 Firms

Firms are either producing tradeables or non-tradeables, and all firms are price and wage takers. The production function of a firm of type \( h = NT, T \) is

\[ y^h_t = \frac{\eta_t}{\beta} \left( L^h_t \right)^{\beta}, \quad 0 < \beta < 1 \] (1.26)

where \( L^h_t \) is labor input, and \( \eta_t \) is a productivity parameter. Firms maximize profits, which yields the following demand for labor

\[ L^h_t = \left( \frac{\eta_t}{W_t} P^h_t \right)^{\frac{1}{1-\beta}} \] (1.27)

where \( W_t \) is the nominal wage rate, and therefore the output supply function is

\[ y^h_t = \frac{1}{\beta} \eta_t^{\frac{1}{1-\beta}} \left( \frac{P^h_t}{W_t} \right)^{\frac{\beta}{1-\beta}} \] (1.28)

with \( h = NT, T \).

1.2.3 Wages

It is assumed that the labor market is competitive, and therefore the (nominal) wage is determined from the market clearing condition

\[ L = N^{NT} \left( \frac{\eta_t P^{NT}}{W_t} \right)^{\frac{1}{1-\beta}} + N^T \left( \frac{\eta_t P^T}{W_t} \right)^{\frac{1}{1-\beta}} \] (1.29)

where \( N^{NT} \) (\( N^T \)) is the number of firms producing non-tradeables (tradeables). Therefore, the equilibrium wage can be written

\[ W_t = W \left( P^{NT}_t, P^T_t, \eta_t \right) \] (1.30)

and using this in the output supply functions yields

\[ y_t^{NT} = y^{NT} \left( P^{NT}_t, P^T_t, \eta_t \right) \] (1.31)
\[ y_t^T = y^T \left( P_{t+1}^{NT}, P^T_t, \eta_t \right) \]  
(1.32)

where the signs of the partial derivatives follow from (1.28) and (1.29). Hence, an increase in the price of tradeables (non-tradeables) decreases the output supply of non-tradeables (tradeables) since the wage increases. However, an increase in the own price increases the output supply, since the wage increases less than proportional to the increase in the price, and thus the sector-specific real wage decreases.

1.2.4 Public sector

The objective of the public sector is to minimize the variance of the consumption bundles. In order to do so, the public sector demands non-tradeables \( g_{t}^{NT} \), and this is financed via lump-sum taxes. It is assumed that the public sector runs a balanced budget, i.e.,

\[ P_{t}^{NT} g_{t}^{NT} = T_t. \]  
(1.33)

Public demand is assumed not to affect directly the utility of households to focus on the pure demand effects.\(^5\) Furthermore, it is assumed, for simplicity, that the public sector does not demand tradeables, as this would only affect the domestic economy through increasing the tax burden and worsening the trade balance.

1.2.5 Equilibrium

The market for non-tradeables is in equilibrium when

\[ y_{t}^{NT} = c_t^{NT, Agg} + g_{t}^{NT}. \]  
(1.34)

The trade balance \( (x_t) \) is determined by the excess supply of tradeables

\[ x_t = y_t^T - c_t^{T, Agg}. \]  
(1.35)

This closes the model.\(^6\)

1.2.6 Consumption risk

In Appendix 1.A it is shown that the expected present value of the household’s wealth can be rewritten as

\[ A_t = \frac{1 - \lambda \alpha}{1 - \alpha} f_t + \frac{1}{1 - \alpha} \sum_{j=0}^{\infty} (1 + \delta)^{-j} \mathbb{E}_t \left( \frac{P_{t+j}^{T} y_{t+j}}{Q_{t+j}} \right) \]  
(1.36)

\(^5\)It would suffice to assume that public demand is separable from private consumption in the household utility function.

\(^6\)Note that it is implicitly assumed that neither firms nor workers are internationally mobile or that they have no incentives to migrate due to fiscal policy or shocks.
and combined with (1.18) this shows that the risk in the private consumption bundle of the non-constrained households arises from variability in the real income generated in the tradeables sector, \( \frac{P^T_{t+j}}{Q^{t+j}} y^{T}_{t+j} \). The same is true for the constrained households. This is seen by combining (1.13) with (see Appendix 1.A for a derivation)

\[
i_t = \pi_0 f_t + \pi_1 \frac{P^T_t}{Q_t} y^T_t + \pi_2 \sum_{j=0}^{\infty} (1 + \delta)^{-j} E_t \left( \frac{P^T_{t+j}}{Q^{t+j}} y^{T}_{t+j} \right)
\]

where \( \pi_0 \equiv \frac{(1-\lambda)\alpha}{1-\alpha} \frac{\delta}{1+\delta} \), \( \pi_1 \equiv \frac{1}{1-\lambda \alpha} \), and \( \pi_2 \equiv \frac{1}{1-\alpha} \frac{(1-\lambda)\alpha}{1-\lambda \alpha} \frac{\delta}{1+\delta} \). Hence, as in Andersen & Holden (2002) consumption risk stems from real income generated in the tradeables sector, and clearly there is a potential role for an active stabilization policy aimed at stabilizing real income from the tradeables sector.

1.3 Steady state

Assuming that the non-constrained households have no initial wealth, \( F_t = f_t = 0 \), the steady state behavior of the non-constrained households is the same as that of the constrained households, which leads to the following proposition.

Proposition 1.1. The (initial) steady state is independent of the fraction of liquidity constrained households, \( \lambda \).

Proof. In steady state we have from (1.17) (dropping the time subscripts) \( A = f + \sum_{j=0}^{\infty} (1 + \delta)^{-j} i = \frac{1+\delta}{\delta} i \), assuming \( f = 0 \). Hence, \( c^{NT,c} = c^{NT,nc} \) using (1.11) and (1.22), and thus \( \frac{\partial c^{NT,Agg}}{\partial \lambda} = \frac{\partial y^{NT}}{\partial \lambda} = 0 \) from (1.24) and (1.34), respectively.

Proposition 1.1 is intuitive, since the non-constrained households with a constant real income flow \( i \) simply choose to spend their current income, as the objective and subjective discount rates are equal and the time horizon is infinite.

Andersen & Holden (1998) show that their model has a well-defined steady state. Proposition 1.1 implies that the model in this paper is identical to their model in steady state, and therefore the same is true for this model. Also, see Andersen & Holden (1998) for comparative static results of the steady state.

1.4 Productivity shocks

This section considers a supply shock, in particular a shock to productivity. Thus, it is assumed that productivity behaves according to

\[ \eta_t = \bar{\eta} + \varepsilon_t \]
where $\varepsilon_t$ is the deviation of productivity from its long-run value, $\bar{\eta}$, and we assume that these deviations are serially uncorrelated and unexpected, i.e., $\mathbb{E}_t [\varepsilon_{t+j}] = 0 \forall j > 0$ and $\mathbb{E}_t [\varepsilon_{t+j}\varepsilon_{t+k}] = 0 \forall j \neq k$.

To obtain analytical solutions we rely on linearizations around the initial steady state. To ease notation let $r x^h_t \equiv \frac{P^h_t x^h_t}{q_t}$ denote the deflated measure of a variable $x^h_t$ with $h = NT, T$, and let $\tilde{r} x^h_t$ denote the deviation of $r x^h_t$ from its steady state value.

It is assumed that public demand for non-tradeables follows the rule

$$\tilde{r} g^T_t = \kappa \varepsilon_t \tag{1.38}$$

where $\kappa$ is the stabilization parameter chosen by the government. Note that the policy rule only specifies the change in government spending following a shock. Below we show that this policy rule can be used to completely stabilize the consumption bundles, and therefore it meets the objective of minimizing the variance of the consumption bundles. This also implies that the balanced budget assumption is not restrictive in this model.

### 1.4.1 No stabilization

At first, consider the case of no stabilization, i.e., $\kappa = 0$. In Appendix 1.B it is shown that real income generated in the tradeables sector evolves as

$$\tilde{r} y^T_t = \chi_1 \left( -\chi_2 f_t - \chi_3 \sum_{j=1}^{\infty} (1 + \delta)^{-j} \mathbb{E}_t [\tilde{r} y^T_{t+j}] + \chi_4 \varepsilon_t \right) \tag{1.39}$$

where the $\chi$'s are defined in the appendix, $\chi_1, \chi_2, \chi_3 > 0$, and $\chi_4 > 0$ for $\kappa = 0$. Hence, the immediate effect of a positive productivity shock is that the real income generated in the tradeables sector exceeds its steady state level, since more is produced of both the tradeable and the non-tradeable good, cf. (1.31) and (1.32), and the latter implies a drop in the relative price of non-tradeables, cf. (1.34).

Since $\frac{\partial \chi_1}{\partial \lambda} < 0$ (and $\frac{\partial \chi_4}{\partial \lambda} = 0$), the direct effect of a productivity shock is diminishing in the fraction being liquidity constrained.\(^7\) Hence, there is a lower variation in real income from the tradeables sector, and thus in the consumption bundles, cf. (1.36) and (1.37), when liquidity constraints are tight. The immediate consequence of this result is that the need for an active stabilization policy following a supply shock is decreasing in the fraction being liquidity constrained.

\(^7\)In general, only the sign of the direct effect can be determined. However, the extreme case where all consumers are liquidity constrained, $\lambda = 1$, sheds some light on the total effect. Since $\frac{d \tilde{r} y^T_t / d \varepsilon_t}{d \lambda} \bigg|_{\lambda=1} = \frac{d \chi_1}{d \lambda} \chi_4 < 0$, a marginal decrease in $\lambda$ increases the volatility in the real income from the tradeables sector following a productivity shock, and thus the variance of the value of the consumption bundle increases, implying a larger need for stabilizing policies.
This result may at first come as a surprise, but intuition is straightforward. Following a positive productivity shock, the supply of tradeables (and non-tradeables) will increase. Hence, to stabilize real income from the tradeables sector, demand for non-tradeables will have to increase as well in order to increase the consumer price index, which can only happen if the price of non-tradeables increases (recall the exogeneity of the price of tradeables). The rise in production, and thus income, following the increase in productivity raises consumption for both constrained and non-constrained households. However, the non-constrained will smooth out this windfall gain, leading to a smaller increase in demand (for both tradeables and non-tradeables), and thus a smaller increase in the relative price of non-tradeables, than when more consumers are consuming in a hand-to-mouth fashion. I.e., the ability for some consumers to smooth out consumption weakens the stabilizing effect stemming from a rise in demand in response to a positive productivity shock. Vice versa for a drop in productivity.

Hence, in this model the presence of hand-to-mouth consumers actually works as a stabilizer when business cycle fluctuations are driven by supply shocks. This is in contrast to the usual view that the presence of hand-to-mouth consumers amplifies business cycle fluctuations.

1.4.2 Active fiscal stabilization

Now, considering the possibility of an active fiscal stabilization policy via the public demand for non-tradeables leads to the following proposition:

**Proposition 1.2.** Following a productivity shock, there exists a choice of the stabilization parameter, \( \kappa = \kappa^* > 0 \), which ensures perfect stabilization of the values of the consumption bundles, i.e., \( \tilde{b}^{nc}_t = \tilde{b}^c_t = 0 \).

**Proof.** See Appendix 1.B.

Hence, the optimal stabilization policy implies that public demand for non-tradeables increases following a positive productivity shock since the relative price of non-tradeables (and thus the wage) has to increase enough to offset the direct effect of the productivity gain on real income generated in the tradeables sector. Since households are risk averse, this stabilization policy will on average increase welfare because consumption is stabilized.

**Proposition 1.3.** The optimal stabilization policy is independent of the fraction of hand-to-mouth consumers, i.e., \( \frac{\partial \kappa^*}{\partial \lambda} = 0 \).

**Proof.** See Appendix 1.B.

The optimal stabilization policy implies that the steady state level of consumption is attained in all periods, and therefore Proposition 1.3 follows directly from Proposition 1.1. Hence,
in this model fiscal policy is effective, but it is not true that fiscal policy is more effective in stabilizing the economy when liquidity constraints are tight.

Finally note that the discussion above assumes that the supply shock affects productivity in both sectors. However, the same qualitative results are obtained in case of asymmetric shocks, i.e., by assuming that only one of the sectors is affected, cf. Appendix 1.B.

1.5 Concluding remarks

This paper introduced hand-to-mouth consumers in a model of a small open economy. Following a productivity shock it was shown that there exists an active fiscal stabilization policy which is able to perfectly stabilize the consumption bundles. This policy is independent of the liquidity constraints in the sense that the optimal reaction of public consumption does not depend on the fraction consuming in a hand-to-mouth fashion. Furthermore, the presence of hand-to-mouth consumers actually reduces the need for an active policy stabilizing productivity shocks.

Hence, the consequences of hand-to-mouth consumption are more involved than previously thought, and there is clearly a need for more work in this area. Also note that some of the usual arguments carry over to this model: From a policy perspective it will be possible to temporarily boost (reduce) aggregate demand by making a positive (negative) temporary transfer from the non-constrained to the constrained households. Note that the policy does not have to be unexpected in order to achieve this reaction in aggregate demand. On the other hand, the temporary nature of the transfer is crucial, since there are no activity effects of permanent transfers, but only redistributive effects.

Finally, it is worth noting that the assumption of flexible wages is crucial for the results in this paper. If wages were completely rigid, there would be no role for fiscal policy in this model (see Appendix 1.B). Furthermore, the model used is quite stylized due to the many assumptions needed to obtain the analytical results. It would therefore be interesting for future research to investigate, whether the results generalize to more advanced small open economy models and to other types of shocks.

\footnote{Formally this is seen by replacing (1.10) with \( i_{t+j}^c = \frac{p_{t+j}^N y_{t+j}^N}{q_{t+j}^N} + \frac{p_{t+j}^T y_{t+j}^T}{q_{t+j}^T} - \frac{T_{t+j}}{q_{t+j}^T} + \frac{1}{\lambda} u_t \) and \( i_{t+j}^{nc} = \frac{P_{t+j}^N y_{t+j}^N}{q_{t+j}^N} + \frac{T_{t+j}}{q_{t+j}^N} - \frac{1}{1-\lambda} u_t \), where \( u_t \) is the total real amount transferred from the non-constrained households to the constrained, and \( \lambda \in (0,1) \). Note that ceteris paribus the transfer does not affect aggregate income, but only the distribution of income. Since the non-constrained households smooth out the temporary windfall gain/loss but the constrained do not, aggregate consumption of non-tradeables will, ceteris paribus, change by \( \alpha \frac{Q_t^N}{\bar{\pi}} \frac{P_{t+j}^N y_{t+j}^N}{q_{t+j}^N} \), cf. (1.11), (1.22) and (1.24).

\footnote{To see this let \( u_{t+j} = \pi \forall j \geq 0 \). Then, the economy will immediately reach a new steady-state, where \( c_t^N \cdot e \) has changed by \( \alpha \frac{Q_t^N}{\bar{\pi}} \frac{1}{\lambda} \), cf. (1.11), and \( c_t^{NT,nc} \) has changed by \( \alpha \frac{Q_t^N}{\bar{\pi}} \frac{1}{\lambda} \), cf. (1.22). Hence, aggregate consumption is unchanged, cf. (1.24).}
1.6 Bibliography


Stähler, N. & C. Thomas (2012): “FiMod – A DSGE Model for Fiscal Policy Simulations,” 
Economic Modelling, 29, 239–261.

Tagkalakis, A. (2008): “The Effects of Fiscal Policy on Consumption in Recessions and 

101, in German.
Appendices

1.A Consumption risk

In this appendix (1.36) and (1.37) are derived. Using (1.10), (1.33) and (1.34) in $i_{t+j} = I_{t+j}/Q_{t+j}$ yields

$$i_{t+j} = \frac{P^{NT}_{t+j}}{Q_{t+j}} + \frac{P^T_{t+j}}{Q_{t+j}}y_{t+j}$$

$$= \lambda \alpha i_{t+j} + (1 - \lambda) \alpha \frac{\delta}{1 + \delta} A_{t+j} + \frac{P^T_{t+j}y_{t+j}}{Q_{t+j}} \Leftrightarrow$$

$$i_{t+j} = \frac{1}{1 - \lambda \alpha} \left[(1 - \lambda) \alpha \frac{\delta}{1 + \delta} A_{t+j} + \frac{P^T_{t+j}y_{t+j}}{Q_{t+j}}\right]$$

(1.40)

where the second line uses (1.11), (1.22) and (1.24). Inserting in (1.17) implies

$$A_t = f_t + \sum_{j=0}^{\infty} (1 + \delta)^{-j} E_t \left[(1 - \lambda) \alpha \frac{\delta}{1 + \delta} A_{t+j} + \frac{P^T_{t+j}y_{t+j}}{Q_{t+j}}\right] \frac{1}{1 - \alpha \lambda}$$

$$= f_t + \frac{(1 - \lambda) \alpha}{1 - \alpha \lambda} A_t + \sum_{j=0}^{\infty} (1 + \delta)^{-j} E_t \left[\frac{P^T_{t+j}y_{t+j}}{Q_{t+j}}\right] \frac{1}{1 - \alpha \lambda} \Leftrightarrow$$

$$A_t = \frac{1 - \lambda \alpha}{1 - \alpha} f_t + \frac{1}{1 - \alpha} \sum_{j=0}^{\infty} (1 + \delta)^{-j} E_t \left[\frac{P^T_{t+j}y_{t+j}}{Q_{t+j}}\right]$$

where the second line uses the random walk property of wealth, (1.20). Finally, inserting in (1.40) yields

$$i_t = \frac{(1 - \lambda) \alpha \delta}{1 - \alpha} f_t + \frac{1}{1 - \lambda \alpha} \frac{P^T_t y_{t+j}}{Q_t y_{t+j}} + \frac{1}{1 - \alpha} \frac{(1 - \lambda) \alpha \delta}{1 - \lambda \alpha} \sum_{j=0}^{\infty} (1 + \delta)^{-j} E_t \left[\frac{P^T_{t+j}y_{t+j}}{Q_{t+j}}\right].$$
1.B Productivity shocks

This appendix proves Propositions 1.2 and 1.3. Using (1.11), (1.22) and (1.24) in (1.34) and rewriting yields

\[ ry_t^N = \lambda \alpha i_t + (1 - \lambda) \alpha \frac{\delta}{1 + \delta} A_t + ry_t^N. \]  

(1.41)

Furthermore, by making a first-order Taylor approximation of (1.31) and (1.32) around the initial steady state, we get

\[ \tilde{r}_y^N t = \gamma_0 \tilde{P}^N t + \gamma_1 \varepsilon_t \]  

(1.42)

\[ \tilde{r}_y T t = -\rho_0 \tilde{P}^N t + \rho_1 \varepsilon_t \]  

(1.43)

where \( \gamma_0, \gamma_1, \rho_0, \rho_1 > 0 \) follows directly from (1.31) and (1.32). Combining the linearized version of (1.41) with (1.42) and inserting (1.38) yields

\[ \tilde{P}^N t = \lambda \alpha \gamma_0 \tilde{i}_t + (1 - \lambda) \alpha \frac{\delta}{1 + \delta} \tilde{A}_t + \frac{\kappa - \gamma_1}{\gamma_0} \varepsilon_t. \]

Inserting this and the linearized versions of (1.36) and (1.37) in (1.43) implies

\[ \tilde{r}_y T t = \chi_1 \left( -\chi_2 \tilde{f}_t - \chi_3 \sum_{j=1}^{\infty} (1 + \delta)^{-j} E_t [\tilde{r}_y T t + j] + \chi_4 \varepsilon_t \right) \]

where \( \chi_1 \equiv \left( 1 + \frac{\omega_0}{\gamma_0} \right) \left( 1 + \frac{1 - \alpha}{1 - \alpha} \right) \left( 1 - \frac{\delta}{1 - \delta} \right) > 0, \chi_2 \equiv \frac{\omega_0}{\gamma_0} (1 - \lambda) \frac{\alpha}{1 - \alpha} \frac{\delta}{1 + \delta} > 0, \chi_3 \equiv \frac{\omega_0}{\gamma_0} \frac{\alpha}{1 - \alpha} \frac{\delta}{1 + \delta} > 0, \text{ and } \chi_4 \equiv \rho_1 - \rho_0 \frac{\kappa - \gamma_1}{\gamma_0} \geq 0. \)

Choosing \( \kappa = \gamma_1 + \frac{\rho_1}{\rho_0} \gamma_0 \equiv \kappa^* > 0 \) implies \( \tilde{r}_y T t = 0. \) To see this note that initially \( \tilde{f}_t = 0. \) Furthermore, \( \tilde{A}_t = \tilde{i}_t = 0 \) from (1.36) and (1.37) when \( E_t [\tilde{r}_y T t + j] = 0 \) \( \forall j > 0. \) To see the latter point, note that \( \tilde{f}_{t+1} = (1 + \delta) \left( \tilde{f}_t + \tilde{i}_t - \tilde{b}_t \right) = 0, \) and therefore there are no effects in future periods. Hence, using these results in (1.13) and (1.18) we have proven \( \tilde{b}_t^c = \tilde{b}_t^nc = 0. \)

Thus, \( \frac{\partial \kappa^*}{\partial \lambda} = \frac{\partial \left( \gamma_1 + \frac{\rho_1}{\rho_0} \gamma_0 \right)}{\partial \lambda} = 0, \) since the fraction of liquidity constrained households does not affect the steady state (recall that the \( \gamma \)'s and the \( \rho \)'s are partial derivatives evaluated in steady state), cf. Proposition 1.1.

Asymmetric shocks can be analyzed either by setting \( \gamma_1 = 0 \) (only shocks to the tradeables sector) or by setting \( \rho_1 = 0 \) (only shocks to the non-tradeables sector), which yields the optimal stabilization parameters \( \kappa^*|_{\gamma_1=0} = \frac{\rho_1}{\rho_0} \gamma_0 > 0 \) and \( \kappa^*|_{\rho_1=0} = \gamma_1 > 0, \) i.e., the qualitative results are not altered.

If, on the other hand, wages are completely rigid, we have \( \rho_0 = 0, \) and therefore \( \tilde{r}_y T t = \rho_1 \varepsilon_t \) always, with no possibility for fiscal policy to stabilize real income from the tradeables sector.
Chapter 2

Business Cycle Dependent Unemployment Benefits with Wealth Heterogeneity and Precautionary Savings
Business Cycle Dependent Unemployment Benefits with Wealth Heterogeneity and Precautionary Savings*

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May 2013

Abstract

In the wake of the financial and economic crisis the discussion about social insurance and optimal stabilization policies has re-blossomed. This paper adds to the literature by studying the effects of a business cycle dependent level of unemployment benefits in a model with labor market matching, wealth heterogeneity, precautionary savings, and aggregate fluctuations in productivity. The results are ambiguous: both procyclical and countercyclical unemployment benefits can increase welfare relative to business cycle invariant benefits. Procyclical benefits are beneficial due to countercyclicality of the distortionary effect (on job creation) from providing unemployment insurance, whereas countercyclical benefits facilitate consumption smoothing. The calibration strategy and the responsiveness of wages to temporary changes in the level of unemployment benefits turn out to be crucial for the results, both qualitatively and quantitatively.

JEL Classification: E32, H3, J65

Keywords: Unemployment insurance; Business cycles; Wealth heterogeneity; Precautionary savings

*Part of this paper was written while the author was visiting Tepper School of Business, Carnegie Mellon University. The author is thankful for their kind hospitality. Comments from Torben M. Andersen, James Costain, Mikkel N. Hermansen, Per Krusell, Nicolas Petrosky-Nadeau, Morten O. Ravn, Rune M. Vejlin, Randall Wright as well as participants in seminars at Aarhus University, in the 2012 annual meeting of the Danish Econometric Society, in the 6th Nordic Summer Symposium in Macroeconomics, in the 2012 Cycles, Adjustment, and Policy Conference, and in the 2012 DGPE Conference are gratefully acknowledged. All remaining errors are, of course, my own.

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2.1 Introduction

In the wake of the financial and economic crisis the discussion about social insurance and optimal stabilization policies has re-blossomed. To let labor market policies, e.g. the unemployment insurance scheme, depend on the position of the business cycle has been emphasized as a way to strengthen both social insurance and the (automatic) stabilization of economic fluctuations, but the literature (both theoretical and empirical) on these subjects is still of modest size.

The optimal design of unemployment insurance (UI) has been studied for several years, see Fredriksson & Holmlund (2006) for a survey of the literature. However, only recently the literature has started to investigate the effects of UI across the business cycle. The firsts to consider UI in a business cycle context were Kiley (2003), Sánchez (2008), and Andersen & Svarer (2011b), who all used static models not allowing for shifts between good and bad times.

Recently, the literature has been fast-growing, and business cycle dependent UI is now being analyzed in dynamic models allowing for shifts between recessions and booms, see\(^1\) Moyen & Stähler (2009), Andersen & Svarer (2010), Landais, Michaillat, & Saez (2010), Kroft & Notowidigdo (2011), Mitman & Rabinovich (2011), Jung & Kuester (2011), Ek (2012), and Schuster (2012). However, the conclusions are open as some papers suggest that unemployment benefits (both level and duration) should be countercyclical, i.e., the UI system should be more generous in bad times than in good times, whereas others suggest procyclical UI generosity.

Importantly, all of the above do not allow for savings, and thus, they neither allow for wealth heterogeneity nor partial self-insurance in the form of precautionary savings.\(^2\)

Accounting for precautionary savings is potentially very important when studying the insurance effects of unemployment benefits. The opportunity for individuals to self-insure has important consequences for optimal UI, as shown by Abdulkadiroğlu, Kuruşçu, & Şahin (2002). They also show that UI schemes which are designed ignoring the possibility of partial self-insurance via savings can actually be harmful to the economy.

Accounting for the heterogeneity of economic agents, e.g. wealth heterogeneity, has proven to be crucial when answering important economic questions. As an example, the welfare costs of business cycles are orders of magnitude larger in models with heterogeneous agents than originally suggested by Lucas (1987; 2003), see e.g. Storesletten, Telmer, & Yaron (2001), and Krusell, Mukoyama, Şahin, & Smith (2009). For surveys of the fast growing literature with heterogeneous agents models see Heathcote, Storesletten, & Violante (2009) and Guvenen (2011).

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\(^1\)For considerations about the practical implementation of business cycle contingent unemployment insurance along with more thorough reviews of this literature (both the theoretical and the empirical) see Andersen & Svarer (2009; 2011a).

\(^2\)Landais et al. (2010) briefly consider self-insurance in the form of home production in their one-period model.
Several papers have studied unemployment insurance in models with savings and heterogeneous agents, see Young (2004), Pollak (2007), Reichling (2007), Lentz (2009), Krusell, Mukoyama, & Şahin (2010), Vejlin (2011), and Mukoyama (2011). However, none of these papers study unemployment insurance in a business cycle context.

In this sense, Costain & Reiter (2005) are more related to this paper. They find that procyclical social security contributions are optimal, while unemployment benefits should be almost constant across states. However, their model is different from the model used in this paper in some respects, for example their asset structure is more simplistic, i.e., the interest rate is fixed, there is no physical capital in the production process, and wages are independent of asset holdings.

This paper studies the effects of a business cycle dependent level of unemployment benefits in a model with wealth heterogeneity and precautionary savings. We use the model with aggregate fluctuations in productivity from Krusell et al. (2010), who only explicitly analyze UI in the steady state version of their model. Since labor supply and search effort are both exogenous, UI does not cause moral hazards on the worker side, and therefore the optimal (linear) UI scheme does not trade off insurance and incentives to work/search, but instead it trades off insurance and job creation. The model is basically a merger between two strands of the literature: i) the Bewley-Huggett-Aiyagari model, where risk-averse consumers face idiosyncratic earnings risks, against which they can only insure partially (through savings), and ii) the Diamond-Mortensen-Pissarides (DMP) search/matching model of the labor market, where equilibrium unemployment and vacancies are determined endogenously.

The model is calibrated to US data, and we find that both procyclical and countercyclical unemployment benefits can increase welfare relative to business cycle invariant benefits. Procyclical UI is beneficial because the distortionary effect of UI (on job creation) is countercyclical, and thus, the employment process is stabilized, whereas countercyclical UI is beneficial because it facilitates consumption smoothing and raises mean consumption, i.e., it stabilizes the labor income process through stabilization of wages. It turns out that there is a non-monotone relationship between these counteracting effects. The largest welfare gain (in consumption equivalent terms) is obtained by having procyclical unemployment benefits when UI benefits are conditioned on (current) productivity. This finding is robust to changing the calibration strategy, but it turns out that the chosen calibration strategy is crucial for the magnitude of the welfare gain obtained by shifting from constant UI across the business cycle to procyclical UI benefits. However, if UI benefits are conditioned on either the unemployment level or lagged productivity instead and the public budget is allowed to work as a buffer, the largest welfare gain is achieved from countercyclical UI generosity. The same conclusion is reached if wages are completely rigid.

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3See Bewley (undated), Huggett (1993), and Aiyagari (1994).
The rest of the paper is structured as follows: The model is presented in Section 2.2, and Section 2.3 explains how the model is solved numerically. Section 2.4 considers the effects of business cycle dependent unemployment benefits, whereas Section 2.5 contains various robustness checks and extensions. Finally, Section 2.6 concludes.

## 2.2 Model

We use the model with aggregate productivity shocks from Krusell et al. (2010) with some minor extensions. Time is discrete. Following Krusell & Smith (1998) it is assumed that aggregate productivity takes on two values, $z = g$ in good periods, and $z = b$ in bad periods (with $g > b > 0$), and it follows a first-order Markov process, where the probability of moving from state $z$ to state $z'$ is denoted $\pi_{zz'} \in [0, 1]$.

### 2.2.1 Matching

The labor market has a Diamond-Mortensen-Pissarides search & matching structure. Thus, unemployed workers and vacant jobs coexist, and existing matches between a worker and a firm are assumed to be separated at the exogenous rate $\sigma \in (0, 1)$.

Unemployed workers and vacant jobs are randomly matched according to the aggregate matching function $M(u,v)$, which exhibits constant returns-to-scale and is increasing in both arguments. $u$ is the number of unemployed workers, and $v$ is the number of vacancies. Thus, the job finding probability $\lambda_w$ is

$$\lambda_w = \frac{M(u,v)}{u} = M\left(1, \frac{v}{u}\right) = M\left(1, \theta\right)$$

where the vacancy-unemployment ratio is defined as $\theta \equiv \frac{v}{u}$. The worker finding probability $\lambda_f$ is

$$\lambda_f = \frac{M(u,v)}{v} = M\left(\frac{u}{v}, 1\right) = M\left(\theta^{-1}, 1\right).$$

Hence, the law of motion for the unemployment rate $u$ is given by

$$u' = (1 - \lambda_w)u + \sigma(1 - u)$$

where a prime (') denotes a next period variable.

### 2.2.2 Asset structure

It is assumed that no markets exist for insurance against idiosyncratic employment shocks. However, there exist two assets, capital $k$ and equity $x$, where capital is used in the production process and the equity is a claim for aggregate firm profits.

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5For notational convenience time subscripts are left out throughout the model description.
The joint distribution of assets and employment across consumers is denoted \( S \), and the aggregate state in any given period is governed by \( (z, S) \).

Next period’s distribution of assets is determined in this period since it depends only on the consumers’ asset accumulations and portfolio choice decisions. Likewise, next period’s distribution of the employment statuses\(^6\) (the fraction being employed and unemployed, respectively) is also determined in this period since it follows from the law of motion of aggregate unemployment in (2.1). Therefore, the joint distribution of assets and employment across consumers in the next period is determined in this period, and we can write

\[
S' = \Omega (z, S). \tag{2.2}
\]

Hence, the aggregate state in the next period is either \((g, S')\) or \((b, S')\).

Let the consumer’s state variable be

\[
a \equiv [1 + r(z, S) - \delta] k + [p(z, S) + d(z, S)] x
\]

which is total asset holdings of the individual, and where \( \delta \) is the depreciation rate of capital; \( r(z, S) \) is the interest rate; \( p(z, S) \) is the equity price; \( d(z, S) \) is the dividend.

Like Krusell et al. (2010) we implement the portfolio choice by considering two Arrow securities, each paying one unit of the consumption good in a given state and nothing in the other state. This implementation is without loss of generality since the two assets, aggregate capital and equity, can be used to create these securities. Investment firms carry out this transformation, see below.

Let \( Q_{z'}(z, S) \) denote the price of an Arrow security that provides one unit of the consumption good in the next period if and only if the next period aggregate productivity is \( z' \) when the current state is \( (z, S) \). Then, the asset prices must satisfy the following no-arbitrage conditions

\[
1 = Q_g(z, S) [1 - \delta + r(g, S')] + Q_b(z, S) [1 - \delta + r(b, S')] \tag{2.3}
\]

\[
p(z, S) = Q_g(z, S) [p(g, S') + d(g, S')] + Q_b(z, S) [p(b, S') + d(b, S')] \tag{2.4}
\]

since we can perfectly track the returns on capital and equity by investing in the two Arrow securities.

### 2.2.3 Consumers

There is a continuum of consumers with mass 1, and these are either employed \((1 - u)\) or unemployed \(u\). The consumers face an exogenous borrowing constraint at \( a \) and are heterogeneous with respect to employment status and asset holdings. Labor supply and search effort are both exogenous.

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\(^6\)Note that only the aggregate distribution of employment statuses is determined this period. Next period’s employment state is still uncertain at the individual level.
2.2.3.1 Unemployed consumers

Let \( a'_z \) denote the consumer’s demand for an Arrow security that pays out one unit of the consumption good in the next period if and only if the next period’s aggregate productivity turns out to be \( z' \). The unemployed worker’s optimization problem is

\[
U (a; z, S) = \max_{c, a'_g \geq 2, a'_b \geq 2} u (c) + \beta \left\{ \pi_{zg} \left[ (1 - \lambda_w (z, S)) U (a'_g; g, S') + \lambda_w (z, S) W (a'_g; g, S') \right] + \pi_{zb} \left[ (1 - \lambda_w (z, S)) U (a'_b; b, S') + \lambda_w (z, S) W (a'_b; b, S') \right] \right\}
\]

subject to

\[
c + Q_g (z, S) a'_g + Q_b (z, S) a'_b = a + h - T \quad \text{and} \quad S' = \Omega (z, S)
\]

where \( u (\cdot) \) is an increasing and strictly concave instantaneous utility function; \( c \) is the consumption level; \( \beta \in (0, 1) \) is the discount factor; \( \lambda_w (z, S) \) is the job finding probability defined above; \( h \) is unemployment benefits before tax; \( T \) is a lump-sum tax paid by all consumers; \( U (a; z, S) \) is the value of being unemployed with asset holding \( a \), and \( W (a; z, S) \) is the value of being employed taking the wage determination into account.

Let the decision rule, i.e., the optimal solution to the optimization problem, for \( a'_z \) be \( \psi^u_z (a; z, S) \).

2.2.3.2 Employed consumers

The employed worker’s optimization problem is

\[
\widetilde{W} (w, a; z, S) = \max_{c, a'_g \geq 2, a'_b \geq 2} u (c) + \beta \left\{ \pi_{zg} \left[ \sigma U (a'_g; g, S') + (1 - \sigma) W (a'_g; g, S') \right] + \pi_{zb} \left[ \sigma U (a'_b; b, S') + (1 - \sigma) W (a'_b; b, S') \right] \right\}
\]

subject to

\[
c + Q_g (z, S) a'_g + Q_b (z, S) a'_b = a + w - T \quad \text{and} \quad S' = \Omega (z, S)
\]

where \( w \) is the wage. Hence, \( \widetilde{W} (w, a; z, S) \) is the value of being employed given the wage \( w \), and \( W (a; z, S) \) is the value of being employed when taking the wage determination into account. Denoting the wage function, i.e., the outcome of the wage determination, as \( w = \omega (a; z, S) \) the relationship is

\[
W (a; z, S) \equiv \widetilde{W} (\omega (a; z, S), a; z, S).
\]

Let the decision rule for \( a'_z \) be \( \psi^e_z (w; a; z, S) \) for a given wage, and define

\[
\psi^e_z (a; z, S) \equiv \psi^e_z (\omega (a; z, S), a; z, S).
\]
2.2.4 Firms

Assume a one-firm-one-job structure. To find a vacant worker the firm posts a vacancy. The value of a vacancy, $V(z,S)$, is

$$V(z,S) = -\xi + Q_g(z,S) \left[ (1 - \lambda_f(z,S)) V(g,S) + \lambda_f(z,S) \int J(\psi^g_a(a;z,S);g,S') \frac{f_u(a;S)}{u} da \right]$$

$$+ Q_b(z,S) \left[ (1 - \lambda_f(z,S)) V(b,S') + \lambda_f(z,S) \int J(\psi^b_a(a;z,S);b,S') \frac{f_u(a;S)}{u} da \right]$$

where $\xi$ is the vacancy cost; $\lambda_f(z,S)$ is the worker finding probability defined above; $f_u(a;S)$ is the population of unemployed workers with asset holdings $a$, and thus, $f_u(a;S)/u$ is the density function of the unemployed workers over $a$; $J(a;z,S)$ is the value of a filled job taking the wage determination into account, and hence, the integrals show the expected value of matching with an unemployed worker (given a future state) taking the wage determination and the individual decision rules into account. The firm discounts future values by the Arrow security prices since these are the rates at which the non-constrained consumers discount future states.\(^7\) There is free entry of firms which implies that firms post vacancies $v(z,S)$ until $V(z,S) = 0$.

A matched firm rents capital from the consumers at a rental rate of $r(z,S)$ and pays the worker a wage $w$. The value of a filled job given the wage $w$, $\bar{J}(w,a;z,S)$, is therefore

$$\bar{J}(w,a;z,S) = \bar{\pi}(w;z,S) + Q_g(z,S) \left[ \sigma V(g,S') + (1 - \sigma) J(\psi^e_g(w,a;z,S);g,S') \right]$$

$$+ Q_b(z,S) \left[ \sigma V(b,S') + (1 - \sigma) J(\psi^e_b(w,a;z,S);b,S') \right]$$

where the instantaneous profit is defined as $\bar{\pi}(w;z,S) \equiv \max_k \{zF(k) - r(z,S)k - w\}$, and $zF(k)$ is the production function, which is increasing and strictly concave in the capital input $k$. Again, we can define

$$J(a;z,S) \equiv \bar{J}(\omega(a;z,S),a;z,S).$$

The first-order condition implies $r(z,S) = zF'(k)$. Symmetry implies that in equilibrium each firm has the same capital stock, and the capital stock per job is $\bar{k} = \hat{k}/(1 - u)$, where $\hat{k}$ is the aggregate capital stock. Therefore, the equilibrium profit can be written as

$$\pi(a;z,S) \equiv zF(\hat{k}) - r(z,S)\bar{k} - \omega(a;z,S).$$

The dividend is then calculated as the total profits minus the total vacancy costs, that is

$$d(z,S) = \int \pi(a;z,S) f_e(a;S) da - \xi v$$

i.e., aggregated profits over all firms (=all jobs) since $f_e(a;S)$ is the population of employed workers with asset holdings $a$.

\(^7\)In the numerical solution of the model it turns out that very few (if any) consumers have a binding borrowing constraint.


2.2.5 Wages

When an unemployed worker and a vacant job get matched, the wage is determined through Nash Bargaining. Thus, the wage in a match including a worker with asset holdings $a$ solves

$$\max_{w} \left( \bar{W}(w; a; z, S) - U(a; z, S) \right)^{\gamma} \left( \bar{J}(w; a; z, S) - V(z, S) \right)^{1-\gamma}$$

where $\gamma \in (0, 1)$ is the bargaining power of the worker. The solution is described by $w = \omega(a; z, S)$.

The bargained wage depends on the asset holdings of the worker, because these affect both the worker’s outside option in the current period and the chosen asset holdings next period (contingent on the future aggregate state).\(^8\)

2.2.6 Investment firms

We envision competitive investment firms who sell contingency claims to consumers by rearranging capital and equity. Asset market clearing requires

$$\int \psi^{\psi_e}_z (a; z, S) f_e (a; S) \text{da} + \int \psi^{\psi_u}_z (a; z, S) f_u (a; S) \text{da} = (1 - \delta + r(z', S')) k' + p(z', S') + d(z', S')$$

for each $z'$, together with the no-arbitrage conditions (2.3) and (2.4).

2.2.7 Government

The government provides (partial) unemployment insurance as it pays out unemployment benefits to the unemployed workers. This is financed via a lump-sum tax levied on all consumers. Assume (for now) that the public budget has to balance each period, that is

$$T = uh$$

(2.8)

i.e., the tax revenue equals total public expenditures.

2.3 Solving the model

This section explains how the model is solved numerically. Furthermore, it briefly discusses, how the economy behaves in the benchmark of constant UI benefits across the business cycle.\(^8\)

\(^8\)Note that the wage is reset every period, also for pre-existing matches. Thus, it is implicitly assumed that firms cannot commit to future wages.
2.3.1 Computation

In Appendix 2.A it is shown that the resource balance condition (goods-market equilibrium condition)

\[ \hat{c} + \left[ \hat{k}' - (1 - \delta) \hat{k} \right] = zF(\hat{k}) \left( 1 - u \right) - \xi v \]  

is fulfilled, where \( \hat{c} \) is aggregate private consumption. That is, aggregate private consumption plus investments equal aggregate output (net of aggregate vacancy costs), where we can define \( \hat{y} \equiv zF(\hat{k}) \left( 1 - u \right) - \xi v \).

Furthermore, for a complete definition of the recursive competitive equilibrium see Krusell et al. (2010). Their appendix also contains a detailed description on how to solve the model numerically, and therefore this section only summarizes the method used.

Using the idea of Krusell & Smith (1998), consumers are assumed to have bounded rational perceptions of the evolutions of key economic variables, i.e., we apply the method of "approximate aggregation". Hence, consumers perceive the next period aggregate capital stock, \( \hat{k}' \), as a (log-linear) function of \((z, \hat{k}, u)\). The same is true for this period’s \( \theta, p, d, \) and \( Q_g \) (or \( Q_b \), see below). Krusell et al. (2010) show that these simple prediction rules are highly accurate with \( R^2 \)s above 0.999, and with very small forecasting and prediction errors.

The numerical solution of the model proceeds as follows (using the \( z = g \) case for illustration): 1) Guess on the law of motion for aggregate capital, i.e., \( \hat{k}' \) as a (log-linear) function of \((z, \hat{k}, u)\). 2) Guess on coefficients of the prediction rules for \( \theta, p, d \) and \( Q_g \) as (log-linear) functions of \((z, \hat{k}, u)\). 3) Calculate \( u' \) from the law of motion in (2.1). 4) Calculate \( Q_b \) using the no-arbitrage condition in (2.3) and the first three steps. 5) Perform the individual maximization and determine the wages from the Nash bargaining. 6) Simulate the economy for many periods using the results from the previous steps, and update the forecasting and prediction rules using the data from the simulation. Iterate until the forecasting and prediction rules converge (gain sufficient accuracy). The resulting forecasting and prediction rules for the standard case are presented in Appendix 2.B.

2.3.2 Calibration

We apply the calibration of Krusell et al. (2010) who calibrate the model to fit US data. A period is chosen to be six weeks. The production function is \( zF(k) = zk^\alpha \). The parameters

---

9 Without this assumption, consumers needed to know the law of motion for the entire distribution of agents, which is an infinite-dimensional object. "Approximate aggregation" assumes that a finite set of moments is sufficient for forecasting future economic variables.

10 We use 60 grid points in the \( a \) direction for the value functions, 15 points in the \( a \) direction for the wage function, 4 points in both the \( \hat{k} \) and the \( u \) direction. We interpolate between grid points using cubic splines in the \( a \) direction and linear interpolation in the other directions.

11 Following Krusell et al. (2010), the economy is simulated for 2000 periods, and we disregard the first 500 periods.
\[ \alpha = 0.3, \delta = 0.01, \text{ and } \beta = 0.995 \] are chosen using three calibration targets: a capital share of 0.3, an investment-output ratio of 0.2, and an annual rate of return on capital of 0.04. Also, the borrowing constraint is chosen as \( g = 0 \). The utility function is \( u(c) = \log(c) \).

Following Cooley & Prescott (1995) the productivity levels are chosen to be \( g = 1.02 \) and \( b = 0.98 \) yielding an unconditional mean productivity of 1. Following Krusell & Smith (1999) and Krusell et al. (2009) the average duration of each boom (or recession) is set to two years, i.e., 16 periods in this model, which implies \( \pi_{bb} = \pi_{gg} = 0.9375 \) with \(^{12}\pi_{bg} = 1 - \pi_{bb} \) and \( \pi_{gb} = 1 - \pi_{gg} \).

In the standard case, the matching parameters are calibrated following Shimer (2005). In the benchmark with constant unemployment benefits across the business cycle we set \( h = 0.99 \), which turns out to be approximately 40% of the average wage. The separation rate is \( \sigma = 0.05 \). The matching function is \( M(u, v) = \chi u^{\eta} v^{1-\eta} \). Aiming for \( \theta = 1 \) in equilibrium pins down \( \chi = 0.6 \). Furthermore, \( \xi = 0.5315 \) is chosen such that \( \theta = 1 \) satisfies the free-entry condition. Finally, \( \eta = \gamma = 0.72 \), again following Shimer (2005).

### 2.3.3 Benchmark

This section briefly discusses the benchmark case of invariant UI benefits across the business cycle. Table 2.1 summarizes the means and fluctuations of the key economic variables across the business cycle.

**Table 2.1: Summary statistics for the benchmark of invariant UI benefits**

<table>
<thead>
<tr>
<th>( z )</th>
<th>( u )</th>
<th>( v )</th>
<th>( \theta )</th>
<th>( k )</th>
<th>( w )</th>
<th>( h )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>mean</strong></td>
<td>1.0000</td>
<td>0.0768</td>
<td>0.0771</td>
<td>1.0039</td>
<td>66.955</td>
<td>2.4818</td>
</tr>
<tr>
<td>( \Delta_g )</td>
<td>+2.00%</td>
<td>-0.63%</td>
<td>+2.36%</td>
<td>+2.99%</td>
<td>+0.43%</td>
<td>+2.01%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( T )</th>
<th>( RR )</th>
<th>( p )</th>
<th>( d )</th>
<th>( r )</th>
<th>( \hat{y} )</th>
<th>( \hat{c} )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>mean</strong></td>
<td>0.0761</td>
<td>0.3801</td>
<td>0.9061</td>
<td>0.0042</td>
<td>0.0150</td>
<td>3.2966</td>
</tr>
<tr>
<td>( \Delta_g )</td>
<td>-0.63%</td>
<td>-2.04%</td>
<td>+2.06%</td>
<td>+56.17%</td>
<td>+1.74%</td>
<td>+2.16%</td>
</tr>
</tbody>
</table>

Note: **mean** denotes the unconditional mean, i.e., \( 0.5(\bar{x}_g + \bar{x}_b) \), where \( \bar{x}_z \) is the average of \( x \) across periods with \( z = \hat{z} \). \( \Delta_g \) is the percentage deviation of the average across good states from the unconditional mean. Thus, per definition \( \Delta_b = -\Delta_u \), and only \( \Delta_g \) is shown. \( \bar{w} \) is the average wage; \( RR \equiv \frac{\bar{w} - \bar{T}}{\bar{T}} \) is the average replacement ratio; \( \hat{y} \) is the aggregate output (net of vacancy costs); \( \hat{c} \) is the aggregate consumption.

From Table 2.1 it is seen that the benchmark economy behaves as expected in several aspects. Vacancy creation, and thereby also the \( \nu-u \) ratio, is procyclical. Hence, the job finding
rate is procyclical, which leads to a countercyclical unemployment rate. Thus, there is a clear negative relationship between aggregate productivity and unemployment, and the correlation is $\text{corr}(z, u) = -0.82$.

Due to the balanced budget requirement, the lump-sum tax is countercyclical as UI expenditures are higher during recessions where more workers are unemployed. The average wage is near-proportional to $z$, and wages are procyclical. This makes the average replacement ratio countercyclical, i.e., the income of an unemployed worker relative to the (average) income of an employed worker is higher in bad times than in good times.

The dividend is highly procyclical because profits are very volatile. Finally, aggregate output (net of vacancy costs) is procyclical, while aggregate consumption is only slightly procyclical, i.e., consumers are to a large extent able to smooth consumption out over the business cycle.

Table 2.1 also reveals that the fluctuations in $u$, $v$, and $\theta$ over the business cycle are very small. This is a well-known result from e.g. Shimer (2005). As a robustness check, in Section 2.5.6 we use a different calibration strategy inspired by Hagedorn & Manovskii (2008), which delivers much more reasonable fluctuations in these key business cycle variables.

Finally, the Gini coefficient for wealth in the benchmark economy is 0.3153 on average across the business cycle, and it is 0.03% higher in good states.

### 2.4 Business cycle dependent unemployment benefits

This section analyzes the effects of allowing unemployment benefits to depend on the position of the business cycle. Similar to Costain & Reiter (2005), we assume that the government is interested in the welfare consequences of a UI scheme where the level of unemployment benefits depends linearly on aggregate productivity, that is

$$h = \bar{h} + \phi_z \frac{z - \bar{z}}{\bar{z}}$$

(2.10)

where $\bar{z}$ is the average aggregate productivity across the business cycle; $\bar{h}$ is the benefit level when aggregate productivity equals its average; $\phi_z$ is the policy choice variable, and it determines the degree of business cycle dependence. A positive (negative) $\phi_z$ implies pro(counter)-cyclical benefits, while $\phi_z = 0$ implies a business cycle independent UI level (the benchmark).

---

13Numerically, the Gini coefficient for wealth is calculated as $G = 1 - \frac{\sum_{i=1}^{n} f(a_i)(\Gamma_i + \Gamma_{i-1})}{\Gamma_n}$, where $\Gamma_i \equiv \sum_{j=1}^{i} f(a_j) a_j$, $\Gamma_0 = 0$, $f(a)$ is the discrete probability density function of asset holdings, and $n$ is the number of grid points in the asset distribution, see e.g. Xu (2004).

14This UI scheme does not allow unemployment benefits to depend on the whole history of shocks but only on the current period shock, which has a practical implementation appeal more than a purely theoretical appeal. Furthermore, note that choosing $\bar{h}$ as in section 2.3.2 implies that the average level of unemployment benefits is unaffected compared to Krusell et al. (2010).
2.4.1 Welfare measure

Consider the welfare consequences of changing $\phi_z$ from the benchmark of invariant unemployment benefits ($\phi_z = 0$). In order to be able to calculate the expected welfare gain for each individual, we will make the experiment of moving an individual along with its asset level and employment status from the benchmark economy to an economy with a different $\phi_z$. As is typical in this literature the welfare consequences of such experiments can be found following Lucas (1987). The welfare gain, $\mu$, can be calculated from

$$
\mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \beta^t \log ((1 + \mu) c_t) \right] = \mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \beta^t \log (c_t^{EXP}) \right]
$$

where $c_t$ is the individual’s consumption under the benchmark case (business cycle invariant benefits), and $c_t^{EXP}$ is the individual’s consumption under the experiment (holding fixed the individual’s asset level and employment status in the initial period). Thus, $\mu$ measures the consumption equivalent, i.e., by how much should the individual be permanently compensated in terms of consumption if not moving to the ”new economy”. Hence, $\mu > 0 (\mu < 0)$ means that the individual gains (loses) from the experiment.

One advantage of the present welfare measure is that one can calculate the expected welfare gain for each individual in the benchmark economy, i.e., it is possible to distinguish between poor and rich workers. A potentially important drawback of this welfare measure is the discrepancy between actual aggregate asset holdings in the economy and the asset holdings aggregated over all the individuals moved to the new economy (one by one). The same applies for actual aggregate employment versus employment aggregated over the moved individuals. Therefore, as a robustness check, one can apply the standard utilitarian welfare measure. Hence, we calculate the mean welfare for different $\phi_z$’s and compare this to the mean welfare for $\phi_z = 0$, where welfare in a given period is defined as

$$
\int W(a; z, S) f_e(a; S) da + \int U(a; z, S) f_u(a; S) da
$$

i.e., we use the actual distribution of asset holdings and employment statuses, and not the benchmark distribution. When solving the model numerically, it has always been the case that the two approaches provide equivalent conclusion, i.e., all results regarding the optimality of pro- and countercyclical UI benefits, respectively, are sustained when applying this alternative, utilitarian welfare measure, and therefore only the former welfare measure will be presented below.

---

15 In practice, $\mu$ is found by rearranging to $\mu = \exp \left[ (V^{EXP} - V) (1 - \beta) \right] - 1$, where $V^{EXP} \equiv \mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \beta^t \log (c_t^{EXP}) \right]$ and $V \equiv \mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \beta^t \log (c_t) \right]$. In the tables we present the welfare gain in percentage, i.e., $100 \cdot \mu$.

16 This welfare measure is also used by e.g. Krusell et al. (2010) and Mukoyama (2011) in models without aggregate shocks, and by Costain & Reiter (2005) and Krusell et al. (2009) in models with aggregate shocks.
2.4.2 Welfare consequences

Table 2.2 shows the welfare consequences of moving the agents along with their employment statuses and asset holdings from the benchmark economy with constant unemployment benefits to an economy with business cycle dependent unemployment benefits and slope parameter $\phi_z$.

The table shows that both procyclical and countercyclical UI benefits can increase welfare relative to constant UI benefits across the business cycle. The mean welfare gain is largest in case of procyclical UI benefits (with $\phi_z = 1.76$). The unemployed and the employed experience almost the same welfare gains on average. Actually, even the poorest unemployed who face a binding borrowing constraint prefer procyclical benefits. Most of the employed gain in both good and bad periods, whereas the unemployed gain more in good periods than they lose in bad periods. On average, every consumer in the economy gain when moving from the benchmark of constant UI benefits to procyclical UI generosity, which is somewhat surprising.

On the other hand, if the cyclicality of benefits is too strong, e.g. $\phi_z = \pm 10$, both pro- and countercyclical UI benefits are harmful to the agents. Consumers facing a binding borrowing constraint will still gain in periods where benefits (and thus consumption) are raised, but they will lose much more in periods where benefits are lowered due to diminishing marginal utility.

The maximum attainable mean welfare gain is 0.002% of consumption, which is small compared to the welfare gains found in other studies. Mitman & Rabinovich (2011) find that the optimal UI scheme, which overall implies procyclicality of both benefit level and duration, yields a mean welfare gain of 0.67% of consumption compared to the current US system. Ek (2012) considers both differentiated taxes and benefit levels, and she finds that taxes should be procyclical whereas benefits should be countercyclical, which yields a mean welfare gain of 0.01% of consumption compared to the optimal uniform system. However, both these studies ignore savings, and therefore they are likely to overrate the welfare gains from business cycle dependent unemployment benefits since self-insurance is not possible.

In a model allowing for savings, Costain & Reiter (2005) calculates the welfare costs of business cycles to be 0.269% of consumption by comparing their static model, i.e., without aggregate fluctuations, to their dynamic benchmark. They find that the optimal policy with strongly procyclical taxes and slightly procyclical benefits eliminates around 70% of the welfare costs of business cycles, i.e., it implies a mean welfare gain of 0.185% of consumption compared to the dynamic benchmark. In contrast to this paper, Costain & Reiter (2005) do not consider aggregate assets, e.g. physical capital and equity, and thus, implicitly they do not allow for insurance against aggregate shocks. However, they are able to calculate the welfare costs of business cycles in the case where ”aggregate insurance” is attained. Using this, the welfare

---

17We present the average $\mu$ across good periods, $\bar{\mu}_g$, and bad periods, $\bar{\mu}_b$, along with the unconditional mean $0.5(\bar{\mu}_g + \bar{\mu}_b)$. In contrast, Krusell et al. (2009) choose to pick a random good period and a random bad period. However, the welfare gains are almost constant conditional on aggregate productivity, and therefore the two methods are (almost) equivalent.
gain from the optimal policy described above is 0.049% of consumption relative to the case with "aggregate insurance". This seems to be the relevant number for comparison with this paper, and their number is much larger for two reasons: i) the calibration strategy, cf. Section 2.5.6, and ii) Costain & Reiter (2005) consider two policy variables, the cyclicality of benefits and the cyclicality of the public deficit, against only one, the cyclicality of benefits, in this paper where the public budget (for now) is required to balance each period. The welfare gains suggested by both Costain & Reiter (2005) and Ek (2012) primarily stem from allowing taxes to vary over the business cycle, since they find that differentiated taxes over the business cycle result in much larger welfare gains than differentiated UI benefits. As this paper focuses only on the latter policy variable, one would therefore expect much smaller welfare gains from the optimal (linear) policy.

Table 2.2: Welfare consequences of different degrees of cyclicality in UI benefits

<table>
<thead>
<tr>
<th>$\phi_z$</th>
<th>$z$</th>
<th>Welfare gains (in %)</th>
<th>Fraction gaining (in %)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>overall</td>
<td>unempl</td>
</tr>
<tr>
<td>-10.00</td>
<td>$g$</td>
<td>-0.0259</td>
<td>-0.0619</td>
</tr>
<tr>
<td>-10.00</td>
<td>$b$</td>
<td>-0.0037</td>
<td>0.0336</td>
</tr>
<tr>
<td>-10.00</td>
<td>mean</td>
<td>-0.0148</td>
<td>-0.0141</td>
</tr>
<tr>
<td>-5.00</td>
<td>$g$</td>
<td>-0.0088</td>
<td>-0.0268</td>
</tr>
<tr>
<td>-5.00</td>
<td>$b$</td>
<td>0.0026</td>
<td>0.0212</td>
</tr>
<tr>
<td>-5.00</td>
<td>mean</td>
<td>-0.0031</td>
<td>-0.0028</td>
</tr>
<tr>
<td>-1.00</td>
<td>$g$</td>
<td>-0.0010</td>
<td>-0.0046</td>
</tr>
<tr>
<td>-1.00</td>
<td>$b$</td>
<td>0.0018</td>
<td>0.0055</td>
</tr>
<tr>
<td>-1.00</td>
<td>mean</td>
<td>0.0004</td>
<td>0.0004</td>
</tr>
<tr>
<td>1.00</td>
<td>$g$</td>
<td>0.0028</td>
<td>0.0064</td>
</tr>
<tr>
<td>1.00</td>
<td>$b$</td>
<td>0.0002</td>
<td>-0.0035</td>
</tr>
<tr>
<td>1.00</td>
<td>mean</td>
<td>0.0015</td>
<td>0.0014</td>
</tr>
<tr>
<td>1.76</td>
<td>$g$</td>
<td>0.0041</td>
<td>0.0104</td>
</tr>
<tr>
<td>1.76</td>
<td>$b$</td>
<td>-0.0001</td>
<td>-0.0066</td>
</tr>
<tr>
<td>1.76</td>
<td>mean</td>
<td>0.0020</td>
<td>0.0019</td>
</tr>
<tr>
<td>5.00</td>
<td>$g$</td>
<td>0.0058</td>
<td>0.0241</td>
</tr>
<tr>
<td>5.00</td>
<td>$b$</td>
<td>-0.0057</td>
<td>-0.0241</td>
</tr>
<tr>
<td>5.00</td>
<td>mean</td>
<td>0.0001</td>
<td>0.0000</td>
</tr>
<tr>
<td>10.00</td>
<td>$g$</td>
<td>0.0029</td>
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</tr>
<tr>
<td>10.00</td>
<td>$b$</td>
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<td>-0.0567</td>
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<tr>
<td>10.00</td>
<td>mean</td>
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</tbody>
</table>

Note: The table shows the welfare gains from moving the agents from an economy with constant UI benefits to an economy with cyclically dependent benefits and slope parameter $\phi_z$. $uC$ and $eC$ denote the unemployed and employed, respectively, with a binding borrowing constraint. $g$ is a good state, $b$ is a bad state, mean denotes the unconditional mean, i.e., $0.5(\bar{\mu}_g + \bar{\mu}_b)$.

$: Rounded to 100.00%. $\phi_z = 1.76$ is the level that maximizes the mean welfare gain.

2.4.3 Effects on key economic variables

Table 2.3 shows how the key economic variables are affected by changing the cyclicality in UI benefits, $\phi_z$. The table reveals some of the counteracting effects working in favor of procyclical and countercyclical UI benefits, respectively. From Table 2.2 we already know that the relative importance of these counteracting effects is non-monotone in $\phi_z$. 34
Table 2.3: Averages of key economic variables

<table>
<thead>
<tr>
<th>$\phi_z$</th>
<th>$z$</th>
<th>$u$ (%)</th>
<th>$v$</th>
<th>$\theta$</th>
<th>$k$</th>
<th>$\bar{w}$</th>
<th>$\bar{h}$</th>
<th>$T$</th>
<th>$RR$</th>
<th>$d$</th>
<th>$\bar{y}$</th>
<th>$\bar{c}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.00</td>
<td>$g$</td>
<td>7.6191</td>
<td>0.0796</td>
<td>1.0448</td>
<td>67.240</td>
<td>2.5302</td>
<td>0.9700</td>
<td>0.0739</td>
<td>0.3648</td>
<td>0.0074</td>
<td>3.3679</td>
<td>2.64375</td>
</tr>
<tr>
<td>-1.00</td>
<td>$b$</td>
<td>7.7515</td>
<td>0.0746</td>
<td>0.9629</td>
<td>66.671</td>
<td>2.4335</td>
<td>1.0100</td>
<td>0.0783</td>
<td>0.3956</td>
<td>0.0009</td>
<td>3.2252</td>
<td>2.62045</td>
</tr>
<tr>
<td>-1.00</td>
<td>mean</td>
<td>7.6853</td>
<td>0.0771</td>
<td>1.0039</td>
<td>66.955</td>
<td>2.4819</td>
<td>0.9900</td>
<td>0.0761</td>
<td>0.3802</td>
<td>0.0042</td>
<td>3.2966</td>
<td>2.63210</td>
</tr>
<tr>
<td>0.00</td>
<td>$g$</td>
<td>7.6364</td>
<td>0.0790</td>
<td>1.0339</td>
<td>67.240</td>
<td>2.5317</td>
<td>0.9900</td>
<td>0.0756</td>
<td>0.3723</td>
<td>0.0066</td>
<td>3.3678</td>
<td>2.64376</td>
</tr>
<tr>
<td>0.00</td>
<td>$b$</td>
<td>7.7332</td>
<td>0.0753</td>
<td>0.9738</td>
<td>66.671</td>
<td>2.4320</td>
<td>0.9900</td>
<td>0.0783</td>
<td>0.3956</td>
<td>0.0018</td>
<td>3.2253</td>
<td>2.62042</td>
</tr>
<tr>
<td>0.00</td>
<td>mean</td>
<td>7.6848</td>
<td>0.0771</td>
<td>1.0039</td>
<td>66.955</td>
<td>2.4818</td>
<td>0.9900</td>
<td>0.0761</td>
<td>0.3801</td>
<td>0.0042</td>
<td>3.2966</td>
<td>2.63209</td>
</tr>
<tr>
<td>1.76 g</td>
<td>$g$</td>
<td>7.0674</td>
<td>0.0778</td>
<td>1.0146</td>
<td>67.236</td>
<td>2.5444</td>
<td>1.0252</td>
<td>0.0786</td>
<td>0.3855</td>
<td>0.0048</td>
<td>3.3676</td>
<td>2.64375</td>
</tr>
<tr>
<td>1.76 b</td>
<td>$b$</td>
<td>7.7017</td>
<td>0.0765</td>
<td>0.9931</td>
<td>66.667</td>
<td>2.4202</td>
<td>0.9548</td>
<td>0.0735</td>
<td>0.3741</td>
<td>0.0035</td>
<td>3.2254</td>
<td>2.62035</td>
</tr>
<tr>
<td>1.76 mean</td>
<td></td>
<td>7.6845</td>
<td>0.0771</td>
<td>1.0039</td>
<td>66.952</td>
<td>2.4818</td>
<td>0.9900</td>
<td>0.0761</td>
<td>0.3798</td>
<td>0.0041</td>
<td>3.2965</td>
<td>2.63204</td>
</tr>
<tr>
<td>5.00 g</td>
<td>$g$</td>
<td>7.7266</td>
<td>0.0757</td>
<td>0.9792</td>
<td>67.230</td>
<td>2.5394</td>
<td>1.0900</td>
<td>0.0842</td>
<td>0.4097</td>
<td>0.0017</td>
<td>3.3671</td>
<td>2.64366</td>
</tr>
<tr>
<td>5.00 b</td>
<td>$b$</td>
<td>7.6457</td>
<td>0.0786</td>
<td>1.0286</td>
<td>66.663</td>
<td>2.4242</td>
<td>0.8900</td>
<td>0.0680</td>
<td>0.3489</td>
<td>0.0065</td>
<td>3.2256</td>
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</tr>
<tr>
<td>5.00 mean</td>
<td></td>
<td>7.6861</td>
<td>0.0771</td>
<td>1.0039</td>
<td>66.946</td>
<td>2.4818</td>
<td>0.9900</td>
<td>0.0761</td>
<td>0.3793</td>
<td>0.0041</td>
<td>3.2963</td>
<td>2.63193</td>
</tr>
</tbody>
</table>

Note: The table shows the averages across good and bad periods, respectively. mean denotes the unconditional mean, i.e., $0.5(\bar{x}_g + \bar{x}_b)$, where $\bar{x}_z$ is the average of $x$ across periods with $z = \bar{z}$. $\bar{w}$ is the average wage; $RR \equiv \frac{\bar{w} - T}{\bar{w}}$ is the average replacement ratio. The aggregate output is $\bar{y} \equiv zF(\bar{k}) (1 - u - \xi v)$. $\phi_z = 1.76$ is the level that maximizes the mean welfare gain.

Compared to the benchmark ($\phi_z = 0$) procyclical unemployment benefits lead to stabilization of most of the economic variables. Thus, unemployment, vacancies, and the $v-u$ ratio are all stabilized with procyclical UI generosity. When benefits are raised in booms, the wage will increase, and therefore firms will post fewer vacancies. This leads to a decrease in the $v-u$ ratio, which lowers the job finding probability, and therefore the unemployment level will increase. Similarly, vacancy creation will be higher in recessions where benefits are lowered, which makes job finding easier and unemployment decreases. Furthermore, procyclical benefits stabilize profits and aggregate output (defined as the right-hand side of (2.9)), whereas wages are destabilized. The aggregate output is stabilized because the number of producers (= the employment level) is stabilized, and because the aggregate vacancy costs are stabilized. In the benchmark, the average replacement ratio is countercyclical due to procyclical UI generosity. However, if the procyclicality of UI benefits is sufficiently strong, the average replacement ratio will be procyclical, which is the case for the optimal degree of procyclicality ($\phi_z = 1.76$), in which case UI benefits are 3.56% higher in good times than on average across the business cycle, and the (average) replacement ratio is 1.50% higher.

For countercyclical UI benefits the exact opposite happens. Most variables are destabilized, e.g., unemployment, but wages are stabilized since they increase during recessions due to a better outside option of the worker, and they decrease during booms where the outside option of the worker is worsened. Hence, agents need to save less for bad times and more for good times. For most agents the former dominates, and therefore consumption increases on average. Furthermore, this leads to a stabilization of aggregate consumption.

Table 2.3 also shows that mean unemployment, which can be thought of as the structural unemployment level, is lowered when moving from the benchmark economy ($\phi_z = 0$) to an economy with (optimal) procyclical benefits ($\phi_z = 1.76$), because unemployment drops more during recessions than it increases during booms. Also, structural unemployment is higher.

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with countercyclical benefits, because unemployment increases more during recessions than it decreases during booms. Hence, the distortionary effects of UI (on job creation) is countercyclical, at least for small absolute values of $\phi_z$, and therefore procyclical UI benefits can be welfare improving. This interpretation is in line with Andersen & Svarer (2011b) who find that benefits should be lowest in the state with most distortions. On the other hand, if the procyclicality of UI benefits is too strong (e.g. $\phi_z = 5$), structural unemployment increases compared to the benchmark, and in fact, the unemployment rate will be higher during booms than during recessions.

To sum up, procyclical UI generosity can be beneficial due to the countercyclical nature of the distortionary effects of UI (on job creation), whereas countercyclical UI generosity can be beneficial as it facilitates consumption smoothing and raises mean consumption. Table 2.1 showed that most consumers are able to smooth consumption fairly well, even without countercyclical UI, and therefore procyclical UI generosity dominates countercyclical UI.

Finally, Figure 2.1 shows a sample path for the unemployment rate in the benchmark case of constant UI, in case of (optimal) procyclical UI, and in the case of countercyclical UI. It confirms that unemployment is stabilized in case of procyclical UI since unemployment increases (relative to the benchmark) in good times, where unemployment is low, and decreases in bad times, where unemployment is high, whereas unemployment is destabilized in case of countercyclical UI.

Figure 2.1: Sample path for unemployment, constant UI versus procyclical UI and countercyclical UI

Note: The figure shows sample paths for the unemployment rate, $u$, in the benchmark case of constant UI across the business cycle (dashed line), in the case of procyclical UI (full line; $\phi_z = 1.76$), and in the case of countercyclical UI (dotted line; $\phi_z = -1.00$).
2.5 Robustness and extensions

In this section various robustness checks and extensions are investigated.

2.5.1 Alternative public budget requirement

In the analysis above the public budget was not allowed to act as a buffer. However, an important argument in favor of UI is that it works as an automatic stabilizer. Therefore, this section allows the public budget to balance only on average.\(^\text{18}\)

The results are presented in Tables 2.4 and 2.5 in Appendix 2.C.1. They show that allowing the public budget to balance only on average does not change the qualitative results, i.e., both procyclical and countercyclical UI benefits can increase welfare, and procyclical unemployment benefits (\(\phi_z = 1.75\)) still yield the largest welfare gains. However, the maximum attainable mean welfare gain across individuals is almost 50% lower than when the public budget always balances. The reason is that under a balance budget requirement the tax increase (decrease) counteracts the increase (decrease) in unemployment benefits and, thus, wages. With procyclical UI benefits, consumers no longer gain in bad periods from a low tax or suffer in good periods from a higher tax – the former turns out to be dominant. For the same reasons the welfare gains from countercyclical benefits are larger when the public budget only balances over time. Hence, as is well-known in the literature, requiring the public budget to balance each period works in favor of procyclical UI generosity, see e.g. Andersen & Svarer (2011b).

The optimal procyclical UI scheme implies that the public budget deficit is also procyclical (0.0025 on average across good states and −0.0025 across bad states), i.e., UI expenditures are higher in good states than in bad states, since the increase in the benefit level dominates the decrease in the number of unemployed workers when moving from a bad to a good state.

2.5.2 Unemployment dependent UI benefits

In practice, it seems easier to implement a scheme where UI benefits are conditioned on the unemployment level. Thus, the level of unemployment benefits is determined from

\[
h = \bar{h} + \phi_u \frac{u - \bar{u}}{\bar{u}}
\]

where \(\bar{u}\) is the mean unemployment level. Note that the interpretation of \(\phi_u\) is now a bit more complicated\(^\text{19}\) since changing \(\phi_u\) will also change \(\bar{u}\). As opposed to the interpretation of \(\phi_z\), a positive (negative) \(\phi_u\) implies counter(pro)-cyclical benefits.

\(^{18}\)To be more precise, the tax now solves \(T = 0.5 \left[ \sum_{z} h_z + \sum_{z} h_{\bar{z}} \right] \), where \(\sum_{z} h_z\) is the average of \(u \cdot h\) in periods with \(z = \bar{z}\), and 0.5 is the unconditional probability of each state. Thus, the tax is now constant over time, and for simplicity we abstract from debt dynamics, i.e., in this sense we take a partial approach.

\(^{19}\)To solve this, \(\bar{u}\) could be replaced by the unemployment level in steady state. However, this specification would imply that in general the average \(h\) is no longer \(\bar{h}\), and therefore it would be a different kind of experiment.
The results are presented in Tables 2.6 and 2.7 (budget balance) and Tables 2.8 and 2.9 (budget balances only on average) in Appendix 2.C.2. Again, we find that both procyclical and countercyclical benefits can increase welfare.

In the case of budget balance, procyclical benefits are optimal, as the welfare gain is maximized for $\phi_u = -5.00$ with a mean welfare gain of 0.0101% of consumption. However, even though UI benefits are 1.79% higher across good times than on average across the business cycle, the (average) replacement ratio is 0.21% lower. Hence, the procyclicality of the UI benefits is not strong enough to revert the countercyclicality of the (average) replacement ratio due to procyclical wages.

Furthermore, when the public budget is allowed to work as a buffer, the welfare gain is maximized in case of countercyclical benefits ($\phi_u = 1.20$) with a mean welfare gain of 0.0085% of consumption. This UI scheme implies that the public budget deficit is countercyclical (−0.0013 on average across good states and 0.0013 across bad states), UI benefits are 0.94% lower across good times than on average across the business cycle and the (average) replacement ratio is 3.09% lower. Thus, conditioning UI on unemployment instead of productivity actually alters the choice between pro- and countercyclical UI.

Overall we find that unemployment dependent UI yields much higher welfare gains than UI conditioned on productivity. This is not surprising since the latter (in our model) restricts UI benefits to jump between two levels only, whereas the former allows UI benefits to differ between, say, mild and deep recessions. However, combined with the fact that it is easier, in practice, to condition UI on the unemployment rate, it is somewhat surprising that the approach of conditioning UI directly on unemployment, to the author’s knowledge, is new to the literature.20

### 2.5.3 UI benefits depending on lagged productivity

In reality, statistics on the position of the business cycle always become available with a certain lag. Therefore, this section studies the consequences of conditioning UI benefits on lagged productivity, instead of current productivity, i.e.,

$$h = \bar{h} + \phi_{z_{-1}} \frac{z_{-1} - \bar{z}_{-1}}{\bar{z}_{-1}}$$

where $z_{-1}$ is previous period’s productivity, and $\bar{z}_{-1}$ is mean lagged productivity.21 To solve the model we need to include $z_{-1}$ as an additional state variable.

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20Kroft & Notowidigdo (2011) estimate how the moral hazard cost and the consumption smoothing gain of UI vary with the unemployment rate. They find that a one standard deviation increase in the unemployment rate increases the optimal replacement rate by 14 to 27 percentage points.

21The mean lagged productivity is calculated as $\bar{z}_{-1} = 0.5 \left( \bar{z}_{g-1} + \bar{z}_{b-1} \right)$ where $\bar{z}_{g-1}$ is the average of lagged productivity across good periods (high current level productivity), and similar for $\bar{z}_{b-1}$ across bad periods. Again, this ensures that mean unemployment benefits are still $\bar{h}$.  

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The results are presented in Tables 2.10 and 2.11 (budget balance) and Tables 2.12 and 2.13 (budget balances only on average) in Appendix 2.C.3. Not surprisingly, both procyclical and countercyclical benefits can increase welfare.

With budget balance, procyclical UI benefits are optimal, as the welfare gain is maximized for \( \phi_{z-1} = 1.25 \) with a mean welfare gain of 0.0012% of consumption. Here, benefits are 2.21% higher across good times than on average across the business cycle, but the (average) replacement ratio is only 0.17% higher. In this case, there is no negative value of \( \phi_{z-1} \) delivering a positive welfare gain compared to invariant benefits.

Again we find that when the public budget is allowed to work as a buffer, the welfare gain is maximized in case of countercyclical benefits (\( \phi_{z-1} = -4.00 \)) with a mean welfare gain of 0.0014% of consumption. This UI scheme implies that the public budget deficit is countercyclical (−0.0066 on average across good states and 0.0066 across bad states), UI benefits are 7.06% lower across good times than on average across the business cycle and the (average) replacement ratio is 9.57% lower.

Hence, conditioning UI benefits on lagged productivity results in the same qualitative conclusions as unemployment dependent UI. The reason is that the distribution of asset holdings and employment statuses is determined in the previous period, i.e., it depends on productivity in the previous period, cf. (2.1) and (2.2), and therefore lagged productivity matters more for the status of the economic agents than current productivity. Recall that in the benchmark the correlation between current productivity and unemployment is \( \text{corr}(z,u) = -0.82 \), whereas the correlation between lagged productivity and unemployment is \( \text{corr}(z_{-1},u) = -0.94 \).

### 2.5.4 Distortionary taxation

So far we assumed that the UI scheme was financed through non-distortionary lump-sum taxation. This section considers the effects of business cycle dependent UI, when benefits are financed through a distortionary payroll tax levied on the firms. In particular, the instantaneous profit is now

\[
\tilde{\pi}(w; z, S) \equiv \max_k \{ zF(k) - r(z, S)k - w(1+\tau) \}
\]

where \( \tau \) is the proportional payroll tax, which distorts vacancy creation, and the equilibrium profit is thus

\[
\pi(a; z, S) \equiv zF(\tilde{k}) - r(z, S)\tilde{k} - \omega(a; z, S)(1+\tau).
\]

For the public budget to balance we require

\[
\int \tau \omega(a; z, S) f_e(a; S) da = uh
\]

to hold in each period.
The results are presented in Tables 2.14 and 2.15 (budget balance) and Tables 2.16 and 2.17 (budget balances only on average) in Appendix 2.C.4. They show that the distortionary payroll tax does not change the main conclusion: both procyclical and countercyclical UI benefits can be welfare improving. However, the welfare gains are much larger than when the UI scheme is financed by a lump-sum tax.

In the case of budget balance, procyclical benefits are optimal, as the welfare gain is maximized for $\phi_z = 0.50$ with a mean welfare gain of 0.0048% of consumption. However, even though UI benefits are 1.01% higher across good times than on average across the business cycle, the (average) replacement ratio is 1.09% lower. Again, the procyclicality of the UI benefits is not strong enough to revert the countercyclicality of the (average) replacement ratio due to procyclical wages.

When the public budget is allowed to work as a buffer, the welfare gain is maximized in case of procyclical benefits ($\phi_z = 1.00$) with a mean welfare gain of 0.0199% of consumption, i.e., more than ten times larger than with lump-sum taxation. Also in this case, the (average) replacement ratio is 0.05% lower across good times than on average across the business cycle, even though UI benefits are 2.02% higher. Hence, the government’s choice of how to finance the UI scheme is important for the effects of business cycle dependent UI.

2.5.5 Cross-sectional inequality

A potentially important shortcoming of the model framework and calibration considered so far is the lack of substantial cross-sectional inequality, which may be important for the effects of unemployment insurance. As mentioned above, the Gini coefficient for wealth is 0.3153 with the standard calibration, whereas it is 0.801 in the US data according to Davies, Sandström, Shorrocks, & Wolff (2010). In this section, we therefore discuss possible calibration routes to increase the degree of wealth inequality in the model.

One possibility in this model would be to lower the overall UI level, $h$, and in this way make the unemployed worse off. However, as explained below a large decrease in the overall UI level would lead to a strengthened precautionary savings motive, and therefore both the employed and the unemployed would save even more, in order to smooth consumption in case of a negative shock to the employment status. In fact, the effect is so strong that the Gini coefficient for wealth drops in response to a decrease in the overall UI level.

An alternative method is to consider a higher job separation rate, $\sigma$, which would increase the unemployment rate. However, as the probability of being hit by a negative employment shock increases for all individuals, they will (as above) respond by increasing their precautionary savings, leading to less (and not more) wealth inequality.

Yet another alternative is a decrease in the effectiveness of the matching process, $\chi$. This change will increase the average duration of an unemployment spell, since job finding is now
harder. Therefore, the consumers will on average experience a longer period of unemployment, where they bring down their stock of savings. However, again it turns out that consumers are able to to build up a larger savings buffer in expectation of more dire consequences of being hit by a negative employment shock, implying that wealth inequality does not rise.

2.5.6 Alternative calibration

Like Krusell et al. (2010) we also consider a different calibration strategy inspired by Hagedorn & Manovskii (2008), who showed that the Diamond-Mortensen-Pissarides model is consistent with key business cycle facts, in particular it matches the empirically observed volatility of unemployment, vacancies, and the ratio between the two at business cycle frequencies, if using an alternative calibration strategy than Shimer (2005). In particular, the value of being unemployed is now much closer to the value of being employed since \( h \) will be much larger. Furthermore, the bargaining power \( \gamma \) will be much smaller, effectively implying that wages are less responsive to productivity shocks.

The vacancy cost is set to \( \xi = 2.165 \) which is approximately 60% of the average labor productivity in the model. Following Hagedorn & Manovskii (2008) the bargaining power is set to \( \gamma = 0.052 \). The matching function is \( M(u, v) = \frac{uv}{(u' + v')^{1/2}} \). With \( \theta = 0.7 \) and a job-finding rate of 0.592 as calibration targets we get \( l = 2.2 \). Finally, \( \bar{h} = 2.29 \) implies that \( \theta = 0.7 \) satisfies the free-entry condition. Therefore, the replacement ratio turns out to be very high, approximately 95%, and the interpretation of the value of non-market activities \( h \) must now cover much more than just UI, e.g. home production and self employment.\(^{22}\)

The summary statistics for the benchmark model are found in Table 2.18 in Appendix 2.C.5. It shows that unemployment volatility is now much higher, whereas wages are fairly rigid.

The results are presented in Tables 2.19 and 2.20 (budget balance) and Tables 2.21 and 2.22 (budget balances only on average) in Appendix 2.C.5. They show that using an alternative calibration strategy does not change the overall conclusion: unemployment benefits should be procyclical when conditioning on productivity. But the welfare gains are much larger since unemployment is much more volatile with the Hagedorn-Manovskii calibration, whereas the wages are fairly rigid. In fact, the welfare gains are almost two orders of magnitude larger than with the standard calibration. Again we find that requiring the public budget to balance each period works in favor of procyclical UI since the welfare gains from procyclical UI are higher in Table 2.19 than in Table 2.21.

\(^{22}\)In particular, \( h = \bar{h}^{\text{UI}} + h^{\text{non-UI}} \), where only \( h^{\text{UI}} \) is tax financed via (2.8). We let \( h^{\text{non-UI}} = 1.30 (= 2.29 - 0.99) \), and \( h^{\text{UI}} \) is determined from (2.10) with \( \bar{h}^{\text{UI}} = 0.99 \). Alternatively (but less realistic), it could be assumed that all income from non-market activities must be financed via taxes. It turns out that this experiment delivers exactly the same qualitative results.
Furthermore, countercyclical UI benefits are no longer welfare improving. The reason is that wages are much less volatile compared to the standard calibration, and therefore there are only small gains from stabilizing wages (and thus consumption). These gains are dominated by the very strong countercyclicality of the distortionary effects of UI (on job creation).

Note, however, that the interpretation of changing $h$ over the business cycle is less clear with this calibration since the welfare calculations implicitly assume that changing UI does not affect the other determinants of the value of non-market activities, which is probably not a realistic assumption. Furthermore, unemployment is now too responsive to changes in UI.23

Hence, the two calibrations considered so far exhibit the puzzle pointed out by Costain & Reiter (2008), i.e., either it underpredicts unemployment volatility over the business cycle (for parameter values resulting in a large match surplus), or it overpredicts the response of unemployment to changes in UI benefits (for parameter values resulting in a low match surplus). One way to solve this problem is to introduce sticky wages, which we turn to next.24

### 2.5.7 Rigid wages

Assuming that wages are rigid makes the match surplus more responsive to productivity shocks, and therefore vacancy creation becomes more volatile, cf. Shimer (2004) and Hall (2005). Therefore, in this section we return to our standard (Shimer) calibration, which does not suffer from unrealistically large responses to UI policies, but we will add the assumption that wages are non-responsive to changes in the business cycle situation and temporary changes in the UI level, i.e., wages are no longer determined through Nash bargaining.25 Since the policy experiment analyzed throughout this paper does not change the average UI level, we are not assuming that wages are unresponsive to the overall UI level, which of course is an unrealistic assumption, but only that wages do not respond to temporary changes in the UI level.

The unresponsiveness of wages to temporary changes in the UI level is also present in several other theoretical studies of business cycle dependent unemployment insurance, see e.g. Andersen & Svarer (2010) and Landais et al. (2010).

The summary statistics for the benchmark model are found in Table 2.23 in Appendix 2.C.6. As expected, volatility is now much larger for unemployment and especially the vacancy rate and the $v-u$ ratio. The replacement ratio is now slightly procyclical in the benchmark since wages and benefits are acyclical whereas taxes are countercyclical.

---

23Calculations show that in the model without aggregate shocks (see Krusell et al. (2010) for details) unemployment increases with 0.16% when benefits are increased by 1% for the standard (Shimer) calibration. This figure is 14% for the alternative (Hagedorn-Manovskii) calibration.

24Another solution would be to introduce match-specific productivity shocks, which is beyond the scope of this paper.

25In particular, we fix the wage at $\omega(a) = \bar{\omega} = 2.45 \forall a$ in all periods. I have tried other levels of $\bar{\omega}$, which deliver similar conclusions.
The results are presented in Tables 2.24 and 2.25 (budget balance) and Tables 2.26 and 2.27 (budget balances only on average) in Appendix 2.C.6.

We see that with rigid wages countercyclical UI benefits always dominate procyclical benefits. This is not surprising as the main effect working in favor of procyclical UI benefits is no longer present, i.e., the countercyclicality of the distortionary effects of UI on vacancy creation. With budget balance the welfare gain is maximized when moving to an economy with $\phi_z = -7$, yielding a mean welfare gain of 0.0003% of consumption. In this case UI benefits are 14.14% lower across good times than on average over the business cycle, and the (average) replacement ratio is 14.23% lower.

When the public budget only balances on average, only countercyclical UI benefits are welfare improving, and the optimal (linear) policy is rather extreme, as the welfare gain is maximized with $\phi_z = -45$, yielding a mean welfare gain of 0.0621% of consumption. The unemployed workers with a binding borrowing constraint suffer greatly from this policy, especially in good times where their income, and thus consumption, is cut drastically. However, these consumers constitute a very small fraction of the population, and therefore the average welfare gain is still positive. With this policy, UI benefits are almost 91% lower in good times than on average over the business cycle, and the replacement ratio is almost 97% lower.

Hence, the responsiveness of wages to temporary changes in UI is crucial when determining the optimal UI policy over the business cycle. This point is emphasized in this paper, where the standard calibration with fixed wages and the alternative (Hagedorn-Manovskii) calibration perform similarly well in explaining key business cycle facts, but lead to completely opposite conclusions in the choice between countercyclical and procyclical UI. It is clear that the differences in wage determination across previous theoretical studies may, in part, explain the different conclusions they reach.

2.5.8 Importance of savings

Since an important contribution of the present paper is to investigate business cycle dependent UI in a framework allowing for savings and wealth heterogeneity, it is relevant to compare the results from above to the similar results in a model without savings. Hence, we consider a simplified version of the model where consumers always spend their current income (exogenously imposed), i.e., they have no savings and zero wealth. For production not to collapse to zero we assume that there is a constant capital stock per job, which, for example, could be owned by foreign agents.

As pointed out in the introduction, Abdulkadiroğlu et al. (2002) show that allowing for savings has important consequences for the optimal UI design. The same is true in this framework. To illustrate this, we first consider the optimal level of UI in the case of invariant benefits, i.e., we find the optimal $\tilde{h}$ given $\phi_z = 0$. In the steady state version of the model, Krusell et al.
(2010) find that the average welfare gain is maximized with $\bar{h} \approx 0.30$. Similarly, in the model with aggregate shocks we find that the optimal $\bar{h}$ is 0.23, which corresponds to an average replacement ratio of around 10%, i.e., much lower than the current US level. The reason is that a low degree of insurance generates substantial precautionary savings, which lead to a much larger capital stock. On the other hand, the optimal $\bar{h}$ is 1.50 in the model without savings, corresponding to an average replacement ratio of around 58%, i.e., higher than the current US level. Thus, the possibility for consumers to save is an important determinant of the optimal UI level.

Similarly, the possibility of savings is important when considering business cycle dependent UI. It turns out that the same basic mechanisms are at play in the model without savings: Pro-cyclical UI lowers structural unemployment due to countercyclical distortions, whereas countercyclical UI facilitates consumption smoothing. However, the welfare gain is now strictly concave in $\phi_z$, i.e., procyclical and countercyclical UI cannot be beneficial at the same time.

With the standard (Shimer) calibration we find that the consumption smoothing mechanism is always dominating, i.e., only countercyclical UI benefits deliver positive welfare gains relative to invariant benefits. This holds for both public budget requirements. With the alternative (Hagedorn-Manovskii) calibration, however, the countercyclical UI distortions are always dominating, i.e., only procyclical UI benefits are welfare improving. The reason is that vacancy creation reacts strongly to changes in the UI scheme with the alternative calibration, as shown in Section 2.5.6, i.e., the distortionary effects of UI are very strong. Thus, the chosen calibration strategy is crucial for the optimal cyclicality of UI benefits. This finding is in line with previous work by Moyen & Stähler (2009) who calibrate their model to both US and European data and find that UI entitlement duration should be countercyclical in the US, where the consumption smoothing effect dominates the negative labor market effects, but not in Europe.

For the standard calibration the maximum attainable average welfare gain is $0.0003\% \ (\phi_z = -0.5)$ with budget balance and $0.0024\% \ (\phi_z = -1)$ when the public budget balances on average. For the alternative calibration the maximum attainable average welfare gains are $0.1591\% \ (\text{budget balance}; \ \phi_z = 5)$ and $0.1243\% \ (\text{budget balances on average}; \ \phi_z = 1)$. Hence, it is important to explicitly allow for precautionary savings when quantifying the effects of cyclical UI.

2.6 Concluding remarks

This paper considers the effects and optimality of business cycle dependent unemployment benefits in a dynamic general equilibrium model with labor market matching, wealth heterogeneity, precautionary savings, and aggregate fluctuations in productivity. Our results suggest that welfare gains can be achieved by both procyclical and countercyclical UI benefits. Procyclical UI
benefits can increase welfare as the distortionary effects of UI (on job creation) are countercyclical, and therefore the employment process is stabilized. On the other hand, countercyclical UI benefits can increase welfare as wages are stabilized, consumption smoothing is facilitated and mean consumption is increased, i.e., the labor income process is stabilized. The non-linear relationship between these two opposing effects is what causes the ambiguous results.

The generosity of the UI scheme should be procyclical when conditioning benefits on (current) productivity. This result is robust to changing the public budget requirement and the chosen calibration strategy. The chosen calibration strategy is very important when quantifying the welfare gains from procyclical UI benefits compared to invariant benefits. However, UI generosity should be countercyclical when conditioning benefits on the unemployment level or lagged productivity and allowing the public budget to balance only on average, and when wages are completely rigid.

One important argument in favor of countercyclical UI is that it strengthens the automatic stabilizers. In this model, however, there is only a very weak transmission channel from ‘aggregate demand’ to the supply side. It would thus be interesting for further research to extend the model to allow for a stronger aggregate demand channel, e.g. by incorporating nominal rigidities. In this spirit, Ravn & Sterk (2013) show that the joint presence of matching frictions, incomplete asset markets, and nominal rigidities may help to explain the amplification mechanism, which has caused the Great Recession. However, with fully flexible prices this mechanism is weak. Furthermore, long-term unemployment is crucial for the result. Hence, a model extended in these directions may emphasize the welfare gains from countercyclical UI, and the demand stabilization may partly remove the incentive for precautionary savings.

The moral hazards caused by providing unemployment insurance play no role on the worker side in this paper since search effort and labor supply are both exogenous. This is also the case in e.g. Costain & Reiter (2005) and Moyen & Stähler (2009). Although this is an important omission, we argue that the present model is still relevant for analyzing UI schemes since we do account for important distortions on the firm side. However, it would be interesting to see how the conclusions are altered when allowing for endogenously determined search effort and/or labor supply. If the distortionary effects of UI on search effort is procyclical as suggested by previous studies, see e.g. Andersen & Svarer (2010), this extension of the model is likely to work in favor of countercyclical UI generosity.
2.7 Bibliography


XU, K. (2004): “How has the Literature on Gini’s Index Evolved in the Past 80 Years?” Mimeo, Dalhousie University.

Appendices

2.A Resource balance condition

This appendix shows that the resource balance condition (goods-market clearing condition)
\[
\hat{c} + \left[ \hat{k}' - (1 - \delta) \hat{k} \right] = zF \left( \hat{k} \right) (1 - u) - \xi v
\] (2.11)
is fulfilled, i.e., private consumption plus investments equal aggregate output (net of aggregated vacancy costs), where \( \hat{c} \) is aggregate private consumption
\[
\hat{c} \equiv \int c_e (a; z, S) f_e (a; S) \, da + \int c_u (a; z, S) f_u (a; S) \, da
\]
and
\[
c_e (a; z, S) \equiv a + \omega (a; z, S) - T - Q_g (z, S) \psi^e_g (a; z, S) - Q_b (z, S) \psi^e_b (a; z, S) \] (2.12)
\[
c_u (a; z, S) \equiv a + h - T - Q_g (z, S) \psi^u_g (a; z, S) - Q_b (z, S) \psi^u_b (a; z, S).
\] (2.13)
The proof follows the idea of Krusell et al. (2010) but allows unemployment benefits to be financed internally (via taxes). Integrating (2.12) and (2.13) over all asset holdings and summing up yields
\[
\hat{c} = \int a f_e (a; S) \, da + \int a f_u (a; S) \, da + \int \omega (a; z, S) f_e (a; S) \, da + hu - T \left( 1 - u \right) - Tu
\]
\[
- Q_g (z, S) \left[ \int \psi^e_g (a; z, S) f_e (a; S) \, da + \int \psi^u_g (a; z, S) f_u (a; S) \, da \right]
\]
\[
- Q_b (z, S) \left[ \int \psi^e_b (a; z, S) f_e (a; S) \, da + \int \psi^u_b (a; z, S) f_u (a; S) \, da \right]
\] (2.14)
since \( \int f_e (a; S) \, da = (1 - u) \) and \( \int f_u (a; S) \, da = u. \)

Firstly, consider the terms \( \int a f_e (a; S) \, da + \int a f_u (a; S) \, da \). Assuming that the decision rules for \( a' \) are increasing in \( a \), the law of motion for the asset distribution can be calculated as
\[
\int_a^{a'} f_e (\tilde{a}; S') \, d\tilde{a} = \lambda_w \int_a^{(\psi^e_n)^{-1}(a'; z, S)} f_u (a; S) \, da + (1 - \sigma) \int_a^{(\psi^e_n)^{-1}(a'; z, S)} f_e (a; S) \, da
\]
\[ \int_{a}^{a'} f_u (\bar{a}; S') \, d\bar{a} = (1 - \lambda_w) \int_{a}^{(\psi_u')^{-1}(a'; z, S)} f_u (a; S) \, da + \sigma \int_{a}^{(\psi_u')^{-1}(a'; z, S)} f_e (a; S) \, da \]

where \((\psi_u')^{-1}(a'; z, S)\) denotes the inverse of the decision rule, i.e., the value of \(a\) that satisfies \(a' = \psi_u^u (a; z, S)\), and similarly for \((\psi_e')^{-1}(a'; z, S)\). Differentiating the upper equation with respect to \(a'\) gives

\[ f_e (a'; S') = \lambda_w \rho_u (a'; z, S) f_u ((\psi_u')^{-1}(a'; z, S); S) + (1 - \sigma) \rho_e (a'; z, S) f_e ((\psi_e')^{-1}(a'; z, S); S) \]

applying Leibniz’s rule, where

\[
\begin{align*}
\rho_u (a'; z, S) &\equiv \frac{d (\psi_u')^{-1}(a'; z, S)}{da'} \\
\rho_e (a'; z, S) &\equiv \frac{d (\psi_e')^{-1}(a'; z, S)}{da'}.
\end{align*}
\]

Multiplying this expression by \(a'\) and integrating yields

\[
\int a' f_e (a'; S') \, da' = \lambda_w \int a' \rho_u (a'; z, S) f_u ((\psi_u')^{-1}(a'; z, S); S) \, da' + (1 - \sigma) \int a' \rho_e (a'; z, S) f_e ((\psi_e')^{-1}(a'; z, S); S) \, da'.
\]

Changing variables on the right-hand side using \(a' = \psi_u^u (a; z, S)\) implying \(a = (\psi_u')^{-1}(a'; z, S)\) in the first term, and \(a' = \psi_e^e (a; z, S)\) implying \(a = (\psi_e')^{-1}(a'; z, S)\) in the second term, yields

\[
\int a' f_e (a'; S') \, da' = \lambda_w \int \psi_u^u (a; z, S) \rho_u (\psi_u^u (a; z, S); z, S) f_u (a; S) \frac{d\psi_u^u (a; z, S)}{da} \, da \\
+ (1 - \sigma) \int \psi_e^e (a; z, S) \rho_e (\psi_e^e (a; z, S); z, S) f_e (a; S) \frac{d\psi_e^e (a; z, S)}{da} \, da
\]

where the last step uses the definitions of \(\rho_u (\cdot)\) and \(\rho_e (\cdot)\).

Similarly, using the law of motion for the asset distribution of the unemployed workers, it can be shown that

\[
\int a' f_u (a'; S') \, da' = (1 - \lambda_w) \int \psi_u^u (a; z, S) f_u (a; S) \, da + \sigma \int \psi_e^e (a; z, S) f_e (a; S) \, da.
\]

Summing up yields

\[
\int a' f_e (a'; S') \, da' + \int a' f_u (a'; S') \, da' = \int \psi_u^u (a; z, S) f_u (a; S) \, da + \int \psi_e^e (a; z, S) f_e (a; S) \, da
\]

\[
= (1 - \delta + r (z', S')) \hat{k}' + p (z', S') + d (z', S')
\]

using the asset market clearing condition (2.7), and lagging this expression one period gives

\[
\int a f_e (a; S) \, da + \int a f_u (a; S) \, da = (1 - \delta + r (z, S)) \hat{k} + p (z, S) + d (z, S).
\] (2.15)
Secondly, using the equilibrium profit (2.5) in the dividend expression (2.6) yields
\[ d(z, S) = \int \left[ zF(\hat{k}) - r(z, S)\hat{k} - \omega(a; z, S) \right] f_e(a; S) da - \xi v \]
\[ = zF(\hat{k})(1 - u) - r(z, S)\hat{k} - \int \omega(a; z, S) f_e(a; S) da - \xi v \]  \hspace{1cm} (2.16)
using \( \int f_e(a; S) da = (1 - u) \) and \( \hat{k} = \hat{k}/(1 - u) \).

Thirdly, using the public budget requirement (2.8) we have
\[ hu - T = 0 \]  \hspace{1cm} (2.17)

Fourthly, using the asset market clearing condition (2.7) we have
\[ Q_g(z, S) \left[ \int \psi_\varepsilon^g(a; z, S) f_e(a; S) da + \int \psi_\nu^g(a; z, S) f_u(a; S) da \right] + \]
\[ Q_b(z, S) \left[ \int \psi_\varepsilon^b(a; z, S) f_e(a; S) da + \int \psi_\nu^b(a; z, S) f_u(a; S) da \right] = Q_g(z, S) \left[ (1 - \delta + r(g, S')) \hat{k}' + p(g, S') + d(g, S') \right] + \]
\[ Q_b(z, S) \left[ (1 - \delta + r(b, S')) \hat{k}' + p(b, S') + d(b, S') \right] = \hat{k}' + p(z, S) \]  \hspace{1cm} (2.18)
where the last step uses the no-arbitrage conditions (2.3) and (2.4).

Finally, we obtain (2.11) by inserting (2.15), (2.16), (2.17), and (2.18) in (2.14).

2.B Law of motion and prediction rules

This appendix shows the forecasting and prediction rules used by the bounded rational agents (with the standard Shimer calibration). The law of motion for the aggregate capital stock is
\[ \log \hat{k}' = 0.0645 + 0.9827 \log \hat{k} - 0.0033 \log u + 0.0451 \log z \]  \hspace{1cm} (\( R^2 = 0.9999995 \))
and the prediction rules for the other aggregate variables are
\[ \log \theta = -2.1097 + 0.5070 \log \hat{k} + 0.0071 \log u + 1.3913 \log z \]  \hspace{1cm} (\( R^2 = 0.9999834 \))
\[ \log (p + d) = -1.8831 + 0.3923 \log \hat{k} - 0.0545 \log u + 1.0510 \log z \]  \hspace{1cm} (\( R^2 = 0.9999993 \))
\[ \log Q_g = -0.6803 - 5.0218 \log \hat{Q}_g + 0.0628 \log \hat{k} + 0.0016 \log u \]  \hspace{1cm} (\( R^2 = 0.9999260 \))
\[ \log Q_b = 0.6134 + 6.5404 \log \hat{Q}_b - 0.0555 \log \hat{k} - 0.0014 \log u \]  \hspace{1cm} (\( R^2 = 0.9999217 \))
where \( \hat{Q}_z \equiv \pi_{zz}/\left(1 - \delta + r(z, \hat{k}', u')\right) \) for \( z = \{g, b\} \). \( \hat{k}' \) is calculated from the law of motion above, whereas \( u' \) is calculated from (2.1).
2.C  Robustness and extensions

2.C.1  Alternative public budget requirement

Table 2.4: Welfare consequences of different degrees of cyclicality in UI benefits – public budget balances on average

<table>
<thead>
<tr>
<th>φ</th>
<th>z</th>
<th>Welfare gains (in %)</th>
<th>Fraction gaining (in %)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>overall</td>
<td>unempl</td>
</tr>
<tr>
<td>−10.00</td>
<td>g</td>
<td>−0.0499</td>
<td>−0.0864</td>
</tr>
<tr>
<td>−10.00</td>
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<td>0.0641</td>
</tr>
<tr>
<td>−10.00</td>
<td>mean</td>
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<tr>
<td>−5.00</td>
<td>g</td>
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<tr>
<td>−5.00</td>
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<td>0.0176</td>
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<td>mean</td>
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<td>−1.00</td>
<td>g</td>
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<td>−1.00</td>
<td>b</td>
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<td>mean</td>
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<td>0.0007</td>
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<td>g</td>
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<td>mean</td>
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<td>−0.0030</td>
</tr>
</tbody>
</table>

Note: The table shows the welfare gains from moving the agents from an economy with constant UI benefits to an economy with cyclically dependent benefits and slope parameter φ. uC and eC denote the unemployed and employed, respectively, with a binding borrowing constraint. g is a good state, b is a bad state, mean denotes the unconditional mean, i.e., 0.5(μ_g + μ_b). ⋄: Rounded to 100.00%. φ = 1.75 is the level that maximizes the mean welfare gain.
Table 2.5: Averages of key economic variables – public budget balances on average

<table>
<thead>
<tr>
<th>$\phi_z$</th>
<th>$z$</th>
<th>$u$ (%)</th>
<th>$v$</th>
<th>$\theta$</th>
<th>$k$</th>
<th>$w$</th>
<th>$h$</th>
<th>$T$</th>
<th>$RR$</th>
<th>$p$</th>
<th>$d$</th>
<th>$g$</th>
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<tbody>
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<td>0.9700</td>
<td>0.0761</td>
<td>0.3643</td>
<td>0.9310</td>
<td>0.0074</td>
<td>3.3675</td>
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<td>1.0100</td>
<td>0.0761</td>
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</tr>
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<td>1.02875</td>
<td>66.686</td>
<td>2.4244</td>
<td>0.8900</td>
<td>0.0761</td>
<td>0.3735</td>
<td>0.9198</td>
<td>0.0066</td>
<td>3.2259</td>
</tr>
<tr>
<td>5.00</td>
<td>mean</td>
<td>7.6853</td>
<td>0.0772</td>
<td>1.00435</td>
<td>67.002</td>
<td>2.4824</td>
<td>0.9900</td>
<td>0.0761</td>
<td>0.3790</td>
<td>0.9063</td>
<td>0.0041</td>
<td>3.2972</td>
</tr>
</tbody>
</table>

Note: The table shows the averages across good and bad periods, respectively. mean denotes the unconditional mean, i.e., $0.5(\bar{x}_g + \bar{x}_b)$, where $\bar{x}_z$ is the average of $x$ across periods with $z = z$. $\bar{w}$ is the average wage; $RR ≡ h - T \bar{w} - T$ is the average replacement ratio. The aggregate output is $\hat{y} ≡ zF(\bar{y}) (1 - u) - \xi v$. $\phi_z = 1.75$ is the level that maximizes the mean welfare gain.

2.C.2 Unemployment dependent UI benefits

Table 2.6: Welfare consequences of different degrees of cyclicality in UI benefits – unemployment dependent UI, budget balance

<table>
<thead>
<tr>
<th>$\phi_u$</th>
<th>$z$</th>
<th>Overall</th>
<th>$uC$</th>
<th>$eC$</th>
<th>Fraction gaining (in %)</th>
</tr>
</thead>
<tbody>
<tr>
<td>−5.00</td>
<td>$g$</td>
<td>0.0111</td>
<td>0.0146</td>
<td>0.0108</td>
<td>0.0252</td>
</tr>
<tr>
<td>−5.00</td>
<td>$b$</td>
<td>0.0090</td>
<td>0.0054</td>
<td>0.0094</td>
<td>-0.0095</td>
</tr>
<tr>
<td>−5.00</td>
<td>mean</td>
<td>0.0101</td>
<td>0.0100</td>
<td>0.0101</td>
<td>0.0079</td>
</tr>
<tr>
<td>−1.00</td>
<td>$g$</td>
<td>0.0041</td>
<td>0.0052</td>
<td>0.0040</td>
<td>0.0087</td>
</tr>
<tr>
<td>−1.00</td>
<td>$b$</td>
<td>0.0043</td>
<td>0.0032</td>
<td>0.0044</td>
<td>-0.0010</td>
</tr>
<tr>
<td>−1.00</td>
<td>mean</td>
<td>0.0042</td>
<td>0.0042</td>
<td>0.0042</td>
<td>0.0039</td>
</tr>
<tr>
<td>1.00</td>
<td>$g$</td>
<td>0.0025</td>
<td>0.0010</td>
<td>0.0027</td>
<td>-0.0031</td>
</tr>
<tr>
<td>1.00</td>
<td>$b$</td>
<td>0.0044</td>
<td>0.0059</td>
<td>0.0042</td>
<td>0.0111</td>
</tr>
<tr>
<td>1.00</td>
<td>mean</td>
<td>0.0035</td>
<td>0.0035</td>
<td>0.0034</td>
<td>0.0030</td>
</tr>
</tbody>
</table>

Note: The table shows the welfare gains from moving the agents from an economy with constant UI benefits to an economy with cyclically dependent benefits and slope parameter $\phi_u$. $uC$ and $eC$ denote the unemployed and employed, respectively, with a binding borrowing constraint. $g$ is a good state, $b$ is a bad state, mean denotes the unconditional mean, i.e., $0.5(\bar{\mu}_g + \bar{\mu}_b)$. Rounded to 100.00%, $\phi_u = -5.00$ is the level that maximizes the mean welfare gain.
Table 2.7: Averages of key economic variables – unemployment dependent UI, budget balance

<table>
<thead>
<tr>
<th>φ_u</th>
<th>z</th>
<th>u (%)</th>
<th>v</th>
<th>θ</th>
<th>k</th>
<th>w</th>
<th>h</th>
<th>T</th>
<th>RR</th>
<th>p</th>
<th>d</th>
<th>ŷ</th>
</tr>
</thead>
<tbody>
<tr>
<td>−5.00</td>
<td>g</td>
<td>7.6569</td>
<td>0.0782</td>
<td>1.0209</td>
<td>67.236</td>
<td>2.5318</td>
<td>1.0077</td>
<td>0.0772</td>
<td>0.3791</td>
<td>0.9172</td>
<td>0.0069</td>
<td>3.3677</td>
</tr>
<tr>
<td>−5.00</td>
<td>b</td>
<td>7.7113</td>
<td>0.0761</td>
<td>0.9871</td>
<td>66.666</td>
<td>2.4319</td>
<td>0.9723</td>
<td>0.0750</td>
<td>0.3807</td>
<td>0.8935</td>
<td>0.0013</td>
<td>3.2253</td>
</tr>
<tr>
<td>−5.00</td>
<td>mean</td>
<td>7.6841</td>
<td>0.0771</td>
<td>1.0040</td>
<td>66.951</td>
<td>2.4819</td>
<td>0.9900</td>
<td>0.0761</td>
<td>0.3799</td>
<td>0.9054</td>
<td>0.0041</td>
<td>3.2965</td>
</tr>
<tr>
<td>−1.00</td>
<td>g</td>
<td>7.6427</td>
<td>0.0787</td>
<td>1.0299</td>
<td>67.239</td>
<td>2.5318</td>
<td>0.9954</td>
<td>0.0761</td>
<td>0.3744</td>
<td>0.9226</td>
<td>0.0066</td>
<td>3.3678</td>
</tr>
<tr>
<td>−1.00</td>
<td>b</td>
<td>7.7264</td>
<td>0.0756</td>
<td>0.9779</td>
<td>66.669</td>
<td>2.4319</td>
<td>0.9846</td>
<td>0.0761</td>
<td>0.3807</td>
<td>0.8875</td>
<td>0.0018</td>
<td>3.2253</td>
</tr>
<tr>
<td>−1.00</td>
<td>mean</td>
<td>7.6848</td>
<td>0.0771</td>
<td>1.0038</td>
<td>66.954</td>
<td>2.4819</td>
<td>0.9900</td>
<td>0.0761</td>
<td>0.3801</td>
<td>0.9061</td>
<td>0.0042</td>
<td>3.2966</td>
</tr>
<tr>
<td>0.00</td>
<td>g</td>
<td>7.6364</td>
<td>0.0790</td>
<td>1.0339</td>
<td>67.240</td>
<td>2.5317</td>
<td>0.9900</td>
<td>0.0761</td>
<td>0.3723</td>
<td>0.9248</td>
<td>0.0065</td>
<td>3.3678</td>
</tr>
<tr>
<td>0.00</td>
<td>b</td>
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<td>0.0753</td>
<td>0.9738</td>
<td>66.669</td>
<td>2.4320</td>
<td>0.9975</td>
<td>0.0766</td>
<td>0.3878</td>
<td>0.8849</td>
<td>0.0018</td>
<td>3.2253</td>
</tr>
<tr>
<td>0.00</td>
<td>mean</td>
<td>7.6845</td>
<td>0.0771</td>
<td>1.0038</td>
<td>66.954</td>
<td>2.4818</td>
<td>0.9900</td>
<td>0.0761</td>
<td>0.3801</td>
<td>0.9063</td>
<td>0.0042</td>
<td>3.2965</td>
</tr>
<tr>
<td>−1.00</td>
<td>g</td>
<td>7.6278</td>
<td>0.0793</td>
<td>1.0394</td>
<td>67.238</td>
<td>2.5317</td>
<td>0.9825</td>
<td>0.0749</td>
<td>0.3694</td>
<td>0.9278</td>
<td>0.0063</td>
<td>3.3678</td>
</tr>
<tr>
<td>−1.00</td>
<td>b</td>
<td>7.7427</td>
<td>0.0750</td>
<td>0.9682</td>
<td>66.669</td>
<td>2.4319</td>
<td>0.9975</td>
<td>0.0772</td>
<td>0.3908</td>
<td>0.8849</td>
<td>0.0021</td>
<td>3.2252</td>
</tr>
<tr>
<td>−1.00</td>
<td>mean</td>
<td>7.6853</td>
<td>0.0771</td>
<td>1.0038</td>
<td>66.954</td>
<td>2.4818</td>
<td>0.9900</td>
<td>0.0761</td>
<td>0.3801</td>
<td>0.9063</td>
<td>0.0042</td>
<td>3.2965</td>
</tr>
</tbody>
</table>

Note: The table shows the averages across good and bad periods, respectively. mean denotes the unconditional mean, i.e., 0.5(\bar{x}_g + \bar{x}_b), where \bar{x}_z is the average of x across periods with z = \tilde{z}. \bar{w} is the average wage; RR ≡ h − T \bar{w} − T is the average replacement ratio. The aggregate output is \hat{y} ≡ zF(\tilde{k}) (1 − u) − ξv. φ_u = −5.00 is the level that maximizes the mean welfare gain.

Table 2.8: Welfare consequences of different degrees of cyclicality in UI benefits – unemployment dependent UI, public budget balances on average

<table>
<thead>
<tr>
<th>φ_u</th>
<th>z</th>
<th>Welfare gains (in %)</th>
<th>Fraction gaining (in %)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>overall unempl empl</td>
<td>aC  cC</td>
</tr>
<tr>
<td></td>
<td></td>
<td>overall unempl empl</td>
<td>aC  cC</td>
</tr>
<tr>
<td>−5.00</td>
<td>g</td>
<td>0.0042 0.0078 0.0039</td>
<td>0.0213 0.0094</td>
</tr>
<tr>
<td>−5.00</td>
<td>b</td>
<td>-0.0091 -0.0129 -0.0088</td>
<td>-0.0317 -0.0170</td>
</tr>
<tr>
<td>−5.00</td>
<td>mean</td>
<td>-0.0024 -0.0025 -0.0024</td>
<td>-0.0052 -0.0038</td>
</tr>
<tr>
<td>−1.00</td>
<td>g</td>
<td>0.0025 0.0037 0.0025</td>
<td>0.0082 0.0045</td>
</tr>
<tr>
<td>−1.00</td>
<td>b</td>
<td>-0.0009 -0.0021 -0.0008</td>
<td>-0.0074 -0.0030</td>
</tr>
<tr>
<td>−1.00</td>
<td>mean</td>
<td>0.0008 0.0008 0.0008</td>
<td>0.0004 0.0008</td>
</tr>
<tr>
<td>1.00</td>
<td>g</td>
<td>0.0042 0.0027 0.0043</td>
<td>-0.0052 0.0008</td>
</tr>
<tr>
<td>1.00</td>
<td>b</td>
<td>0.0109 0.0125 0.0107</td>
<td>0.0189 0.0135</td>
</tr>
<tr>
<td>1.00</td>
<td>mean</td>
<td>0.0075 0.0076 0.0075</td>
<td>0.0069 0.0072</td>
</tr>
<tr>
<td>1.20</td>
<td>g</td>
<td>0.0052 0.0033 0.0054</td>
<td>-0.0062 0.0013</td>
</tr>
<tr>
<td>1.20</td>
<td>b</td>
<td>0.0118 0.0138 0.0116</td>
<td>0.0218 0.0150</td>
</tr>
<tr>
<td>1.20</td>
<td>mean</td>
<td>0.0085 0.0085 0.0085</td>
<td>0.0078 0.0081</td>
</tr>
</tbody>
</table>

Note: The table shows the welfare gains from moving the agents from an economy with constant UI benefits to an economy with cyclically dependent benefits and slope parameter φ_u. aC and cC denotes the unemployed and employed, respectively, with a binding borrowing constraint. g is a good state, b is a bad state, mean denotes the unconditional mean, i.e., 0.5(\tilde{μ}_g + \tilde{μ}_b).

*: Rounded to 100.00%. φ_u = 1.20 is the level that maximizes the mean welfare gain.

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Table 2.9: Averages of key economic variables – unemployment dependent UI, public budget balances on average

<table>
<thead>
<tr>
<th>$\phi_u$</th>
<th>$z$</th>
<th>$\alpha$ (%)</th>
<th>$\theta$</th>
<th>$k$</th>
<th>$w$</th>
<th>$h$</th>
<th>$T$</th>
<th>$RR$</th>
<th>$p$</th>
<th>$d$</th>
<th>$\hat{y}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.00</td>
<td>$g$</td>
<td>7.6427</td>
<td>0.0787</td>
<td>1.0299</td>
<td>67.239</td>
<td>2.5318</td>
<td>0.9954</td>
<td>0.0761</td>
<td>0.3744</td>
<td>0.9226</td>
<td>0.0066</td>
</tr>
<tr>
<td>-1.00</td>
<td>$b$</td>
<td>7.7264</td>
<td>0.0756</td>
<td>0.9780</td>
<td>66.669</td>
<td>2.4319</td>
<td>0.9846</td>
<td>0.0761</td>
<td>0.3856</td>
<td>0.8895</td>
<td>0.0016</td>
</tr>
<tr>
<td>-1.00</td>
<td>mean</td>
<td>7.6845</td>
<td>0.0771</td>
<td>1.0039</td>
<td>66.954</td>
<td>2.4819</td>
<td>0.9900</td>
<td>0.0761</td>
<td>0.3800</td>
<td>0.9061</td>
<td>0.0041</td>
</tr>
<tr>
<td>0.00</td>
<td>$g$</td>
<td>7.6364</td>
<td>0.0789</td>
<td>1.0338</td>
<td>67.233</td>
<td>2.5317</td>
<td>0.9900</td>
<td>0.0761</td>
<td>0.3744</td>
<td>0.9226</td>
<td>0.0066</td>
</tr>
<tr>
<td>0.00</td>
<td>$b$</td>
<td>7.7333</td>
<td>0.0753</td>
<td>0.9738</td>
<td>66.665</td>
<td>2.4319</td>
<td>0.9900</td>
<td>0.0761</td>
<td>0.3856</td>
<td>0.8895</td>
<td>0.0016</td>
</tr>
<tr>
<td>0.00</td>
<td>mean</td>
<td>7.6849</td>
<td>0.0771</td>
<td>1.0038</td>
<td>66.949</td>
<td>2.4818</td>
<td>0.9900</td>
<td>0.0761</td>
<td>0.3801</td>
<td>0.9061</td>
<td>0.0042</td>
</tr>
<tr>
<td>1.00</td>
<td>$g$</td>
<td>7.6280</td>
<td>0.0793</td>
<td>1.0406</td>
<td>67.222</td>
<td>2.5315</td>
<td>0.9825</td>
<td>0.0761</td>
<td>0.3692</td>
<td>0.9277</td>
<td>0.0062</td>
</tr>
<tr>
<td>1.00</td>
<td>$b$</td>
<td>7.7429</td>
<td>0.0750</td>
<td>0.9681</td>
<td>66.656</td>
<td>2.4318</td>
<td>0.9975</td>
<td>0.0761</td>
<td>0.3911</td>
<td>0.8843</td>
<td>0.0021</td>
</tr>
<tr>
<td>1.00</td>
<td>mean</td>
<td>7.6854</td>
<td>0.0771</td>
<td>1.0037</td>
<td>66.939</td>
<td>2.4816</td>
<td>0.9900</td>
<td>0.0761</td>
<td>0.3802</td>
<td>0.9065</td>
<td>0.0042</td>
</tr>
<tr>
<td>1.20</td>
<td>$g$</td>
<td>7.6259</td>
<td>0.0794</td>
<td>1.0406</td>
<td>67.219</td>
<td>2.5315</td>
<td>0.9807</td>
<td>0.0761</td>
<td>0.3684</td>
<td>0.9287</td>
<td>0.0062</td>
</tr>
<tr>
<td>1.20</td>
<td>$b$</td>
<td>7.7455</td>
<td>0.0749</td>
<td>0.9665</td>
<td>66.653</td>
<td>2.4318</td>
<td>0.9993</td>
<td>0.0761</td>
<td>0.3919</td>
<td>0.8843</td>
<td>0.0021</td>
</tr>
<tr>
<td>1.20</td>
<td>mean</td>
<td>7.6857</td>
<td>0.0771</td>
<td>1.0036</td>
<td>66.936</td>
<td>2.4816</td>
<td>0.9900</td>
<td>0.0761</td>
<td>0.3802</td>
<td>0.9065</td>
<td>0.0042</td>
</tr>
</tbody>
</table>

Note: The table shows the averages across good and bad periods, respectively. $\text{mean}$ denotes the unconditional mean, i.e., $0.5(\bar{x}_g + \bar{x}_b)$, where $\bar{x}$ is the average of $x$ across periods with $z = \hat{z}$. $\bar{w}$ is the average wage; $RR \equiv \frac{\bar{h} - T}{\bar{w} - T}$ is the average replacement ratio. The aggregate output is $\hat{y} \equiv zF(k) \left(1 - u \right) - \xi_v$. $\phi_u = 1.20$ is the level that maximizes the mean welfare gain.

2.C.3 UI benefits depending on lagged productivity

Table 2.10: Welfare consequences of different degrees of cyclicality in UI benefits – UI depending on lagged productivity, budget balance

<table>
<thead>
<tr>
<th>$\phi_{z-1}$</th>
<th>$z$</th>
<th>Welfare gains (in %)</th>
<th>Fraction gaining (in %)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>overall</td>
<td>unempl</td>
<td>empl</td>
</tr>
<tr>
<td>-1.00</td>
<td>$g$</td>
<td>-0.0015</td>
<td>-0.0049</td>
</tr>
<tr>
<td>-1.00</td>
<td>$b$</td>
<td>0.0008</td>
<td>0.0043</td>
</tr>
<tr>
<td>-1.00</td>
<td>mean</td>
<td>-0.0004</td>
<td>-0.0003</td>
</tr>
<tr>
<td>1.00</td>
<td>$g$</td>
<td>0.0201</td>
<td>0.0055</td>
</tr>
<tr>
<td>1.00</td>
<td>$b$</td>
<td>-0.0000</td>
<td>-0.0035</td>
</tr>
<tr>
<td>1.00</td>
<td>mean</td>
<td>0.0011</td>
<td>0.0010</td>
</tr>
<tr>
<td>1.25</td>
<td>$g$</td>
<td>0.0026</td>
<td>0.0068</td>
</tr>
<tr>
<td>1.25</td>
<td>$b$</td>
<td>-0.0001</td>
<td>-0.0045</td>
</tr>
<tr>
<td>1.25</td>
<td>mean</td>
<td>0.0012</td>
<td>0.0012</td>
</tr>
</tbody>
</table>

Note: The table shows the welfare gains from moving the agents from an economy with constant UI benefits to an economy with cyclically dependent benefits and slope parameter $\phi_{z-1}$. $uC$ and $eC$ denote the unemployed and employed, respectively, with a binding borrowing constraint. $g$ is a good state, $b$ is a bad state, $\text{mean}$ denotes the unconditional mean, i.e., $0.5(\bar{\mu}_g + \bar{\mu}_b)$. $\phi_{z-1} = 1.25$ is the level that maximizes the mean welfare gain.

$\diamond$: Rounded to 100.00%. $\phi_{z-1} = 1.25$ is the level that maximizes the mean welfare gain.
Table 2.11: Averages of key economic variables – UI depending on lagged productivity, budget balance

<table>
<thead>
<tr>
<th>$\phi_{z-1}$</th>
<th>$z$</th>
<th>$u$ (%)</th>
<th>$v$</th>
<th>$\theta$</th>
<th>$k$</th>
<th>$w$</th>
<th>$h$</th>
<th>$T$</th>
<th>$RR$</th>
<th>$p$</th>
<th>$d$</th>
<th>$\hat{y}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.00</td>
<td>$g$</td>
<td>7.6171</td>
<td>0.0797</td>
<td>1.0401</td>
<td>67.240</td>
<td>2.5311</td>
<td>0.0725</td>
<td>0.0741</td>
<td>0.3657</td>
<td>0.9321</td>
<td>0.0066</td>
<td>3.3679</td>
</tr>
<tr>
<td>-1.00</td>
<td>$b$</td>
<td>7.7536</td>
<td>0.0746</td>
<td>0.9616</td>
<td>66.671</td>
<td>2.4325</td>
<td>1.0075</td>
<td>0.0781</td>
<td>0.3947</td>
<td>0.8805</td>
<td>0.0019</td>
<td>3.2252</td>
</tr>
<tr>
<td>-1.00 mean</td>
<td></td>
<td>7.6854</td>
<td>0.0771</td>
<td>1.0039</td>
<td>66.956</td>
<td>2.4818</td>
<td>0.9900</td>
<td>0.0761</td>
<td>0.3802</td>
<td>0.9063</td>
<td>0.0042</td>
<td>3.2966</td>
</tr>
<tr>
<td>0.00</td>
<td>$g$</td>
<td>7.6358</td>
<td>0.0790</td>
<td>1.0342</td>
<td>67.239</td>
<td>2.5319</td>
<td>0.9900</td>
<td>0.0756</td>
<td>0.3723</td>
<td>0.9250</td>
<td>0.0063</td>
<td>3.3678</td>
</tr>
<tr>
<td>0.00</td>
<td>$b$</td>
<td>7.7331</td>
<td>0.0753</td>
<td>0.9738</td>
<td>66.670</td>
<td>2.4317</td>
<td>0.9900</td>
<td>0.0766</td>
<td>0.3879</td>
<td>0.8876</td>
<td>0.0020</td>
<td>3.2253</td>
</tr>
<tr>
<td>0.00 mean</td>
<td></td>
<td>7.6845</td>
<td>0.0771</td>
<td>1.0040</td>
<td>66.955</td>
<td>2.4818</td>
<td>0.9900</td>
<td>0.0761</td>
<td>0.3801</td>
<td>0.9063</td>
<td>0.0042</td>
<td>3.2966</td>
</tr>
<tr>
<td>1.00</td>
<td>$g$</td>
<td>7.6854</td>
<td>0.0783</td>
<td>1.0222</td>
<td>67.236</td>
<td>2.5327</td>
<td>1.0075</td>
<td>0.0771</td>
<td>0.3789</td>
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<td>3.3677</td>
</tr>
<tr>
<td>1.00</td>
<td>$b$</td>
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<td>0.0761</td>
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<td>66.668</td>
<td>2.4309</td>
<td>0.9725</td>
<td>0.0750</td>
<td>0.3947</td>
<td>0.8947</td>
<td>0.0022</td>
<td>3.2254</td>
</tr>
<tr>
<td>1.00 mean</td>
<td></td>
<td>7.6839</td>
<td>0.0772</td>
<td>1.0012</td>
<td>66.935</td>
<td>2.4818</td>
<td>0.9900</td>
<td>0.0766</td>
<td>0.3879</td>
<td>0.9063</td>
<td>0.0041</td>
<td>3.2965</td>
</tr>
<tr>
<td>1.25</td>
<td>$g$</td>
<td>7.6598</td>
<td>0.0781</td>
<td>1.0192</td>
<td>67.233</td>
<td>2.5329</td>
<td>1.0118</td>
<td>0.0775</td>
<td>0.3903</td>
<td>0.9160</td>
<td>0.0060</td>
<td>3.3676</td>
</tr>
<tr>
<td>1.25</td>
<td>$b$</td>
<td>7.7080</td>
<td>0.0762</td>
<td>0.9891</td>
<td>66.665</td>
<td>2.4307</td>
<td>0.9862</td>
<td>0.0746</td>
<td>0.3793</td>
<td>0.8965</td>
<td>0.0022</td>
<td>3.2253</td>
</tr>
<tr>
<td>1.25 mean</td>
<td></td>
<td>7.6839</td>
<td>0.0772</td>
<td>1.0012</td>
<td>66.935</td>
<td>2.4818</td>
<td>0.9900</td>
<td>0.0761</td>
<td>0.3879</td>
<td>0.9062</td>
<td>0.0041</td>
<td>3.2965</td>
</tr>
</tbody>
</table>

Note: The table shows the averages across good and bad periods, respectively. $\text{mean}$ denotes the unconditional mean, i.e., $0.5(\bar{x}_g + \bar{x}_b)$, where $\bar{x}_z$ is the average of $x$ across periods with $z = \hat{z}$. $\bar{w}$ is the average wage; $RR \equiv \frac{\bar{h} - T}{\bar{w} - T}$ is the average replacement ratio. The aggregate output is $\hat{y} \equiv zF(\hat{k}) (1 - u) - \xi v$. $\phi_{z-1} = 1.25$ is the level that maximizes the mean welfare gain. For numerical reasons only, the benchmark is slightly different from comparable tables, since lagged productivity is included as an additional state variable, which changes the interpolation outcomes as well as the prediction rules.

Table 2.12: Welfare consequences of different degrees of cyclicality in UI benefits – UI depending on lagged productivity, public budget balances on average

<table>
<thead>
<tr>
<th>$\phi_{z-1}$</th>
<th>$z$</th>
<th>Welfare gains (in %)</th>
<th>Fraction gaining (in %)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>overall unempl empl</td>
<td>overall unempl empl</td>
</tr>
<tr>
<td>-4.00</td>
<td>$g$</td>
<td>-0.0148 -0.0285 -0.0137 -0.0927 -0.0398</td>
<td>1.86 3.47 1.72</td>
</tr>
<tr>
<td>-4.00</td>
<td>$b$</td>
<td>0.0176 0.0319 0.0164 0.0842 0.0353</td>
<td>99.69 98.65 99.78</td>
</tr>
<tr>
<td>-4.00 mean</td>
<td></td>
<td>0.0014 0.0017 0.0013 0.0042 0.0023</td>
<td>87.87 94.11 87.36</td>
</tr>
<tr>
<td>-1.00</td>
<td>$g$</td>
<td>-0.0034 -0.0068 -0.0031 -0.0220 -0.0093</td>
<td>2.27 4.20 2.10</td>
</tr>
<tr>
<td>-1.00</td>
<td>$b$</td>
<td>0.0048 0.0084 0.0045 0.0222 0.0096</td>
<td>99.87 99.14 99.93</td>
</tr>
<tr>
<td>-1.00 mean</td>
<td></td>
<td>0.0007 0.0008 0.0007 0.0001 0.0001</td>
<td>100.00 100.00 100.00</td>
</tr>
<tr>
<td>0.25</td>
<td>$g$</td>
<td>0.0011 0.0020 0.0010 0.0057 0.0026</td>
<td>99.24 98.96 99.26</td>
</tr>
<tr>
<td>0.25</td>
<td>$b$</td>
<td>-0.0018 -0.0008 -0.0008 -0.0054 -0.0021</td>
<td>0.88 3.07 0.70</td>
</tr>
<tr>
<td>0.25 mean</td>
<td></td>
<td>0.0001 0.0001 0.0001 0.0002 0.0002</td>
<td>92.29 87.80 92.66</td>
</tr>
<tr>
<td>1.00</td>
<td>$g$</td>
<td>0.0039 0.0074 0.0037 0.0220 0.0096</td>
<td>98.95 97.78 99.05</td>
</tr>
<tr>
<td>1.00</td>
<td>$b$</td>
<td>-0.0041 -0.0077 -0.0038 -0.0220 -0.0090</td>
<td>0.33 1.68 0.22</td>
</tr>
<tr>
<td>1.00 mean</td>
<td></td>
<td>-0.0001 -0.0001 -0.0001 -0.0000 0.0003</td>
<td>42.09 34.20 42.74</td>
</tr>
</tbody>
</table>

Note: The table shows the welfare gains from moving the agents from an economy with constant UI benefits to an economy with cyclically dependent benefits and slope parameter $\phi_{z-1}$. $uC$ and $eC$ denote the unemployed and employed, respectively, with a binding borrowing constraint. $g$ is a good state, $b$ is a bad state, $\text{mean}$ denotes the unconditional mean, i.e., $0.5(\bar{\mu}_g + \bar{\mu}_b)$. $\phi_{z-1} = -4.00$ is the level that maximizes the mean welfare gain.

Rounded to 100.00%. $\phi_{z-1} = -4.00$ is the level that maximizes the mean welfare gain.
Table 2.13: Averages of key economic variables – UI depending on lagged productivity, public budget balances on average

<table>
<thead>
<tr>
<th>$\phi_z$</th>
<th>$z$</th>
<th>$\bar{x} (%)$</th>
<th>$\bar{w}$</th>
<th>$\bar{h}$</th>
<th>$\bar{T}$</th>
<th>$\bar{RR}$</th>
<th>$\bar{p}$</th>
<th>$\bar{d}$</th>
<th>$\bar{y}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-1.00$</td>
<td>$g$</td>
<td>7.5638</td>
<td>0.0818</td>
<td>1.0812</td>
<td>67.157</td>
<td>2.5227</td>
<td>0.0291</td>
<td>0.0763</td>
<td>3.9730</td>
</tr>
<tr>
<td>$-1.00$</td>
<td>$b$</td>
<td>7.8181</td>
<td>0.0723</td>
<td>0.9247</td>
<td>66.628</td>
<td>2.4345</td>
<td>1.0599</td>
<td>0.0763</td>
<td>2.3242</td>
</tr>
<tr>
<td>$-1.00$</td>
<td>mean</td>
<td>7.6909</td>
<td>0.0770</td>
<td>1.0030</td>
<td>66.893</td>
<td>2.4811</td>
<td>0.9900</td>
<td>0.0763</td>
<td>2.3956</td>
</tr>
<tr>
<td>$-1.00$</td>
<td>$g$</td>
<td>7.6175</td>
<td>0.0797</td>
<td>0.9738</td>
<td>66.657</td>
<td>2.4371</td>
<td>0.9900</td>
<td>0.0761</td>
<td>2.3767</td>
</tr>
<tr>
<td>$-1.00$</td>
<td>$b$</td>
<td>7.7538</td>
<td>0.0771</td>
<td>0.9615</td>
<td>66.657</td>
<td>2.4818</td>
<td>0.9900</td>
<td>0.0761</td>
<td>2.3956</td>
</tr>
<tr>
<td>$-1.00$</td>
<td>mean</td>
<td>7.6856</td>
<td>0.0771</td>
<td>1.0040</td>
<td>66.936</td>
<td>2.4816</td>
<td>0.9900</td>
<td>0.0761</td>
<td>2.3967</td>
</tr>
<tr>
<td>$0.00$</td>
<td>$g$</td>
<td>7.6359</td>
<td>0.0790</td>
<td>1.0341</td>
<td>67.233</td>
<td>2.5318</td>
<td>0.9900</td>
<td>0.0761</td>
<td>2.3767</td>
</tr>
<tr>
<td>$0.00$</td>
<td>$b$</td>
<td>7.7332</td>
<td>0.0753</td>
<td>0.9247</td>
<td>66.628</td>
<td>2.4371</td>
<td>0.9900</td>
<td>0.0761</td>
<td>2.3767</td>
</tr>
<tr>
<td>$0.00$</td>
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<td>0.0771</td>
<td>1.0042</td>
<td>66.949</td>
<td>2.4816</td>
<td>0.9900</td>
<td>0.0761</td>
<td>2.3967</td>
</tr>
<tr>
<td>$1.00$</td>
<td>$g$</td>
<td>7.6547</td>
<td>0.0783</td>
<td>1.0224</td>
<td>67.249</td>
<td>2.5328</td>
<td>0.9900</td>
<td>0.0761</td>
<td>2.3767</td>
</tr>
<tr>
<td>$1.00$</td>
<td>$b$</td>
<td>7.7129</td>
<td>0.0761</td>
<td>0.9861</td>
<td>66.665</td>
<td>2.4371</td>
<td>0.9900</td>
<td>0.0761</td>
<td>2.3767</td>
</tr>
<tr>
<td>$1.00$</td>
<td>mean</td>
<td>7.6838</td>
<td>0.0771</td>
<td>1.0042</td>
<td>66.961</td>
<td>2.4816</td>
<td>0.9900</td>
<td>0.0761</td>
<td>2.3967</td>
</tr>
</tbody>
</table>

Note: The table shows the averages across good and bad periods, respectively. mean denotes the unconditional mean, i.e., $0.5(\bar{x}_g + \bar{x}_b)$, where $\bar{x}_z$ is the average of $x$ across periods with $z = \hat{z}$. $\bar{w}$ is the average wage; $\bar{RR} \equiv \bar{h} - \bar{T} \bar{w} - \bar{T}$ is the average replacement ratio. The aggregate output is $\tilde{y} \equiv zF(\tilde{k}(1 - u) - \xi v)$. $\phi_z = -4.00$ is the level that maximizes the mean welfare gain. For numerical reasons only, the benchmark is slightly different from comparable tables, since lagged productivity is included as an additional state variable, which changes the interpolation outcomes as well as the prediction rules.

2.C.4 Distortionary taxation

Table 2.14: Welfare consequences of different degrees of cyclicality in UI benefits – distortionary taxation, budget balance

<table>
<thead>
<tr>
<th>$\phi_z$</th>
<th>$z$</th>
<th>Welfare gains (in %)</th>
<th>Fraction gaining (in %)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>overall</td>
<td>unempl</td>
</tr>
<tr>
<td>$-1.00$</td>
<td>$g$</td>
<td>-0.0006</td>
<td>-0.0047</td>
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<tr>
<td>$-1.00$</td>
<td>$b$</td>
<td>0.0025</td>
<td>0.0066</td>
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<tr>
<td>$-1.00$</td>
<td>mean</td>
<td>0.0009</td>
<td>0.0010</td>
</tr>
<tr>
<td>$0.50$</td>
<td>$g$</td>
<td>0.0053</td>
<td>0.0073</td>
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<td>$0.50$</td>
<td>$b$</td>
<td>0.0043</td>
<td>0.0023</td>
</tr>
<tr>
<td>$0.50$</td>
<td>mean</td>
<td>0.0048</td>
<td>0.0048</td>
</tr>
<tr>
<td>$1.00$</td>
<td>$g$</td>
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<td>0.0080</td>
</tr>
<tr>
<td>$1.00$</td>
<td>$b$</td>
<td>0.0013</td>
<td>-0.0028</td>
</tr>
<tr>
<td>$1.00$</td>
<td>mean</td>
<td>0.0026</td>
<td>0.0026</td>
</tr>
</tbody>
</table>

Note: The table shows the welfare gains from moving from an economy with constant UI benefits to an economy with cyclically dependent benefits and slope parameter $\phi_z$. $uC$ and $eC$ denote the unemployed and employed, respectively, with a binding borrowing constraint. $g$ is a good state, $b$ is a bad state, mean denotes the unconditional mean, i.e., $0.5(\tilde{\mu}_g + \tilde{\mu}_b)$. $\phi_z = 0.50$ is the level that maximizes the mean welfare gain.
Table 2.15: Averages of key economic variables – distortionary taxation, budget balance

<table>
<thead>
<tr>
<th>φz</th>
<th>z</th>
<th>u (%)</th>
<th>ν</th>
<th>θ</th>
<th>k</th>
<th>w</th>
<th>h</th>
<th>RR</th>
<th>τ</th>
<th>p</th>
<th>d</th>
<th>g</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.00</td>
<td>g</td>
<td>7.6623</td>
<td>0.0784</td>
<td>1.0227</td>
<td>67.186</td>
<td>2.4501</td>
<td>0.9700</td>
<td>0.3959</td>
<td>0.0329</td>
<td>0.9303</td>
<td>0.0075</td>
<td>3.3667</td>
</tr>
<tr>
<td>-1.00</td>
<td>b</td>
<td>7.8024</td>
<td>0.0732</td>
<td>0.9383</td>
<td>66.616</td>
<td>2.3488</td>
<td>1.0100</td>
<td>0.4300</td>
<td>0.0364</td>
<td>0.8786</td>
<td>0.0007</td>
<td>3.2239</td>
</tr>
<tr>
<td>-1.00</td>
<td>mean</td>
<td>7.7324</td>
<td>0.0758</td>
<td>0.9805</td>
<td>66.901</td>
<td>2.3995</td>
<td>0.9900</td>
<td>0.4130</td>
<td>0.0346</td>
<td>0.9044</td>
<td>0.0041</td>
<td>3.2953</td>
</tr>
</tbody>
</table>

Note: The table shows the averages across good and bad periods, respectively. mean denotes the unconditional mean, i.e., \( \bar{\mu} \equiv \frac{\bar{\mu}_g + \bar{\mu}_b}{2} \), where \( \bar{\mu}_z \) is the average of \( \mu \) across periods with \( z = \hat{z} \). \( w \) is the average wage; \( RR \equiv \bar{w} \) is the average replacement ratio. The aggregate output is \( \hat{y} \equiv zF \left( \hat{k} \right) (1 - u) - \xi v \). \( \phi_z = 0.50 \) is the level that maximizes the mean welfare gain.

Table 2.16: Welfare consequences of different degrees of cyclicality in UI benefits – distortionary taxation, public budget balances on average

<table>
<thead>
<tr>
<th>φz</th>
<th>z</th>
<th>Welfare gains (in %)</th>
<th>Fraction gaining (in %)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>overall</td>
<td>unempl</td>
</tr>
<tr>
<td>-1.00</td>
<td>g</td>
<td>0.0020</td>
<td>-0.0017</td>
</tr>
<tr>
<td>-1.00</td>
<td>b</td>
<td>0.0095</td>
<td>0.0133</td>
</tr>
<tr>
<td>-1.00</td>
<td>mean</td>
<td>0.0057</td>
<td>0.0058</td>
</tr>
<tr>
<td>1.00</td>
<td>g</td>
<td>0.0236</td>
<td>0.0273</td>
</tr>
<tr>
<td>1.00</td>
<td>b</td>
<td>0.0161</td>
<td>0.0123</td>
</tr>
<tr>
<td>1.00</td>
<td>mean</td>
<td>0.0199</td>
<td>0.0198</td>
</tr>
</tbody>
</table>

Note: The table shows the welfare gains from moving the agents from an economy with constant UI benefits to an economy with cyclical dependent benefits and slope parameter \( \phi_z \). \( uC \) and \( eC \) denote the unemployed and employed, respectively, with a binding borrowing constraint. \( g \) is a good state, \( b \) is a bad state, mean denotes the unconditional mean, i.e., \( 0.5(\bar{\mu}_g + \bar{\mu}_b) \). Rounding to 100.00%. \( \phi_z = 1.00 \) is the level that maximizes the mean welfare gain.
Table 2.17: Averages of key economic variables – distortionary taxation, public budget balances on average

<table>
<thead>
<tr>
<th>φ_{z}</th>
<th>z</th>
<th>u (%)</th>
<th>v</th>
<th>θ</th>
<th>k</th>
<th>w</th>
<th>h</th>
<th>RR</th>
<th>τ</th>
<th>p</th>
<th>d</th>
<th>θ</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.00</td>
<td>g</td>
<td>7.6632</td>
<td>0.0783</td>
<td>1.0220</td>
<td>67.141</td>
<td>2.4456</td>
<td>0.9700</td>
<td>0.3966</td>
<td>0.0346</td>
<td>0.9307</td>
<td>0.0074</td>
<td>3.3660</td>
</tr>
<tr>
<td>-1.00</td>
<td>b</td>
<td>7.8001</td>
<td>0.0733</td>
<td>0.9396</td>
<td>66.593</td>
<td>2.3526</td>
<td>1.0100</td>
<td>0.4293</td>
<td>0.0346</td>
<td>0.8801</td>
<td>0.0007</td>
<td>3.2236</td>
</tr>
<tr>
<td>-1.00</td>
<td>mean</td>
<td>7.7316</td>
<td>0.0758</td>
<td>0.9808</td>
<td>66.867</td>
<td>2.3991</td>
<td>0.9900</td>
<td>0.4130</td>
<td>0.0346</td>
<td>0.9054</td>
<td>0.0041</td>
<td>3.2948</td>
</tr>
</tbody>
</table>

Note: The table shows the averages across good and bad periods, respectively. \( \text{mean} \) denotes the unconditional mean, i.e., \( \bar{x}_{g} + \bar{x}_{b} \), where \( \bar{x} \) is the average of \( x \) across periods with \( z = \tilde{z} \). \( \bar{w} \) is the average wage; \( RR \equiv \hat{h} \bar{w} \) is the average replacement ratio. The aggregate output is \( \hat{y} \equiv zF(\hat{k})(1 - u) - \xi v \). \( \phi_{z} = 1 \) is the level that maximizes the mean welfare gain.

2.C.5 Alternative calibration

For the alternative (Hagedorn-Manovskii) calibration we include the second moments in some of the prediction rules. In the benchmark, the law of motion for the aggregate capital stock is

\[
\log \hat{k}' = 0.0904 + 0.9765 \log \hat{k} - 0.0032 \log u + 0.0310 \log z \quad (R^2 = 0.9999993)
\]

and the prediction rules for the other aggregate variables are

\[
\log \theta = -154.7377 + 70.7565 \log \hat{k} + 2.4886 \log u + 192.3435 \log z - 8.0931 \left( \log \hat{k} \right)^2 -0.0063 (\log u)^2 - 44.3079 \log \hat{k} \log z - 1.0605 \log u \log z - 0.5941 \log \hat{k} \log u \quad (R^2 = 0.9999736)
\]

\[
\log (p + d) = -5.0678 + 1.3788 \log \hat{k} - 0.0572 \log u + 3.3249 \log z \quad (R^2 = 0.9999860)
\]

\[
\log Q_g = 0.1165 + 1.7180 \log \tilde{Q}_g - 0.0271 \log \hat{k} - 0.0032 \log u + 0.0026 \left( \log \hat{k} \right)^2 -0.0001 (\log u)^2 + 0.0006 \log \hat{k} \log u \quad (R^2 = 0.9999953)
\]

\[
\log Q_b = -0.1201 - 0.0529 \log \tilde{Q}_b + 0.0196 \log \hat{k} + 0.0089 \log u - 0.0019 \left( \log \hat{k} \right)^2 +0.0002 (\log u)^2 - 0.0018 \log \hat{k} \log u \quad (R^2 = 0.9999585)
\]

where \( \tilde{Q}_z \equiv \pi_{zz} \left( 1 - \delta + r \left( z, \hat{k}', u' \right) \right) \) for \( z = \{g, b\} \). \( \hat{k}' \) is calculated from the law of motion above, whereas \( u' \) is calculated from (2.1).
Table 2.18: Summary statistics for the benchmark of invariant UI benefits – alternative calibration, budget balance

<table>
<thead>
<tr>
<th>$z$</th>
<th>$u$</th>
<th>$v$</th>
<th>$\theta$</th>
<th>$k$</th>
<th>$w$</th>
<th>$h$</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>1.0000</td>
<td>0.0792</td>
<td>0.0552</td>
<td>0.7138</td>
<td>66.5861</td>
<td>2.3859</td>
</tr>
<tr>
<td>$\Delta_g$</td>
<td>+2.00%</td>
<td>−9.89%</td>
<td>+9.52%</td>
<td>+19.10%</td>
<td>+0.38%</td>
<td>+0.75%</td>
</tr>
</tbody>
</table>

$\bar{x}$ denotes the unconditional mean, i.e., $\frac{1}{5} \left( \sum_{x} x \right)$, where $x$ is the average across periods with $z = z$. $\Delta_g$ is the percentage deviation of the average across good states from the unconditional mean. Thus, per definition $\Delta_b = -\Delta_g$, and only $\Delta_g$ is shown. $\bar{w}$ is the average wage; $\bar{RR} \equiv \bar{h} \cdot \bar{w}$ is the average replacement ratio; $\bar{y}$ is the aggregate output (net of vacancy costs); $\bar{c}$ is the aggregate consumption. Note that $d$ can become negative.

Table 2.19: Welfare consequences of different degrees of cyclicality in UI benefits – alternative calibration, budget balance

<table>
<thead>
<tr>
<th>$\phi_z$</th>
<th>$z$</th>
<th>Welfare gains (in %)</th>
<th>$\mu$</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>$g$</td>
<td>overall</td>
<td>-0.023</td>
<td>-0.1030</td>
</tr>
<tr>
<td>1.00</td>
<td>$b$</td>
<td>overall</td>
<td>-0.0900</td>
<td>-0.0904</td>
</tr>
<tr>
<td>1.00</td>
<td>mean</td>
<td>overall</td>
<td>-0.0966</td>
<td>-0.0967</td>
</tr>
<tr>
<td>1.00</td>
<td>$g$</td>
<td>overall</td>
<td>0.0768</td>
<td>0.0775</td>
</tr>
<tr>
<td>1.00</td>
<td>$b$</td>
<td>overall</td>
<td>0.0636</td>
<td>0.0630</td>
</tr>
<tr>
<td>1.00</td>
<td>mean</td>
<td>overall</td>
<td>0.0702</td>
<td>0.0703</td>
</tr>
<tr>
<td>2.00</td>
<td>$g$</td>
<td>overall</td>
<td>0.1321</td>
<td>0.1334</td>
</tr>
<tr>
<td>2.00</td>
<td>$b$</td>
<td>overall</td>
<td>0.1029</td>
<td>0.1018</td>
</tr>
<tr>
<td>2.00</td>
<td>mean</td>
<td>overall</td>
<td>0.1175</td>
<td>0.1176</td>
</tr>
</tbody>
</table>

Note: The table shows the welfare gains from moving the agents from an economy with constant UI benefits to an economy with cyclically dependent benefits and slope parameter $\phi_z$. $\mu_C$ and $\sigma_C$ denote the unemployed and employed, respectively, with a binding borrowing constraint. $g$ is a good state, $b$ is a bad state, mean denotes the unconditional mean, i.e., $0.5(\bar{\mu}_g + \bar{\mu}_b)$. Convergence is not achieved for values of $\phi_z$ higher than 2.1.
Table 2.20: Averages of key economic variables – alternative calibration, budget balance

<table>
<thead>
<tr>
<th>φz</th>
<th>z</th>
<th>u (%)</th>
<th>v</th>
<th>θ</th>
<th>k</th>
<th>w</th>
<th>h</th>
<th>T</th>
<th>RR</th>
<th>p</th>
<th>d</th>
<th>ŷ</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.00</td>
<td>g</td>
<td>7.0150</td>
<td>0.0623</td>
<td>0.8911</td>
<td>66.89</td>
<td>2.3894</td>
<td>2.2734</td>
<td>0.0680</td>
<td>0.9500</td>
<td>2.5786</td>
<td>0.0354</td>
<td>3.2825</td>
</tr>
<tr>
<td>-1.00</td>
<td>b</td>
<td>9.1879</td>
<td>0.0482</td>
<td>0.5290</td>
<td>66.195</td>
<td>2.3815</td>
<td>2.3134</td>
<td>0.0928</td>
<td>0.9702</td>
<td>2.1325</td>
<td>-0.0116</td>
<td>3.1179</td>
</tr>
<tr>
<td>-1.00</td>
<td>mean</td>
<td>8.1015</td>
<td>0.0553</td>
<td>0.7101</td>
<td>66.442</td>
<td>2.3855</td>
<td>2.2934</td>
<td>0.0804</td>
<td>0.9601</td>
<td>2.3555</td>
<td>0.0119</td>
<td>3.2002</td>
</tr>
<tr>
<td>0.00</td>
<td>g</td>
<td>7.1373</td>
<td>0.0605</td>
<td>0.8501</td>
<td>66.841</td>
<td>2.4037</td>
<td>2.2934</td>
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<td>0.9527</td>
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<td>0.0285</td>
<td>3.2856</td>
</tr>
<tr>
<td>0.00</td>
<td>b</td>
<td>8.7046</td>
<td>0.0500</td>
<td>0.5775</td>
<td>66.331</td>
<td>2.3681</td>
<td>2.3134</td>
<td>0.0928</td>
<td>0.9702</td>
<td>2.2058</td>
<td>-0.0048</td>
<td>3.1267</td>
</tr>
<tr>
<td>0.00</td>
<td>mean</td>
<td>8.1015</td>
<td>0.0553</td>
<td>0.7101</td>
<td>66.442</td>
<td>2.3855</td>
<td>2.2934</td>
<td>0.0804</td>
<td>0.9601</td>
<td>2.3555</td>
<td>0.0119</td>
<td>3.2002</td>
</tr>
<tr>
<td>1.00</td>
<td>g</td>
<td>8.1015</td>
<td>0.0553</td>
<td>0.7101</td>
<td>66.442</td>
<td>2.3855</td>
<td>2.2934</td>
<td>0.0804</td>
<td>0.9601</td>
<td>2.3555</td>
<td>0.0119</td>
<td>3.2002</td>
</tr>
<tr>
<td>1.00</td>
<td>b</td>
<td>9.1879</td>
<td>0.0482</td>
<td>0.5290</td>
<td>66.195</td>
<td>2.3815</td>
<td>2.3134</td>
<td>0.0928</td>
<td>0.9702</td>
<td>2.1325</td>
<td>-0.0116</td>
<td>3.1179</td>
</tr>
<tr>
<td>1.00</td>
<td>mean</td>
<td>8.1015</td>
<td>0.0553</td>
<td>0.7101</td>
<td>66.442</td>
<td>2.3855</td>
<td>2.2934</td>
<td>0.0804</td>
<td>0.9601</td>
<td>2.3555</td>
<td>0.0119</td>
<td>3.2002</td>
</tr>
<tr>
<td>2.00</td>
<td>g</td>
<td>7.9210</td>
<td>0.0552</td>
<td>0.7138</td>
<td>66.586</td>
<td>2.3681</td>
<td>2.2934</td>
<td>0.0862</td>
<td>0.9673</td>
<td>2.2663</td>
<td>0.0024</td>
<td>3.1353</td>
</tr>
<tr>
<td>2.00</td>
<td>b</td>
<td>8.7046</td>
<td>0.0500</td>
<td>0.5775</td>
<td>66.331</td>
<td>2.3105</td>
<td>0.0517</td>
<td>0.0362</td>
<td>0.9444</td>
<td>2.2058</td>
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<td>3.1267</td>
</tr>
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<td>2.00</td>
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<td>8.1015</td>
<td>0.0553</td>
<td>0.7101</td>
<td>66.442</td>
<td>2.3855</td>
<td>2.2934</td>
<td>0.0804</td>
<td>0.9601</td>
<td>2.3555</td>
<td>0.0119</td>
<td>3.2002</td>
</tr>
</tbody>
</table>

Note: The table shows the averages across good and bad periods, respectively. mean denotes the unconditional mean, i.e., \( \bar{x}_g + \bar{x}_b \), where \( \bar{x}_z \) is the average of \( x \) across periods with \( z = \hat{z} \). \( \bar{w} \) is the average wage; \( RR \equiv h - T \bar{w} - T \) is the average replacement ratio. The aggregate output is \( \hat{y} \equiv zF(\hat{k}) (1 - u) - \xi v \).

Table 2.21: Welfare consequences of different degrees of cyclicality in UI benefits – alternative calibration, public budget balances on average

<table>
<thead>
<tr>
<th>φz</th>
<th>z</th>
<th>Welfare gains (in percentage)</th>
<th>Fraction gaining (in %)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>overall</td>
<td>unemployed</td>
</tr>
<tr>
<td>-1.00</td>
<td>g</td>
<td>-0.1031</td>
<td>-0.1037</td>
</tr>
<tr>
<td>-1.00</td>
<td>b</td>
<td>-0.0713</td>
<td>-0.0708</td>
</tr>
<tr>
<td>-1.00</td>
<td>mean</td>
<td>-0.0872</td>
<td>-0.0873</td>
</tr>
<tr>
<td>1.00</td>
<td>g</td>
<td>0.0704</td>
<td>0.0711</td>
</tr>
<tr>
<td>1.00</td>
<td>b</td>
<td>0.0380</td>
<td>0.0375</td>
</tr>
<tr>
<td>1.00</td>
<td>mean</td>
<td>0.0542</td>
<td>0.0543</td>
</tr>
<tr>
<td>2.00</td>
<td>g</td>
<td>0.1110</td>
<td>0.1123</td>
</tr>
<tr>
<td>2.00</td>
<td>b</td>
<td>0.0461</td>
<td>0.0450</td>
</tr>
<tr>
<td>2.00</td>
<td>mean</td>
<td>0.0786</td>
<td>0.0787</td>
</tr>
</tbody>
</table>

Note: The table shows the welfare gains from moving the agents from an economy with constant UI benefits to an economy with cyclically dependent benefits and slope parameter φz. uC and eC denote the unemployed and employed, respectively, with a binding borrowing constraint. g is a good state, b is a bad state, mean denotes the unconditional mean, i.e., \( 0.5(\bar{\mu}_g + \bar{\mu}_b) \). Convergence is not achieved for values of φz higher than 2.1.
Table 2.22: Averages of key economic variables – alternative calibration, public budget balances on average

<table>
<thead>
<tr>
<th>φ</th>
<th>z</th>
<th>u (%)</th>
<th>v</th>
<th>θ</th>
<th>k</th>
<th>w</th>
<th>h</th>
<th>T</th>
<th>RR</th>
<th>p</th>
<th>d</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.00</td>
<td>g</td>
<td>7.0355</td>
<td>0.0620</td>
<td>0.8839</td>
<td>66.578</td>
<td>2.3884</td>
<td>2.2734</td>
<td>0.0806</td>
<td>0.9502</td>
<td>2.7839</td>
<td>0.0359</td>
<td>3.2809</td>
</tr>
<tr>
<td>-1.00</td>
<td>b</td>
<td>9.2071</td>
<td>0.0482</td>
<td>0.5262</td>
<td>66.125</td>
<td>2.3814</td>
<td>2.3134</td>
<td>0.0806</td>
<td>0.9705</td>
<td>2.1460</td>
<td>-0.0119</td>
<td>3.1165</td>
</tr>
<tr>
<td>-1.00</td>
<td>mean</td>
<td>8.1213</td>
<td>0.0551</td>
<td>0.7050</td>
<td>66.352</td>
<td>2.3849</td>
<td>2.2934</td>
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<td>0.0120</td>
<td>3.1987</td>
</tr>
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<td>g</td>
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<td>0.0603</td>
<td>0.8441</td>
<td>66.749</td>
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<td>2.2934</td>
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</tr>
<tr>
<td>0.00</td>
<td>mean</td>
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<td>0.0551</td>
<td>0.7092</td>
<td>66.501</td>
<td>2.3853</td>
<td>2.2934</td>
<td>0.0786</td>
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<td>2.3134</td>
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<td>0.0516</td>
<td>0.6216</td>
<td>66.333</td>
<td>2.3540</td>
<td>2.2734</td>
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<td>0.9646</td>
<td>2.2650</td>
<td>0.0023</td>
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</tr>
<tr>
<td>1.00</td>
<td>mean</td>
<td>8.1213</td>
<td>0.0551</td>
<td>0.7120</td>
<td>66.601</td>
<td>2.3856</td>
<td>2.2934</td>
<td>0.0774</td>
<td>0.9601</td>
<td>2.3767</td>
<td>0.0119</td>
<td>3.2097</td>
</tr>
<tr>
<td>2.00</td>
<td>g</td>
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<td>0.0568</td>
<td>0.7586</td>
<td>66.961</td>
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<td>0.0141</td>
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</tr>
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<td>0.0533</td>
<td>0.6682</td>
<td>66.387</td>
<td>2.3402</td>
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<td>0.0767</td>
<td>0.9617</td>
<td>2.3106</td>
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<td>3.1390</td>
</tr>
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<td>2.00</td>
<td>mean</td>
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<td>0.0551</td>
<td>0.7134</td>
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<td>0.0767</td>
<td>0.9600</td>
<td>2.3667</td>
<td>0.0120</td>
<td>3.2123</td>
</tr>
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</table>

Note: The table shows the averages across good and bad periods, respectively. mean denotes the unconditional mean, i.e., 0.5(¯x_g + ¯x_b), where ¯x_z is the average of x across periods with z = ¯z. w is the average wage; RR ≡ h − T ¯w − T is the average replacement ratio. The aggregate output is ˆy ≡ z F( ˜k ) (1 − u) − ξ v.

2.C.6 Rigid wages

Table 2.23: Summary statistics for the benchmark of invariant UI benefits – rigid wages, budget balance

<table>
<thead>
<tr>
<th>φ</th>
<th>z</th>
<th>u (%)</th>
<th>v</th>
<th>θ</th>
<th>k</th>
<th>w</th>
<th>h</th>
<th>T</th>
<th>RR</th>
<th>p</th>
<th>d</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>1.0000</td>
<td>0.0655</td>
<td>0.1263</td>
<td>1.981</td>
<td>67.5924</td>
<td>2.4500</td>
<td>0.9900</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Δ_g</td>
<td>+2.00%</td>
<td>-6.80%</td>
<td>+23.72%</td>
<td>+29.86%</td>
<td>+0.39%</td>
<td>0.00%</td>
<td>0.00%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>mean</td>
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<td>3.3090</td>
<td>2.6427</td>
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</tr>
<tr>
<td>Δ_y</td>
<td>-6.80%</td>
<td>+0.29%</td>
<td>+11.61%</td>
<td>+474.61%</td>
<td>+2.06%</td>
<td>+2.02%</td>
<td>+0.36%</td>
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</tbody>
</table>

Note: mean denotes the unconditional mean, i.e., 0.5(¯x_g + ¯x_b), where ¯x_z is the average of x across periods with z = ¯z. Δ_g is the percentage deviation of the average across good states from the unconditional mean. Thus, per definition Δ_b = −Δ_g, and only Δ_g is shown. w is the average wage; RR ≡ h − T ¯w − T is the average replacement ratio; ˆy is the aggregate output (net of vacancy costs); ˆc is the aggregate consumption. Note that d can become negative.
Table 2.24: Welfare consequences of different degrees of cyclicality in UI benefits – rigid wages, budget balance

<table>
<thead>
<tr>
<th>$\phi_z$</th>
<th>$z$</th>
<th>Overall unemp</th>
<th>empl</th>
<th>$uC$</th>
<th>$eC$</th>
<th>Fraction gaining (in %)</th>
</tr>
</thead>
<tbody>
<tr>
<td>−7.00 g</td>
<td>0.0003</td>
<td>−0.0392</td>
<td>0.0224</td>
<td>0.0097</td>
<td>0.0003</td>
<td>93.39</td>
</tr>
<tr>
<td>−7.00 b</td>
<td>0.0004</td>
<td>0.0344</td>
<td>−0.0021</td>
<td>0.0979</td>
<td>0.0034</td>
<td>7.23</td>
</tr>
<tr>
<td>−7.00 mean</td>
<td>0.0003</td>
<td>0.0026</td>
<td>0.0000</td>
<td>0.0020</td>
<td>0.0018</td>
<td>51.13</td>
</tr>
</tbody>
</table>

Note: The table shows the welfare gains from moving the agents from an economy with constant UI benefits to an economy with cyclically dependent benefits and slope parameter $\phi_z$. $uC$ and $eC$ denote the unemployed and employed, respectively, with a binding borrowing constraint. $g$ is a good state, $b$ is a bad state, mean denotes the unconditional mean, i.e., $0.5(\bar{\mu}_g + \bar{\mu}_b)$. $\cdot$: Rounded to 100.00%. $\phi_z = −7.00$ is the level that maximizes the mean welfare gain.

Table 2.25: Averages of key economic variables – rigid wages, budget balance

<table>
<thead>
<tr>
<th>$\phi_z$</th>
<th>$z$</th>
<th>$u$ (%)</th>
<th>$\theta$</th>
<th>$k$</th>
<th>$\bar{w}$</th>
<th>$h$</th>
<th>$hT$</th>
<th>$RR$</th>
<th>$p$</th>
<th>$d$</th>
<th>$\hat{y}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>−7.00 g</td>
<td>6.1018</td>
<td>0.1562</td>
<td>2.5719</td>
<td>67.85</td>
<td>2.4500</td>
<td>0.8500</td>
<td>0.0519</td>
<td>0.3328</td>
<td>2.5479</td>
<td>0.0376</td>
<td>3.3758</td>
</tr>
<tr>
<td>−7.00 b</td>
<td>6.9920</td>
<td>0.0963</td>
<td>1.3887</td>
<td>67.32</td>
<td>2.4500</td>
<td>1.1300</td>
<td>0.0790</td>
<td>0.4433</td>
<td>2.0178</td>
<td>−0.0245</td>
<td>3.2422</td>
</tr>
<tr>
<td>−7.00 mean</td>
<td>6.5469</td>
<td>0.1263</td>
<td>1.9803</td>
<td>67.59</td>
<td>2.4500</td>
<td>0.9900</td>
<td>0.0654</td>
<td>0.3880</td>
<td>2.2828</td>
<td>0.0065</td>
<td>3.3090</td>
</tr>
<tr>
<td>−1.00 g</td>
<td>6.1013</td>
<td>0.1562</td>
<td>2.5724</td>
<td>67.86</td>
<td>2.4500</td>
<td>0.9900</td>
<td>0.0692</td>
<td>0.3948</td>
<td>2.0178</td>
<td>−0.0245</td>
<td>3.2422</td>
</tr>
<tr>
<td>−1.00 b</td>
<td>6.9913</td>
<td>0.0963</td>
<td>1.3892</td>
<td>67.30</td>
<td>2.4500</td>
<td>0.9900</td>
<td>0.0692</td>
<td>0.3879</td>
<td>2.2828</td>
<td>0.0065</td>
<td>3.3090</td>
</tr>
<tr>
<td>−1.00 mean</td>
<td>6.5464</td>
<td>0.1263</td>
<td>1.9808</td>
<td>67.59</td>
<td>2.4500</td>
<td>0.9900</td>
<td>0.0692</td>
<td>0.3879</td>
<td>2.2828</td>
<td>0.0065</td>
<td>3.3090</td>
</tr>
<tr>
<td>0.00 g</td>
<td>6.1014</td>
<td>0.1562</td>
<td>2.5724</td>
<td>67.86</td>
<td>2.4500</td>
<td>0.9900</td>
<td>0.0692</td>
<td>0.3879</td>
<td>2.2828</td>
<td>0.0065</td>
<td>3.3090</td>
</tr>
<tr>
<td>0.00 b</td>
<td>6.9911</td>
<td>0.0963</td>
<td>1.3893</td>
<td>67.30</td>
<td>2.4500</td>
<td>0.9900</td>
<td>0.0692</td>
<td>0.3879</td>
<td>2.2828</td>
<td>0.0065</td>
<td>3.3090</td>
</tr>
<tr>
<td>0.00 mean</td>
<td>6.5463</td>
<td>0.1263</td>
<td>1.9809</td>
<td>67.59</td>
<td>2.4500</td>
<td>0.9900</td>
<td>0.0692</td>
<td>0.3879</td>
<td>2.2828</td>
<td>0.0065</td>
<td>3.3090</td>
</tr>
<tr>
<td>1.00 g</td>
<td>6.1014</td>
<td>0.1562</td>
<td>2.5723</td>
<td>67.85</td>
<td>2.4500</td>
<td>0.9900</td>
<td>0.0692</td>
<td>0.3879</td>
<td>2.2828</td>
<td>0.0065</td>
<td>3.3090</td>
</tr>
<tr>
<td>1.00 b</td>
<td>6.9911</td>
<td>0.0963</td>
<td>1.3893</td>
<td>67.30</td>
<td>2.4500</td>
<td>0.9900</td>
<td>0.0692</td>
<td>0.3879</td>
<td>2.2828</td>
<td>0.0065</td>
<td>3.3090</td>
</tr>
<tr>
<td>1.00 mean</td>
<td>6.5463</td>
<td>0.1263</td>
<td>1.9808</td>
<td>67.59</td>
<td>2.4500</td>
<td>0.9900</td>
<td>0.0692</td>
<td>0.3879</td>
<td>2.2828</td>
<td>0.0065</td>
<td>3.3090</td>
</tr>
</tbody>
</table>

Note: The table shows the averages across good and bad periods, respectively. mean denotes the unconditional mean, i.e., $0.5(\bar{x}_g + \bar{x}_b)$, where $\bar{x}_z$ is the average of $x$ across periods with $z = z$. $\bar{w}$ is the average wage; $RR \equiv hT \bar{w}$ is the average replacement ratio. The aggregate output is $\hat{y} \equiv zF(\tilde{k})(1 - u) - \xi v$. $\phi_z = −7.00$ is the level that maximizes the mean welfare gain.
Table 2.26: Welfare consequences of different degrees of cyclicality in UI benefits – rigid wages, public budget balances on average

<table>
<thead>
<tr>
<th>( \phi_z )</th>
<th>( z )</th>
<th>Welfare gains (in %)</th>
<th>Fraction gaining (in %)</th>
<th>Overall</th>
<th>Unempl</th>
<th>Empl</th>
<th>Overall</th>
<th>Unempl</th>
<th>Empl</th>
</tr>
</thead>
<tbody>
<tr>
<td>-45.00</td>
<td>( g )</td>
<td>-0.0022</td>
<td>-0.2676</td>
<td>0.0112</td>
<td>-2.5266</td>
<td>-0.1558</td>
<td>43.43</td>
<td>1.06</td>
<td>46.13</td>
</tr>
<tr>
<td>-45.00</td>
<td>( b )</td>
<td>0.1263</td>
<td>0.3478</td>
<td>0.1095</td>
<td>0.6317</td>
<td>0.1609</td>
<td>95.88</td>
<td>100.00</td>
<td>95.55</td>
</tr>
<tr>
<td>-45.00</td>
<td>mean</td>
<td>0.0621</td>
<td>0.0701</td>
<td>0.0604</td>
<td>-0.9474</td>
<td>0.0026</td>
<td>99.98</td>
<td>99.61</td>
<td>100.00</td>
</tr>
<tr>
<td>-1.00</td>
<td>( g )</td>
<td>-0.0013</td>
<td>-0.0031</td>
<td>-0.0016</td>
<td>0.0132</td>
<td>-0.0001</td>
<td>90.44</td>
<td>5.18</td>
<td>95.99</td>
</tr>
<tr>
<td>-1.00</td>
<td>( b )</td>
<td>0.0043</td>
<td>0.0094</td>
<td>0.0040</td>
<td>-0.0218</td>
<td>-0.0064</td>
<td>99.75</td>
<td>100.00</td>
<td>99.73</td>
</tr>
<tr>
<td>-1.00</td>
<td>mean</td>
<td>0.0028</td>
<td>0.0031</td>
<td>0.0028</td>
<td>0.0039</td>
<td>0.0031</td>
<td>100.00</td>
<td>100.00</td>
<td>100.00</td>
</tr>
</tbody>
</table>

Note: The table shows the welfare gains from moving the agents from an economy with constant UI benefits to an economy with cyclically dependent benefits and slope parameter \( \phi_z \). \( uC \) and \( eC \) denote the unemployed and employed, respectively, with a binding borrowing constraint. \( g \) is a good state, \( b \) is a bad state, mean denotes the unconditional mean, i.e., \( 0.5(\bar{\mu}_g + \bar{\mu}_b) \). \( \phi_z = -45.00 \) is the level that maximizes the mean welfare gain.

Table 2.27: Averages of key economic variables – rigid wages, public budget balances on average

<table>
<thead>
<tr>
<th>( \phi_z )</th>
<th>( z )</th>
<th>( v )</th>
<th>( \theta )</th>
<th>( k )</th>
<th>( \hat{w} )</th>
<th>( h )</th>
<th>( T )</th>
<th>( RR )</th>
<th>( p )</th>
<th>( d )</th>
<th>( \hat{y} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-45.00</td>
<td>( g )</td>
<td>6.2314</td>
<td>0.1449</td>
<td>2.3264</td>
<td>67.316</td>
<td>2.4500</td>
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<td>0.0604</td>
<td>0.0124</td>
<td>2.3368</td>
<td>0.0387</td>
</tr>
<tr>
<td>-45.00</td>
<td>( b )</td>
<td>6.9428</td>
<td>0.0987</td>
<td>1.4217</td>
<td>67.260</td>
<td>2.4500</td>
<td>1.8900</td>
<td>0.0648</td>
<td>0.3886</td>
<td>2.1086</td>
<td>0.0060</td>
</tr>
<tr>
<td>-45.00</td>
<td>mean</td>
<td>6.5871</td>
<td>0.1218</td>
<td>1.8740</td>
<td>67.288</td>
<td>2.4500</td>
<td>0.9900</td>
<td>0.0648</td>
<td>0.3886</td>
<td>2.2814</td>
<td>0.0066</td>
</tr>
<tr>
<td>-1.00</td>
<td>( g )</td>
<td>6.1214</td>
<td>0.1546</td>
<td>2.5340</td>
<td>67.585</td>
<td>2.4500</td>
<td>1.0000</td>
<td>0.0692</td>
<td>0.3952</td>
<td>2.0186</td>
<td>-0.0247</td>
</tr>
<tr>
<td>-1.00</td>
<td>( b )</td>
<td>7.0032</td>
<td>0.0958</td>
<td>1.3770</td>
<td>67.275</td>
<td>2.4500</td>
<td>0.9900</td>
<td>0.0648</td>
<td>0.3879</td>
<td>2.2814</td>
<td>0.0066</td>
</tr>
<tr>
<td>-1.00</td>
<td>mean</td>
<td>6.5623</td>
<td>0.1252</td>
<td>1.9558</td>
<td>67.530</td>
<td>2.4500</td>
<td>0.9900</td>
<td>0.0648</td>
<td>0.3879</td>
<td>2.2814</td>
<td>0.0066</td>
</tr>
<tr>
<td>0.00</td>
<td>( g )</td>
<td>6.1188</td>
<td>0.1548</td>
<td>2.5398</td>
<td>67.795</td>
<td>2.4500</td>
<td>0.9900</td>
<td>0.0648</td>
<td>0.3879</td>
<td>2.2814</td>
<td>0.0066</td>
</tr>
<tr>
<td>0.00</td>
<td>( b )</td>
<td>7.0044</td>
<td>0.0957</td>
<td>1.3765</td>
<td>67.276</td>
<td>2.4500</td>
<td>0.9900</td>
<td>0.0648</td>
<td>0.3868</td>
<td>2.2817</td>
<td>0.0066</td>
</tr>
<tr>
<td>0.00</td>
<td>mean</td>
<td>6.5616</td>
<td>0.1251</td>
<td>1.9582</td>
<td>67.530</td>
<td>2.4500</td>
<td>0.9900</td>
<td>0.0648</td>
<td>0.3879</td>
<td>2.2817</td>
<td>0.0066</td>
</tr>
<tr>
<td>1.00</td>
<td>( g )</td>
<td>6.1162</td>
<td>0.1551</td>
<td>2.5453</td>
<td>67.807</td>
<td>2.4500</td>
<td>1.0100</td>
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<td>0.0378</td>
</tr>
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<td>1.3761</td>
<td>67.278</td>
<td>2.4500</td>
<td>0.9700</td>
<td>0.0692</td>
<td>0.3784</td>
<td>2.0178</td>
<td>-0.0246</td>
</tr>
<tr>
<td>1.00</td>
<td>mean</td>
<td>6.5608</td>
<td>0.1254</td>
<td>1.9607</td>
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<td>2.4500</td>
<td>0.9900</td>
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<td>0.3879</td>
<td>2.2823</td>
<td>0.0066</td>
</tr>
</tbody>
</table>

Note: The table shows the averages across good and bad periods, respectively. mean denotes the unconditional mean, i.e., \( 0.5(\bar{x}_g + \bar{x}_b) \), where \( \bar{x}_z \) is the average of \( x \) across periods with \( z = \bar{z} \). \( \hat{w} \) is the average wage; \( RR \equiv \frac{h - T}{\bar{w}} \) is the average replacement ratio. The aggregate output is \( \hat{y} \equiv zF(\hat{k})(1 - u) - \xi v \). \( \phi_z = -45.00 \) is the level that maximizes the mean welfare gain.
Benefit Reentitlement Conditions in Unemployment Insurance Schemes
Benefit Reentitlement Conditions in Unemployment Insurance Schemes*

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CEPR, CESifo, and IZA  CAM, IZA

May 2013

Abstract

The role of employment requirements to qualify for unemployment benefits is considered in a search-matching equilibrium. We show that the reentitlement requirement can work as a substitute to the duration of unemployment benefits. The economic structure and preferences, captured by productivity and risk aversion, respectively, are found to have important consequences for the optimal design of the unemployment insurance (UI) scheme, and this may in part explain the variation in UI schemes across OECD countries. Finally, we consider a business cycle version of our model in which the optimal UI scheme turns out to exhibit countercyclical generosity.

JEL Classification: E32, H3, J65

Keywords: Reentitlement effects; Unemployment insurance; Business cycle

*The authors gratefully acknowledge comments from Birthe Larsen, Morten O. Ravn, and Allan Sørensen. All remaining errors are, of course, our own.

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3.1 Introduction

Most unemployment insurance systems have a finite benefit period. The implication is that unemployed who are not able to find employment before their unemployment benefits expire lose their benefits. They may then be entitled to social assistance which typically exhibits a lower replacement rate and may be conditional on wealth and family situation. To regain the right to unemployment benefits the unemployed has to accumulate a certain amount of employment within a given period, and in the following we will refer to this as a reentitlement requirement. The spell of employment necessary to re-qualify for benefits varies across countries and reflects different parameters which in sum reflect the generosity of the benefit system; i.e., benefit level, duration of benefits and benefit profile.

Venn (2012) gives a detailed description of unemployment benefit systems across OECD countries. She constructs quantitative indicators for the strictness of eligibility criteria for unemployment benefits across 36 OECD and/or EU members, showing large differences across fairly similar countries. The quantitative indicator for the strictness of entitlement conditions (covering minimum employment and contribution record as well as sanctions for voluntary unemployment) is negatively correlated with initial net replacement rate of unemployment benefits (correlation of $-0.05$ but insignificant), with net replacement averaged over 5 years (correlation of $-0.36$ and significant at the 5% level), and with the maximum duration of unemployment benefits (also a correlation of $-0.36$ and significant at the 5% level). That is, countries with relatively soft entitlement requirements tend to have more generous unemployment insurance schemes.

In the literature on optimal design of unemployment insurance systems (see e.g. Fredriksson & Holmlund (2006) and Tatsiramos & van Ours (2012) for surveys), it is only recently that an employment requirement is included as a policy instrument (see e.g. Hopenhayn & Nicolini (2009), Ortega & Rioux (2010), Pan & Zhang (2012), and Zhang & Faig (2012)). Hopenhayn & Nicolini (2009) argue that when it is not possible to distinguish between quits and layoffs the optimal benefit system conditions benefits paid to the unemployed on their employment history. Ortega & Rioux (2010), on the other hand, emphasize that an employment criterion can support job creation since unemployed who have exhausted their benefits are willing to accept lower wages to regain the right to unemployment benefits. In this paper we emphasize an additional argument for having an employment requirement, namely to strengthen search incentives. In the literature on two-tier benefit structures (e.g. Mortensen (1977) and Fredriksson & Holmlund (2001)) the entitlement effect that induces unemployment benefit recipients to increase their search activity when benefits expire is driving the optimality conditions for a two-tier benefit system. In this paper we show that by having an employment requirement it is possible to affect the search margin of both the unemployed receiving benefits and those receiving social assistance.
This finding enables us to discuss the trade-off between different instruments in the benefit system. In particular we focus on the trade-off between the length of the benefit period and the employment period required to regain the right to benefits. Our study is related to Ortega & Rioux (2010). They, however, ignore the search effects of changing the parameters of the benefit system and thereby potentially miss an essential part of the effects of different benefit configurations. Our first contribution is therefore to illustrate the optimal policy mix in a search-matching model where both search and wage setting are affected. We show that the reentitlement requirement can work as a substitute to the benefit duration. Furthermore, we show that the economic structure and preferences, captured by productivity and risk aversion, respectively, are found to have important consequences for the optimal design of the UI scheme, and this may in part explain the variation in UI schemes across OECD countries.

Our second contribution is to illustrate how the optimal structure of the UI scheme varies over the business cycle. Recently, a number of papers have studied optimal unemployment insurance over the business cycle (e.g. Moyen & Stähler (2009), Andersen & Svarer (2010; 2011), Landais, Michaillat, & Saez (2010), Kroft & Notowidigdo (2011), Mitman & Rabinovich (2011), and Ek (2012)). The current analysis will investigate how the reentitlement requirement optimally varies with the business cycle and relate it to the findings in the concurrent literature. We find – like most of the related literature – that the optimal UI scheme over the business cycle exhibits countercyclical generosity.

The optimal UI scheme over the business cycle does not use the different UI instruments symmetrically. In general, benefit levels seem to dominate benefit duration, which again dominates the reentitlement requirement in the sense that the dominated instruments are used to mitigate some of the negative labor market effects of the stronger instruments.

The structure of the paper is as follows: In Section 3.2 we introduce a search-matching model with non-automatic eligibility for unemployment benefits. The effects of reentitlement requirement and benefit duration on job search and labor market outcomes are analyzed in Section 3.3. The optimal UI scheme is considered in Section 3.4, while Section 3.5 discusses the optimal UI scheme over the business cycle. Concluding remarks are given in Section 3.6.

### 3.2 Benefit entitlement in a search-matching model

Consider a search-matching model of the labor market where workers can be in one of four states: i) possessing a job and fulfilling the requirements for benefit eligibility in case of involuntary job-separation (state $E$), ii) possessing a job but not being entitled to unemployment benefits but rather social benefits in case of involuntary job separation (state $N$), iii) being unemployed and entitled to unemployment benefits (state $U$), UIB-unemployed, and iv) being unemployed and not entitled to unemployment benefits but social assistance (state $K$),
There is a continuum of workers with mass one. All workers are assumed to own an equal share of the firms, and therefore firm profits are distributed among workers, which secures that our model can be given a general equilibrium interpretation. All employed may lose their job by the exogenous separation rate $p_{U,E}$. Employed not entitled to unemployment benefits gain eligibility at the rate $p_{E,N}$, and thus the expected work requirement before regaining UIB eligibility is $1/p_{E,N}$. Unemployed eligible for benefits search for jobs at the rate $s_U$ and find a job at the rate $\alpha s_U$, where $\alpha$ is the job-finding rate (see below). Unemployed eligible for benefits lose eligibility at the rate $1/p_{K,U}$, and thus the expected potential duration of benefit receipt is $1/p_{K,U}$. Finally, unemployed non-eligible for benefits search at the rate $s_K$ and thus find a job at the rate $\alpha s_K$. Note that there are only involuntary job-separations in the model.

The instantaneous utility to an employed is

$$h(I_i, 1 - l); i = E, N$$

where $I_i$ is income consisting of labor income after tax $w_i [1 - \tau]$ and its share of profit $\Pi$. Time endowment is normalized to unity and working hours are $l$ (exogenous). The function $h$ has the standard properties.

The instantaneous utility for unemployed is

$$g(I_j, 1 - s_j); j = U, K$$

where income is the sum of the transfer ($b_U$ or $b_K$) and the profit share $\Pi$, and $s_j$ is the amount of time spent searching for a job.

The value functions associated with the four possible labor market states are

$$\rho V_E = h(w_E [1 - \tau] + \Pi, 1 - l) + p_{U,E} [V_U - V_E]$$
$$\rho V_N = h(w_N [1 - \tau] + \Pi, 1 - l) + p_{U,E} [V_K - V_N] + p_{E,N} [V_E - V_N]$$
$$\rho V_U = g(b_U + \Pi, 1 - s_U) + \alpha s_U [V_E - V_U] + p_{K,U} [V_K - V_U]$$
$$\rho V_K = g(b_K + \Pi, 1 - s_K) + \alpha s_K [V_N - V_K]$$

where $\rho$ is the discount rate, $b_U > b_K$ (see below).

We want to focus on differences or asymmetries arising solely from the design of the social safety net, and hence we essentially assume that all workers are identical except for their labor market history and thus possibly their benefit entitlement. In this spirit we have assumed that

---

1 That is, we follow Fredriksson & Holmlund (2001) who show that a fixed time duration can be approximated by a system in which there is a stochastic transition from one benefit level to another.

2 For notational reasons, we allow the instantaneous utility function of the unemployed to differ from that of the employed. However, our results will not hinge on this asymmetry, and in the numerical illustrations $h(\cdot) = g(\cdot)$. 

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\[ p_{K,N} = p_{U,E}; \text{i.e., the job separation rate is the same for eligible and non-eligible workers, and they have the same working hours (exogenous).} \]

Note also that the participation constraints:

\[ V_E \geq V_U, V_N \geq V_K, V_E \geq V_N, V_U \geq V_K \]

are assumed fulfilled in the following.

In the following this short-hand notation will be used

\[ h_i (\cdot) \equiv h (w_i [1 - \tau] + \Pi, 1 - l) \text{ for } i = E, N \]
\[ g_j (\cdot) \equiv g (b_j + \Pi, 1 - s_j) \text{ for } j = U, K. \]

The individual takes all variables except the search level to be beyond its own control (i.e., to be unaffected by its decisions), and thus the optimal search level for the two types of unemployed is determined by (note that standard assumptions on \( g \) ensure that the second order condition is fulfilled; i.e., \( g \) is strictly concave with respect to the second argument)

\[ \frac{\partial g_U (\cdot)}{\partial (1 - s_U)} = \alpha [V_E - V_U] \]
\[ \frac{\partial g_K (\cdot)}{\partial (1 - s_K)} = \alpha [V_N - V_K]. \]

Denoting the share of the population receiving unemployment benefits and social assistance by \( u \) and \( k \), respectively, we have that total search is given as

\[ s \equiv s_U u + s_K k. \]

A standard constant returns to scale matching function defined over total search and vacancies (\( v \)) is assumed

\[ m (s, v). \]

The function \( m \) has the usual properties. It follows that the job-finding rate is given by

\[ \alpha = \frac{m (s, v)}{s} = m (1, \theta) \]

where \( \theta \equiv \frac{\zeta}{s} \) is market tightness, and hence \( \alpha = \alpha (\theta), \alpha' (\theta) > 0 \). The job filling rate is

\[ q = \frac{m (s, v)}{v} = m (\theta^{-1}, 1) \]

and thus \( q = q (\theta), q' (\theta) < 0 \).

Firms post vacancies to find vacant workers, and in the hiring process they are not able to distinguish workers by their eligibilities in the social safety net. Thus, the value of a vacancy (\( J_V \)) is expressed in terms of the expected value of a filled job (\( J_{EXP} \)); i.e.,

\[ \rho J_V = -\kappa + q [J_{EXP} - J_V] \]

\[ ^3 \text{We impose this symmetry to focus on the question whether re-entitlement conditions can be motivated as a means to improve the trade-off between incentives and insurance in the social safety net.} \]
where $\kappa$ is the flow vacancy cost. The free-entry-condition, $J_V = 0$, then implies

$$J_{\text{EXP}} = \frac{\kappa}{q}$$

(3.3)

where $J_{\text{EXP}}$ is the expected value of a filled job; that is

$$J_{\text{EXP}} \equiv \frac{\alpha_s u J_E + \alpha_s K k J_N}{\alpha_s u + \alpha_s K k} = \frac{s u J_E + s K k J_N}{s u + s K k}.$$  

After a firm and a worker are matched, the firm knows whether the candidate is entitled to UIB, and therefore the wage depends on the worker’s UIB eligibility. The value of a job filled with a UIB eligible worker is

$$\rho J_E = y - w_E + p_{U,E} [J_V - J_E],$$

and the value of a job filled with a worker non-eligible for unemployment benefits is

$$\rho J_N = y - w_N + p_{E,N} [J_E - J_N] + p_{U,E} [J_V - J_N].$$

Wages are determined through Nash bargaining; i.e.,

$$w_E = \arg \max_{w_E} (V_E - V_U)\beta (J_E - J_V)^{1-\beta}$$

and

$$w_N = \arg \max_{w_N} (V_N - V_K)\beta (J_N - J_V)^{1-\beta}$$

with the associated first-order conditions (second order conditions assumed to be fulfilled)

$$\beta \frac{\partial J_E}{\partial w_E} + (1 - \beta) \frac{\partial J_V}{\partial w_E} = 0$$

(3.4)

$$\beta \frac{\partial J_N}{\partial w_N} + (1 - \beta) \frac{\partial J_V}{\partial w_N} = 0.$$  

(3.5)

Profits are given as

$$\Pi = [y - w_E] e + [y - w_N] n - v \kappa$$

where $e$ and $n$ denote the number of eligible and non-eligible workers, respectively. The inflow and outflow equations read (where $e = 1 - u - k - n$)

$$U : \quad ep_{U,E} = \alpha s_u u + p_{K,U} u$$

(3.6)

$$K : \quad np_{U,E} + p_{K,U} u = \alpha s_K k$$

(3.7)

$$N : \quad \alpha s_K k = p_{E,N} n + p_{E,N} n$$

(3.8)

Note that workers gaining eligibility for UIB experience an immediate change in the wage since the worker and the firm are implicitly assumed to renegotiate the wage promptly. This assumption may be empirically questionable, but we make it for tractability reasons.
for unemployment, social assistance and non-eligible jobs, respectively.

For later reference note that the fraction of the population in the various labor market states can be written (recall that $1 = e + n + u + k$)

$$\begin{align*}
e &= e(\alpha, s_U, s_K, p_{E,N}, p_{K,U}, p_{U,E}) \\
n &= n(\alpha, s_U, s_K, p_{E,N}, p_{K,U}, p_{U,E}) \\
u &= u(\alpha, s_U, s_K, p_{E,N}, p_{K,U}, p_{U,E}) \\
k &= k(\alpha, s_U, s_K, p_{E,N}, p_{K,U}, p_{U,E}).
\end{align*}$$

Finally, the public budget constraint reads

$$\tau (w_E e + w_N n) = b_U u + b_K k. \tag{3.9}$$

To sum up, the social safety net is characterized by two benefit levels ($b_U, b_K$), the transition out of UIB unemployment ($p_{K,U}$) which is the inverse of benefit duration, and the entry into UIB eligibility ($p_{E,N}$) which captures reentitlement requirements. Below we consider how the various elements of the social safety net affect labor market performance both in the long run (steady-state) and over the business cycle.

In summary, the equilibrium to the model is characterized by unemployed choosing search effort according to (3.1) and (3.2), firms creating vacancies according to (3.3), wages determined by (3.4) and (3.5), the tax rate determined from (3.9) and the flow equations (3.6), (3.7), and (3.8). In Appendix 3.A we show that the resource balance condition (or goods-market equilibrium condition) is fulfilled; i.e., aggregate output (net of vacancy costs) equals aggregate consumption.

**Calibration**

In the next section we derive some partial, analytical results on the effects on job search and gross unemployment from changing benefit duration and the entitlement requirement. In general, however, the model cannot be solved analytically and therefore we use a numerical illustration when necessary.

As explained below we use the baseline calibration of Ortega & Rioux (2010) as well as their functional forms (where possible). The time period is a month. The job separation rate ($p_{U,E}$) is 0.00977. Productivity\(^5\) ($y$) is 1.517 in the benchmark (medium productivity), and the flow cost of vacancies ($\kappa$) is 0.56181. The discount rate ($\rho$) is 0.01. We assume the following functional forms for instantaneous utilities

$$h (I_i, 1 - l) = \frac{1}{\eta} l_i^\eta + \log (1 - l) \quad ; \quad i = E, N$$

---

\(^5\)Productivity and the vacancy cost are rescaled (by 1/1000) compared to Ortega & Rioux (2010) to ensure that the choice of search intensities delivers interior solutions. This simple rescaling has no qualitative effects.
with $\eta = -0.5$, and workers are assumed to spend 40% of their time at work; i.e., $l = 0.4$.

The matching function is Cobb-Douglas

$$m(s,v) = As^\epsilon v^{1-\epsilon}$$

with $A = 0.05$ and $\epsilon = 0.5$. To avoid introducing arbitrary inefficiencies we set $\beta = \epsilon = 0.5$.

### 3.3 Search and properties of the UIB system

We start out by clarifying the role of benefit duration ($p_{K,U}$) and benefit entitlement ($p_{E,N}$) for given benefit levels ($b_U, b_K$) for labor market performance; that is, we consider the implication for a given macroeconomic environment, i.e., wages ($w_E, w_N$), taxes ($\tau$), and job-finding rate ($\alpha$). This clarifies the incentive/search effects of the two instruments as part of the social safety net. The complexity of the overall model makes it impossible to arrive at any analytical results at the general level, but this partial approach gives some important insights on the role of the various elements in the unemployment insurance system, and also for the interpretation of the numerical results presented below.

In Appendix 3.B we show that for a given macroeconomic environment

$$\frac{ds_U}{dp_{K,U}} > 0 ; \quad \frac{ds_K}{dp_{K,U}} < 0$$

$$\frac{ds_U}{dp_{E,N}} < 0 ; \quad \frac{ds_K}{dp_{E,N}} > 0$$

i.e., shorter benefit duration (higher $p_{K,U}$) implies that the UIB-unemployed search more to enhance the chance of finding a job in light of the more dire consequences if this is not successful, and for the SA-unemployed search becomes less attractive since the value of finding a job leading to UIB entitlement is now lower. A more lax entitlement requirement (higher $p_{E,N}$) makes the non-entitled unemployed search more since the value of a job is now higher since it more easily leads to UIB-entitlement, oppositely the eligible unemployed search less since it is less critical to lose entitlement as it can more easily be regained.

Note that the two instruments, benefit duration and entitlement conditions, have different implications for the two types of unemployed. Considering the marginal rate of substitution between the two instruments leaving the utility gain from finding a job unchanged for entitled and non-entitled, respectively, we have (for proof and notation see Appendix 3.B)

$$\frac{dp_{K,U}}{dp_{E,N}} |_{[V_E-V_U]=constant} = \frac{p_{K,U} \alpha s_K}{\rho} B_1 \left[ \frac{1}{\rho} B_2 - [V_N - V_K] \left[ 1 + \frac{1}{\rho} \alpha s_K \right] \right] > 0$$

$$\frac{1}{\rho} A_2 - [V_E - V_U] \left( 1 + \frac{p_{U,E}}{\rho} \right)$$
\[
\frac{dp_{E,N}}{dp_{K,U}} \mid [V_N - V_K] = \text{constant} = \frac{p_{E,N} p_{U,E} \left[ \frac{1}{\rho} A_2 - [V_E - V_U] (1 + \frac{p_{U,E}}{\rho}) \right]}{\left[ \frac{1}{\rho} B_2 - [V_N - V_K] \left[ 1 + \frac{1}{\rho} \alpha s_K \right] \right]} > 0.
\]

Note that search levels are unchanged for utility gains being constant \((V_E - V_U = \text{constant},\) and \(V_N - V_K = \text{constant}\)). The above thus gives the combinations of \(p_{K,U}\) and \(p_{E,N}\), leaving search \((s_U\) and \(s_K\) respectively) unchanged. Both types’ iso-search curves are upward sloping in the \((p_{K,U}, p_{E,N})\)-spaces; that is, a higher rate at which non-entitled become entitled to UIB (easier UIB entitlement) has to be accompanied with a higher rate at which UIB-unemployed lose their UIB entitlement (shorter benefit duration) to leave search unchanged, i.e., the two instruments are substitutes. This applies for both types of unemployed, but the marginal rates of substitution are different, which suggests that it may be desirable to let the UI scheme feature both elements (see below). Intuitively, using two instruments seems to dominate using only one since there are two search levels which can be affected, and the two instruments are not perfect substitutes, cf. the difference in the marginal rates of substitution.

Finally, note that (for proof see Appendix 3.B) an increase in \(p_{K,U}\) leads to a decrease in \(V_E\) and a decrease in \(V_K\), while there is an ambiguous effect on \(V_N\) and \(V_U\). An increase in \(p_{E,N}\) leads to an increase in \(V_E\) and an increase in \(V_K\), while there is an ambiguous effect on \(V_N\) and \(V_U\).

**Labor market outcomes**

In the following we consider the role of benefit duration and employment eligibility conditions by considering the combinations of the two instruments delivering the same gross level of unemployment \((u + k)\).

From the flow equations we have

\[ [e + n] p_{U,E} = \alpha s_U u + \alpha s_K k = \alpha s \]

i.e., for a given job-separation rate \(p_{U,E}\) and job-finding rate \(\alpha\), total employment \((e + n)\) is monotonously increasing in aggregate search. Since \(1 = e + n + u + k\) it follows that \(u + k\) is monotonously decreasing in \(s\).

We show in Appendix 3.B that the gross unemployment rate \((u + k = 1 - (e + n))\) can be written in implicit form as

\[ u + k = F(s_U (p_{K,U}, p_{E,N}), s_K (p_{K,U}, p_{E,N}), p_{K,U}, p_{E,N}). \]

It can be shown that (see Appendix 3.B)

\[ \frac{\partial F (\cdot)}{\partial s_U} < 0 ; \quad \frac{\partial F (\cdot)}{\partial s_K} < 0 \]
\[ \text{sign} \left( \frac{\partial F(\cdot)}{\partial p_{K,U}} \right) = \text{sign}(s_U - s_K) \]

\[ \text{sign} \left( \frac{\partial F(\cdot)}{\partial p_{E,N}} \right) = -\text{sign} \left( \frac{\partial F(\cdot)}{\partial p_{K,U}} \right). \]

It is an implication that for given search there is always a trade-off between benefit duration and benefit entitlement; i.e., if benefit entitlement becomes more easy (higher \( p_{E,N} \)), then benefit duration should be reduced (higher \( p_{K,U} \)) to maintain an unchanged gross unemployment rate. In general search responses imply that the slope of the iso-employment locus cannot be unambiguously signed. Therefore, we turn to a numerical illustration.

Figure 3.1 shows iso-gross unemployment curves in the \((p_{E,N}, p_{K,U})\)-plane for the full model (see above for the numerical details). We see that the iso-gross unemployment curves are positively sloped, which reveals a trade-off between easier reentitlement and shorter UIB duration in sustaining a certain level of \( u+k \). Furthermore, gross unemployment increases when we move to the South-East, i.e., longer UIB duration and/or faster reentitlement, since SA-unemployed turn out to search more intensively than UIB-unemployed in our numerical specification \( s_K > s_U \).

Figure 3.1: Iso-gross unemployment curves, trade-off between UIB duration and reentitlement requirement
3.4 Optimal social safety net

In the following we consider the optimal design of the social safety net assuming a utilitarian social welfare function. In Appendix 3.C we show that the social welfare function can be written

$$\Omega = e\rho V_E + n\rho V_N + u\rho V_U + k\rho V_K$$

$$= eh_E (\cdot) + nh_N (\cdot) + ug_U (\cdot) + kg_K (\cdot). \quad (3.10)$$

Note that profits are distributed to workers such that the above captures the sum of utility generated in the economy under a given policy package \((b_U, b_K, p_{K,U}, p_{E,N})\).

As a prelude it is useful to consider some special cases. The standard case considered in the literature assumes that employment automatically gives entitlement to unemployment benefits in the case of lay-offs corresponding to \(p_{E,N} \to \infty\), in which case \(n \to 0\). A simple one-tier benefit scheme in this case arises if \(p_{K,U} \to 0\) (infinite benefit duration) implying \(k \to 0\). Note also that \(p_{E,N} \to 0\) implies that it is not possible to transit to a job providing entitlement to unemployment benefits in the case of lay-off, and hence\(^6\) \(u \to 0\) and \(e \to 0\); i.e., this case corresponds to a one-tier benefit scheme where the only two states are \(N\) and \(K\).

Hence, finding the optimal social safety net poses two key questions:

i) Is it optimal to have two tiers, i.e., to have a finite duration of unemployment benefits after which unemployed are offered a lower social assistance, corresponding to \(p_{K,U} > 0\)?

ii) Is it optimal to include an employment condition as part of the eligibility conditions in the benefit scheme, i.e., to have \(0 < p_{E,N} < \infty\)?

A sufficient\(^7\) condition for the optimal UI scheme to have two tiers is

$$\lim_{p_{K,U} \to 0} \frac{\partial \Omega}{\partial p_{K,U}} > 0$$

whereas a sufficient condition for the optimality of a reentitlement condition is

$$\lim_{p_{E,N} \to \infty} \frac{\partial \Omega}{\partial p_{E,N}} < 0$$

conditional on two tiers being optimal. In Appendix 3.D we investigate the problem of designing the optimal social safety net, and in particular these two conditions, in more analytical detail. However, the overall complexity of the model prevents us from analytically deriving clear answers to the two key questions, and therefore we resort to a numerical illustration below.

\(^6\)Note that the flow equations imply \(np_{E,N} = p_{K,U}u\).

\(^7\)Rigorously speaking, the sufficient condition should also include \(\lim_{p_{K,U} \to \infty} \frac{\partial \Omega}{\partial p_{K,U}} < 0\). However, in the limit there is only notational difference between a one-tier scheme with only SA-unemployed and a one-tier scheme with only UIB-unemployed.
This suggests that it is by no means trivial whether the optimal scheme includes an entitlement condition. Numerical solutions of the model underpin this ambiguity since a two-tier benefit scheme with non-automatic entitlement for UIB is by no means a universal solution to the problem of choosing the optimal structure of the social safety net. To proceed we adopt the calibration of Ortega & Rioux (2010), see details above, in which case the optimal social safety net exhibits two tiers and non-automatic UIB-entitlement, but it should be clear that this result is not, in general, robust to large changes in functional forms and parameter values.

3.4.1 Numerical illustrations

Optimal UI scheme

The numerical solution reveals that in this case the optimal UI scheme has two tiers and non-automatic entitlement for UIB, i.e., $b_U > b_K$, $0 < p_{K,U} < \infty$, and $0 < p_{E,N} < \infty$. In particular, welfare defined in (3.10) is maximized with $b_U = 0.587$, $b_K = 0.207$, $p_{K,U} = 0.057$, and $p_{E,N} = 0.023$. Gross unemployment $(u+k)$ is approximately 16%. The tax rate is $\tau = 0.065$. The wage of UIB eligible workers is $w_E = 1.080$ yielding a replacement rate ($RR_U \equiv \frac{b_U}{w_E(1-\tau)}$) of 58%, while it is only 23% for non-eligible workers ($RR_K \equiv \frac{b_K}{w_N(1-\tau)}$) since $w_N = 0.953$.

An interesting question is whether our model can explain the large differences in UI schemes across OECD countries documented by e.g. Venn (2012). To this end we find the optimal UI scheme for countries that differ along two dimensions, namely economic structure and preferences. Economic structure here refers to underlying differences in the economies which generate variation in employment rates, and below we implement this as differences in the aggregate productivity level $y$. Also, differences in preferences can explain why the optimal trade-off between incentives and insurance vary across countries. One important preference parameter is the degree of risk aversion, and below we thus find the optimal UI schemes for different $\eta$’s.

Productivity level

An important determinant of the optimal social safety net is productivity $y$, which is here meant to capture underlying structural differences across countries. Differences in $y$ across countries can be interpreted as reflecting income differences; i.e., rich countries have a high $y$, whereas relatively poor countries have a low $y$. To see how the optimal UI scheme depends on the productivity level, Table 3.1 illustrates the optimal policy package for various levels of productivity relative to the benchmark (medium productivity level).

Table 3.1 shows that both the UIB level and the SA level should be higher when productivity is high, whereas the UIB duration (inverse $p_{K,U}$) and the entitlement requirement (inverse $p_{E,N}$) should be lower. Hence, the optimal UI scheme is more generous in three out of the four dimensions when productivity is high. Similarly, the scheme is less generous in the same three
Table 3.1: Optimal UI policy package for different levels of productivity (medium productivity is index 100)

<table>
<thead>
<tr>
<th>y</th>
<th>Low</th>
<th>Medium</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_U$</td>
<td>87.9</td>
<td>100.0</td>
<td>112.3</td>
</tr>
<tr>
<td>$b_K$</td>
<td>88.2</td>
<td>100.0</td>
<td>111.8</td>
</tr>
<tr>
<td>$p_{K,U}$</td>
<td>95.5</td>
<td>100.0</td>
<td>104.2</td>
</tr>
<tr>
<td>$p_{E,N}$</td>
<td>99.3</td>
<td>100.0</td>
<td>100.6</td>
</tr>
<tr>
<td>$\tau$</td>
<td>104.2</td>
<td>100.0</td>
<td>96.3</td>
</tr>
<tr>
<td>$w_E$</td>
<td>88.8</td>
<td>100.0</td>
<td>111.3</td>
</tr>
<tr>
<td>$w_N$</td>
<td>88.7</td>
<td>100.0</td>
<td>111.4</td>
</tr>
<tr>
<td>$RR_U$</td>
<td>99.2</td>
<td>100.0</td>
<td>100.6</td>
</tr>
<tr>
<td>$RR_K$</td>
<td>99.7</td>
<td>100.0</td>
<td>100.1</td>
</tr>
<tr>
<td>$u + k$</td>
<td>104.7</td>
<td>100.0</td>
<td>96.0</td>
</tr>
</tbody>
</table>

Note: $y^{Low} = 0.9 \cdot y^{Medium}$ and $y^{High} = 1.1 \cdot y^{Medium}$.

dimensions when productivity is low. While the UIB and SA levels change a lot in response to a change in the productivity level, the replacement rates are almost unaffected since wages are near-proportional to productivity and taxes are lower when productivity (and thus the employment rate) is high. However, the replacement rates do (slightly) increase in response to an increase in productivity.

Clearly, changing the benefit levels dominates changes in the two remaining instruments. Thus, to focus on the pure trade-off between UIB duration and reentitlement requirement, in Table 3.2 we present the optimal level of these two UI instruments when the UIB level and the SA level are fixed (at their optimal values in the benchmark).

Table 3.2: Optimal UIB duration and entitlement requirement for different levels of productivity when UIB and SA levels are fixed (medium productivity is index 100)

<table>
<thead>
<tr>
<th>y</th>
<th>Low</th>
<th>Medium</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_{K,U}$</td>
<td>125.0</td>
<td>100.0</td>
<td>82.7</td>
</tr>
<tr>
<td>$p_{E,N}$</td>
<td>98.5</td>
<td>100.0</td>
<td>101.7</td>
</tr>
</tbody>
</table>

Note: $y^{Low} = 0.9 \cdot y^{Medium}$ and $y^{High} = 1.1 \cdot y^{Medium}$.

From Table 3.2 we see that also in this case the optimal UI scheme is more generous when productivity is high since the optimal UIB duration is now higher than in the benchmark and the entitlement requirement is still lower. Hence, structural differences, here captured by differences in productivity, may go some way in explaining the negative correlation between the length of UIB duration and the reentitlement requirement documented by Venn (2012).

Furthermore, the responses in both these instruments to changes in the productivity level
are much larger than when UIB and SA levels are allowed to respond as well.

To study how the optimal UI scheme trades off the two instruments, UIB duration and entitlement requirement, in Table 3.3 we present the optimal UIB duration (left panel) when the other three UI instruments are fixed (at the optimal level from the benchmark), and similarly for the optimal entitlement requirement (right panel).

Table 3.3: Optimal UIB duration (left panel) and optimal entitlement requirement (right panel) for different levels of productivity when the three remaining UI instruments are fixed (medium productivity is index 100)

<table>
<thead>
<tr>
<th>y</th>
<th>Low</th>
<th>Medium</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_{K,U}$</td>
<td>125.5</td>
<td>100.0</td>
<td>82.4</td>
</tr>
<tr>
<td>$p_{E,N}$</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>y</th>
<th>Low</th>
<th>Medium</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_{K,U}$</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
</tr>
<tr>
<td>$p_{E,N}$</td>
<td>78.4</td>
<td>100.0</td>
<td>120.2</td>
</tr>
</tbody>
</table>

Note: $y^{\text{Low}} = 0.9 \cdot y^{\text{Medium}}$ and $y^{\text{High}} = 1.1 \cdot y^{\text{Medium}}$.

Comparing Tables 3.2 and 3.3 we see that indeed the optimal UI scheme trades off the two instruments since both react stronger to changes in the productivity level when the other instrument is kept fixed. Especially the entitlement requirement reacts much stronger to productivity changes when this is the only instrument used, which shows that the responses of UIB and SA levels (in Table 3.1) and UIB duration (in Tables 3.1 and 3.2) dampen the response of the entitlement requirement.

Tables 3.1-3.3 all show that the optimal UI scheme is more generous when productivity is high, and less generous when productivity is low. When productivity is high the employment level is also high, and therefore a more generous social safety net is more easily affordable. Hence, this result stems from the balanced budget requirement, a well-known result from the literature, cf. Andersen & Svarer (2011).

One may therefore view the analysis above as adding to the literature on business cycle dependent labor market policies. However, the analysis so far is not well-suited for discussing how the optimal UI scheme responds to temporary changes in productivity. In Section 3.5 we therefore study a business cycle version of our model, in which the public budget is only required to balance on average across the aggregate states. As expected this has important consequences for the optimal response of the UI scheme to changes in productivity.

The role of risk aversion

A potential explanation for the substantial variation in UI schemes across countries is differences in preferences. One important preference parameter is the degree of risk aversion. Therefore, in this section we study the importance of risk aversion for the optimal UI scheme.

Table 3.4 shows the optimal UI policy package for different degrees of risk aversion, $\eta$, relative to the optimal package in the benchmark ($\eta = -0.5$).
We see that the generosity of the optimal UI scheme is increasing in the level of relative risk aversion \((\eta - 1)\) in most dimensions, which is intuitive since the insurance value of UI is higher if workers are more risk averse. For example, the UIB level should be 12% higher when \(\eta = -3\) compared to the benchmark, which results in the replacement rate being 22% higher (an increase of 13 percentage points), whereas it should be lowered for \(\eta = -0.25\).

For the SA level the effects are qualitatively the same, but their magnitudes are even larger since this benefit level should be 87% higher for \(\eta = -3\), which leads to almost a doubling of the replacement rate (an increase of 22 percentage points). These large increases in benefit levels following an increase in relative risk aversion are partially offset by a decrease in the UIB duration \((1/p_{K,U})\), which indicates that higher benefit levels dominate longer UIB duration, and therefore a decrease in the latter can partially finance (even larger) increases in the former ones.

For the reentitlement requirement the results are more complex since the employment condition is loosened for both higher and lower levels of \(\eta\). For \(\eta\) equal to \(-0.75\) or \(-3\) this makes sense as a generous UI scheme is more appreciated, whereas the result for \(\eta = -0.25\) has a similar interpretation as the shortened UIB duration for high risk aversion; i.e., lowering the benefit levels dominates the employment condition when decreasing the generosity of the UI scheme, and therefore cuts in benefit levels make easy accessibility to UI cheaper.

To isolate the trade-off between UIB duration and reentitlement requirement, in Table 3.5 we show the optimal policy packages when benefit levels are fixed (at the optimal levels from the benchmark).

Now, the optimal UIB duration is longer when consumers are more risk averse, whereas the reentitlement requirement is still lower; i.e., the UI scheme is more generous in both dimensions.

---

Table 3.4: Optimal UI policy package for different levels of risk aversion (the benchmark solution \((\eta = -0.50)\) is index 100)

<table>
<thead>
<tr>
<th>(\eta)</th>
<th>-3.00</th>
<th>-0.75</th>
<th>-0.50</th>
<th>-0.25</th>
</tr>
</thead>
<tbody>
<tr>
<td>(b_U)</td>
<td>112.4</td>
<td>102.3</td>
<td>100.0</td>
<td>97.1</td>
</tr>
<tr>
<td>(b_K)</td>
<td>187.1</td>
<td>114.5</td>
<td>100.0</td>
<td>82.9</td>
</tr>
<tr>
<td>(p_{K,U})</td>
<td>131.2</td>
<td>103.1</td>
<td>100.0</td>
<td>97.2</td>
</tr>
<tr>
<td>(p_{E,N})</td>
<td>121.2</td>
<td>100.2</td>
<td>100.0</td>
<td>100.8</td>
</tr>
<tr>
<td>(\tau)</td>
<td>132.1</td>
<td>105.6</td>
<td>100.0</td>
<td>93.1</td>
</tr>
<tr>
<td>(w_E)</td>
<td>94.2</td>
<td>99.3</td>
<td>100.0</td>
<td>100.7</td>
</tr>
<tr>
<td>(w_N)</td>
<td>97.9</td>
<td>99.8</td>
<td>100.0</td>
<td>100.1</td>
</tr>
<tr>
<td>(RR_U)</td>
<td>122.0</td>
<td>103.4</td>
<td>100.0</td>
<td>96.0</td>
</tr>
<tr>
<td>(RR_K)</td>
<td>195.6</td>
<td>115.1</td>
<td>100.0</td>
<td>82.4</td>
</tr>
<tr>
<td>(u + k)</td>
<td>94.2</td>
<td>99.3</td>
<td>100.0</td>
<td>100.7</td>
</tr>
</tbody>
</table>

---

This indicates that there exists a critical value for risk aversion above which the optimal UI scheme only has one tier.
Table 3.5: Optimal UIB duration and entitlement requirement for different levels of risk aversion when UIB and SA levels are fixed (the benchmark solution ($\eta = -0.50$) is index 100)

<table>
<thead>
<tr>
<th>$\eta$</th>
<th>-3.00</th>
<th>-0.75</th>
<th>-0.50</th>
<th>-0.25</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_{K,U}$</td>
<td>33.7</td>
<td>86.1</td>
<td>100.0</td>
<td>120.1</td>
</tr>
<tr>
<td>$p_{E,N}$</td>
<td>200.9</td>
<td>100.8</td>
<td>100.0</td>
<td>102.6</td>
</tr>
</tbody>
</table>

when risk aversion is high. Note that the UIB duration reacts differently than when benefit levels are also chosen optimally, and thus clearly there exists a trade-off between higher benefit levels and longer UIB duration for high risk aversion and low levels and short duration for low risk aversion, which seems always to be dominated by the benefit level.

Hence, preference differences, here captured by differences in risk aversion, yield an alternative explanation of the negative correlation between the length of UIB duration and the reentitlement requirement documented by Venn (2012).

Furthermore, the results for low risk aversion are still complex since UIB duration is shorter (less generous), but also the reentitlement requirement is shorter (more generous). Again, the dominating instrument (here UIB duration) can be changed more when the other instrument (here reentitlement requirement) partially offsets the harmful effects.

To illustrate once again the trade-off between UIB duration and reentitlement requirement, Table 3.6 shows the optimal UIB duration (left panel) when the other three UI instruments are fixed (at the optimal level from the benchmark), and similarly for the optimal entitlement requirement (right panel).

Table 3.6: Optimal UIB duration (left panel) and optimal entitlement requirement (right panel) for different levels of risk aversion when the three remaining UI instruments are fixed (the benchmark solution ($\eta = -0.50$) is index 100)

<table>
<thead>
<tr>
<th>$\eta$</th>
<th>-3.00</th>
<th>-0.75</th>
<th>-0.50</th>
<th>-0.25</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_{K,U}$</td>
<td>30.7</td>
<td>85.9</td>
<td>100.0</td>
<td>119.3</td>
</tr>
<tr>
<td>$p_{E,N}$</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
</tr>
<tr>
<td>$\eta$</td>
<td>-3.00</td>
<td>-0.75</td>
<td>-0.50</td>
<td>-0.25</td>
</tr>
<tr>
<td>$p_{K,U}$</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
</tr>
<tr>
<td>$p_{E,N}$</td>
<td>689.0</td>
<td>115.1</td>
<td>100.0</td>
<td>85.3</td>
</tr>
</tbody>
</table>

We see that the qualitative changes in UIB duration and reentitlement requirement are the same as when both these instruments are varied according to the degree of risk aversion, at least for higher levels of risk aversion. However, the magnitudes are larger for high risk aversion when only one of the instruments is used, underlining that the two instruments work as substitutes. In particular, $p_{E,N}$ is almost seven times larger for $\eta = -3$ than in the benchmark. For $\eta = -0.25$, however, the UIB duration is shortened by less than before since the reentitlement requirement can no longer be used to mitigate some of the negative effects from this, and for the same reason the reentitlement requirement is now higher.
3.5 Business cycles and the social safety net

In this section we consider a business cycle version of our model. The economy is assumed to shift between a good ($G$) and a bad ($B$) state, where a good state is characterized by a higher productivity level, i.e., $y^G > y^B$. The transition rate from the good to the bad state is denoted by $\pi^G$, whereas $\pi^B$ denotes the transition rate from the bad to the good state. Hence, the expected duration of a good (bad) state is $1/\pi^G$ ($1/\pi^B$). For simplicity we follow Ek (2012) and focus solely on steady states; i.e., the economy is assumed to jump directly between the two states. The tax rate is set to ensure that the public budget balances on average across states. The model can be found in Appendix 3.F.

We set $y^G$ to 10% above the benchmark productivity level, and $y^B$ to 10% below. Since we assume that the economy jumps directly between steady states, our model is not well-suited for analyzing the exact transitions between aggregate states. Therefore, we set the $\pi$’s very low ($\pi^G = \pi^B = \frac{1}{1000}$) to avoid having unrealistic transition dynamics driving our results.

**Optimal uniform UI scheme**

At first we find the optimal uniform UI scheme; i.e., the policy that maximizes expected welfare across the cycle conditional on $b^G_U = b^B_U$, $b^G_K = b^B_K$, $p^G_{K,U} = p^B_{K,U}$ and $p^G_{E,N} = p^B_{E,N}$. Expected welfare is defined as

$$\Omega = \frac{\pi^B}{\pi^G + \pi^B} \Omega^G + \frac{\pi^G}{\pi^G + \pi^B} \Omega^B$$

where $\Omega^i = e^i \rho V^i_E + n^i \rho V^i_N + u^i \rho V^i_K + k^i \rho V^i_K$ for $i = G, B$.

The optimal uniform UI scheme is $b^G_U = 0.583$, $b^B_U = 0.204$, $p^G_{K,U} = 0.056$ and $p^B_{E,N} = 0.023$ with $i = G, B$. This policy implies $RR^G_U = 0.53$, $RR^B_U = 0.64$, $RR^G_K = 0.21$, $RR^B_K = 0.26$; i.e., the replacement rates are countercyclical due to procyclical wages. Gross unemployment, $u + k$, is 14.3% in good times and 17.4% in bad times. This gives an unconditional mean gross unemployment, defined as

$$\frac{\pi^B}{\pi^G + \pi^B} (u^G + k^G) + \frac{\pi^G}{\pi^G + \pi^B} (u^B + k^B),$$

of 15.8%, which can be interpreted as the structural gross unemployment rate.

**Optimal UI scheme with business cycle variability**

Table 3.7 shows the optimal UI scheme (relative to the optimal uniform UI scheme) when the policy is not restricted to be uniform; i.e., the instruments are allowed to vary across the business cycle.

We see from Table 3.7 that the optimal UI policy implies a countercyclical UIB level ($b^B_U > b^G_U$), a slightly procyclical SA level ($b^G_K > b^B_K$), a countercyclical UIB duration ($p^B_{K,U} < p^G_{K,U}$), and a countercyclical entitlement requirement ($p^B_{E,N} < p^G_{E,N}$); i.e., the optimal UI scheme is more generous in bad times in two of the four policy dimensions, namely UIB level and duration, but less generous in the remaining two dimensions, SA level and entitlement requirement.
Table 3.7: Optimal business cycle dependent UI scheme (the optimal uniform policy is index 100)

<table>
<thead>
<tr>
<th>$b_U^G$</th>
<th>$b_U^B$</th>
<th>$b_K^G$</th>
<th>$b_K^B$</th>
<th>$p_{K,U}^G$</th>
<th>$p_{K,U}^B$</th>
<th>$p_{E,N}^G$</th>
<th>$p_{E,N}^B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>96.8</td>
<td>103.1</td>
<td>101.4</td>
<td>99.4</td>
<td>105.9</td>
<td>97.0</td>
<td>107.0</td>
<td>95.6</td>
</tr>
</tbody>
</table>

Note, however, that both replacement rates are still countercyclical, $RR_U^G = 0.51$, $RR_U^B = 0.66$, $RR_K^G = 0.21$, $RR_K^B = 0.25$, since the procyclicality of wages for non-entitled workers more than offsets the procyclicality of the SA level.

Furthermore, this optimal UI scheme implies that the structural gross unemployment rate is (slightly) lowered relative to the optimal uniform policy. However, this decrease in the unconditional mean of $u + k$ is very small, namely 0.002%.

The public budget has a deficit of 0.0125 during recessions and a surplus of 0.0125 during booms; i.e., the semi-elasticity of the budget surplus with respect to productivity is approximately 0.00125, which measures the change in the budget surplus for a 1% increase in productivity. This is an increase of almost 16% compared to the optimal uniform UI scheme where the semi-elasticity is 0.00108.

Again, we find the optimal UIB duration and entitlement requirement when the UIB and SA levels are fixed (at the optimal uniform levels). The results are in Table 3.8.

Table 3.8: Optimal business cycle dependent UIB duration and entitlement requirement when UIB and SA levels are fixed (the optimal uniform policy is index 100)

<table>
<thead>
<tr>
<th>$p_{K,U}^G$</th>
<th>$p_{K,U}^B$</th>
<th>$p_{E,N}^G$</th>
<th>$p_{E,N}^B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>107.8</td>
<td>95.1</td>
<td>105.6</td>
<td>96.5</td>
</tr>
</tbody>
</table>

Thus, also when UIB and SA levels are business cycle invariant, the optimal UIB duration and entitlement requirement are both countercyclical. Furthermore, the response of the UIB duration is stronger than when benefit levels are also adjusted, whereas the response of the entitlement requirement is weaker.

To investigate the trade-off between UIB duration and reentitlement requirement in our business cycle model, in Tables 3.9 and 3.10 we find the optimal business cycle dependent UIB duration and entitlement requirement, respectively, when the three remaining instruments are invariant.

Table 3.9: Optimal business cycle dependent UIB duration when the other three instruments are invariant (the optimal uniform policy is index 100)

<table>
<thead>
<tr>
<th>$p_{K,U}^G$</th>
<th>$p_{K,U}^B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>106.4</td>
<td>95.9</td>
</tr>
</tbody>
</table>
Table 3.10: Optimal business cycle dependent entitlement requirement when the other three instruments are invariant (the optimal uniform policy is index 100)

<table>
<thead>
<tr>
<th>( p_{E,N}^B )</th>
<th>( p_{E,N}^P )</th>
</tr>
</thead>
<tbody>
<tr>
<td>97.7</td>
<td>101.7</td>
</tr>
</tbody>
</table>

From Table 3.9 we see that the optimal UIB duration is also countercyclical when only this instrument is state contingent, whereas Table 3.10 shows that the optimal entitlement requirement is procyclical when benefit levels and UIB duration are fixed. Thus, as in the steady state version of our model the UIB duration dominates the reentitlement requirement, and therefore in Table 3.8 the latter is used to mitigate some of the negative labor market consequences of a (even more) countercyclical UIB duration.

The UI scheme in Table 3.9 where only the duration of benefits depends on the business cycle situation closely resembles the US system, in which the benefit duration can be extended when the (local) unemployment rate is high. The Canadian system, on the other hand, is more sophisticated in this respect since it operates with business cycle contingencies in three dimensions (level, duration, and eligibility), and therefore Table 3.7 is the relevant comparison.

To sum up, Tables 3.7-3.10 all show that in the business cycle version of our model, where the public budget balances across states, the optimal UI scheme has countercyclical generosity, independently of which instruments are used.

**Distortions across the business cycle**

To gain more insights on the results from the previous section, Table 3.16 in Appendix 3.G shows the effects on (gross) unemployment and search efforts of a 1% increase in the eight policy instruments, one at a time. Hence, we investigate the distortionary effects of the four elements of the UI scheme, and how these distortions vary across the business cycle.

We see that aggregate search increases when the generosity of the UI system is increased, i.e., higher benefit levels, longer UIB duration, or faster reentitlement for UIB, mainly because gross unemployment increases.

The average search intensities across the business cycle are distorted more for both UIB-unemployed and SA-unemployed when increasing UI generosity in bad times compared to good times, no matter which of the four elements of the UI scheme we consider. Furthermore, the distortionary effects from making the UI scheme more generous are countercyclical in the sense that higher benefits levels, longer duration, or faster reentitlement in bad times distort UIB-search intensity in bad times more than increased UI generosity in good times distorts UIB-search intensity in good times. Hence, in this respect our results are consistent with Ek (2012) but in contrast to Andersen & Svarer (2010) who find that the distortionary effect of UIB on search intensity is procyclical. The reason is that in our model as well as in Ek (2012)
unemployed search harder in good times than in bad times, whereas the opposite is the case in Andersen & Svarer (2010).

The countercyclical nature of UI distortions on search intensities causes the distortions on average UIB-unemployment, average SA-unemployment and average gross unemployment to be countercyclical as well. With this in mind, it may be surprising that the optimal business cycle dependent UI scheme causes the structural gross unemployment rate to drop compared to the optimal uniform UI scheme. However, as we saw in Tables 3.7-3.10, the optimal UI policy is asymmetric; i.e., the generosity of the UI scheme is decreased more in good times than it is increased in bad times, and therefore the average level of \( u + k \) can still fall.

**The role of risk aversion**

As in the steady state version of our model, we are interested in the importance of preferences, in particular risk aversion, for the optimal UI scheme. Therefore, Table 3.11 shows the optimal uniform UI scheme, with business cycle invariant instruments, for different levels of risk aversion, \( \eta \), relative to the optimal uniform UI scheme in the benchmark (\( \eta = -0.5 \)).

Table 3.11: Optimal uniform UI scheme for different levels of risk aversion (the similar policy in the benchmark is index 100)

<table>
<thead>
<tr>
<th>( \eta )</th>
<th>-3.00</th>
<th>-0.75</th>
<th>-0.50</th>
<th>-0.25</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b_{U} )</td>
<td>111.7</td>
<td>102.2</td>
<td>100.0</td>
<td>97.2</td>
</tr>
<tr>
<td>( b_{K} )</td>
<td>189.8</td>
<td>115.0</td>
<td>100.0</td>
<td>82.3</td>
</tr>
<tr>
<td>( p_{K,U} )</td>
<td>135.7</td>
<td>103.2</td>
<td>100.0</td>
<td>97.2</td>
</tr>
<tr>
<td>( p_{E,N} )</td>
<td>120.1</td>
<td>100.1</td>
<td>100.0</td>
<td>100.9</td>
</tr>
</tbody>
</table>

with \( i = G, B \).

Our findings are consistent with, and actually quite similar to, those from the steady state model; that is, optimal benefit levels are increasing in the degree of risk aversion, the UIB duration is decreasing in \( \eta \), and the reentitlement requirement is looser for both higher and lower degrees of risk aversion.

Turning to the case where the UI instruments are allowed to vary over the business cycle, an interesting pattern emerges. In Tables 3.17-3.20 in Appendix 3.G we present the optimal UI schemes over the business cycle for different degrees of risk aversion, relative to the optimal uniform policy with the same \( \eta \). We see that in most cases the qualitative response of the UI instruments to business cycle fluctuations is independent of the degree of risk aversion. Hence, UIB benefits should be higher in bad times and lower in good times, UIB duration is longer across bad times, and reentitlement conditions are tighter in bad times when all UI instruments are allowed to vary with the business cycle. However, the SA level is countercyclical for high levels of risk aversion (\(-3\) or \(-0.75\)), but procyclical for low levels (\(-0.5\) or \(-0.25\)) since for
high degrees of risk aversion the insurance effect of SA dominates the negative labor market effects (distortions). Also when benefit levels are kept fixed, the qualitative change in UIB duration and reentitlement requirement over the business cycle is independent of $\eta$.

However, the magnitudes of these responses are highly dependent on the degree of risk aversion, and in all cases the dependence of the UI instruments on the business cycle situation should be increasing in the degree of risk aversion; e.g., with $\eta = -3$ the UIB level is 5.8% higher in bad times than the optimal uniform policy, but only 2.6% higher with $\eta = -0.25$.

This way of presenting the results is informative about the response of the eight UI policy instruments to business cycle fluctuations, i.e., sign and magnitude of response, but makes it hard to compare the UI policies across different levels of risk aversion. Since the main purpose of this section is to compare UI schemes for different $\eta$’s, below we show the same optimal UI schemes but now presented relative to the similar policy in the benchmark ($\eta = -0.5$).

Table 3.12 shows the optimal business cycle dependent UI scheme when all instruments are allowed to vary.

<table>
<thead>
<tr>
<th>$\eta$</th>
<th>$-3.00$</th>
<th>$-0.75$</th>
<th>$-0.50$</th>
<th>$-0.25$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b^G_U$</td>
<td>106.4</td>
<td>101.6</td>
<td>100.0</td>
<td>97.9</td>
</tr>
<tr>
<td>$b^B_U$</td>
<td>114.6</td>
<td>102.7</td>
<td>100.0</td>
<td>96.7</td>
</tr>
<tr>
<td>$b^G_K$</td>
<td>170.6</td>
<td>112.7</td>
<td>100.0</td>
<td>84.7</td>
</tr>
<tr>
<td>$b^B_K$</td>
<td>200.8</td>
<td>116.3</td>
<td>100.0</td>
<td>81.1</td>
</tr>
<tr>
<td>$p^G_{K,U}$</td>
<td>144.6</td>
<td>104.0</td>
<td>100.0</td>
<td>96.4</td>
</tr>
<tr>
<td>$p^B_{K,U}$</td>
<td>120.1</td>
<td>102.2</td>
<td>100.0</td>
<td>98.0</td>
</tr>
<tr>
<td>$p^G_{E,N}$</td>
<td>130.1</td>
<td>100.6</td>
<td>100.0</td>
<td>100.5</td>
</tr>
<tr>
<td>$p^B_{E,N}$</td>
<td>114.8</td>
<td>100.0</td>
<td>100.0</td>
<td>100.9</td>
</tr>
</tbody>
</table>

We see that similar to the optimal uniform UI scheme, optimal benefit levels are increasing in the degree of risk aversion, the UIB duration is decreasing in $\eta$, and the reentitlement requirement is looser for both higher and lower degrees of risk aversion. For the benefit levels the percentage changes are larger in bad times than in good times, and as discussed above for the SA level this asymmetry is sufficiently strong to overturn the procyclicality from the benchmark. Hence, the countercyclicality of UI generosity is strengthened for higher risk aversion. On the other hand, the percentage changes in UIB duration and employment condition are smaller in bad times than in good times.

Finally, to isolate the trade-off between UIB duration and reentitlement requirement, Tables 3.13-3.15 show the optimal UI scheme over the business cycle when only these two instruments are state contingent, when only the UIB duration is, and when only the reentitlement requirement is, respectively.
Table 3.13: Optimal business cycle dependent UIB duration and entitlement requirement when UIB and SA levels are fixed for different levels of risk aversion (the similar policy in the benchmark is index 100)

<table>
<thead>
<tr>
<th>$\eta$</th>
<th>-3.00</th>
<th>-0.75</th>
<th>-0.50</th>
<th>-0.25</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_{K,U}^G$</td>
<td>186.4</td>
<td>107.0</td>
<td>100.0</td>
<td>93.8</td>
</tr>
<tr>
<td>$p_{K,U}^B$</td>
<td>102.2</td>
<td>100.7</td>
<td>100.0</td>
<td>99.4</td>
</tr>
<tr>
<td>$p_{G}^E,N$</td>
<td>140.2</td>
<td>100.9</td>
<td>100.0</td>
<td>100.2</td>
</tr>
<tr>
<td>$p_{B,E,N}^E$</td>
<td>117.0</td>
<td>100.0</td>
<td>100.0</td>
<td>101.1</td>
</tr>
</tbody>
</table>

Table 3.14: Optimal business cycle dependent UIB duration when the other three instruments are invariant for different levels of risk aversion (the similar policy in the benchmark is index 100)

<table>
<thead>
<tr>
<th>$\eta$</th>
<th>-3.00</th>
<th>-0.75</th>
<th>-0.50</th>
<th>-0.25</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_{K,U}^G$</td>
<td>180.9</td>
<td>106.8</td>
<td>100.0</td>
<td>94.0</td>
</tr>
<tr>
<td>$p_{K,U}^B$</td>
<td>102.7</td>
<td>100.7</td>
<td>100.0</td>
<td>99.4</td>
</tr>
</tbody>
</table>

Table 3.15: Optimal business cycle dependent entitlement requirement when the other three instruments are invariant for different levels of risk aversion (the similar policy in the benchmark is index 100)

<table>
<thead>
<tr>
<th>$\eta$</th>
<th>-3.00</th>
<th>-0.75</th>
<th>-0.50</th>
<th>-0.25</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_{E,N}^G$</td>
<td>88.0</td>
<td>97.0</td>
<td>100.0</td>
<td>103.9</td>
</tr>
<tr>
<td>$p_{E,N}^B$</td>
<td>155.2</td>
<td>102.4</td>
<td>100.0</td>
<td>98.7</td>
</tr>
</tbody>
</table>

All three tables confirm the results from above, i.e., the reentitlement requirement is the dominated instrument, and therefore it is countercyclical when both instruments are state contingent in order to mitigate some of the negative labor market effects of countercyclical UIB duration, but procyclical when this is the only state contingent UI instrument.

Interestingly, the strengthened countercyclicality of the UIB duration for higher levels of risk aversion is achieved by lowering the duration in both aggregate states, but with a much larger drop in good times than in bad times. This, however, does not imply that the UI scheme is less generous for higher degrees of risk aversion since the benefit levels (although invariant across the cycle) are much higher, cf. Table 3.11.

### 3.6 Concluding remarks

In this paper we have considered the role of unemployment benefit reentitlement conditions when designing the optimal unemployment insurance (UI) scheme in a search-matching framework. We have shown that a reentitlement requirement can work as a substitute to the benefit
duration. Furthermore, the underlying economic structure and preferences, captured by differences in productivity and risk aversion, respectively, may in part explain the variation in UI schemes across OECD countries. In a business cycle version of our model, the optimal UI scheme is business cycle dependent and provides better insurance during recessions without leading to a rise in structural unemployment; i.e., the trade-off between incentives and insurance can be improved compared to a business cycle invariant UI scheme.

An important issue which we do not address in this paper is the matter of implementing a business cycle dependent UI scheme in practice. As mentioned above, such schemes are already implemented in e.g. the US and Canada, but there is almost no literature on the practical implementation issues, for example how to choose the trigger for extending/reducing the reentitlement requirement and the benefit duration. The optimal trigger may very well vary for the different UI instruments.

Furthermore, we have restricted attention to a particular shock, namely a productivity shock, but in practice it may be close to impossible to condition the UI scheme on a particular type of shock, and therefore more research is needed to analyze how the optimal UI scheme responds to different kinds of shocks.
3.7 Bibliography


Appendices

3.A Resource balance condition

In this appendix we show that the resource balance condition (or goods-market equilibrium condition) is fulfilled.

First, aggregate output net of vacancy costs is found by aggregating over all firms, that is

\[ ey + ny - v\kappa. \]

Second, aggregate consumption is

\[ e[w_E(1 - \tau) + \Pi] + n[w_N(1 - \tau) + \Pi] + u[b_U + \Pi] + k[b_K + \Pi] \]

and using the public budget requirement (3.9) we can rewrite aggregate consumption to

\[ ew_E + nw_N + \Pi. \]

Inserting for \( \Pi \equiv e[y - w_E] + n[y - w_N] - v\kappa \) yields

\[ ey + ny - v\kappa \]

and hence, we have shown that aggregate output net of vacancy costs equals aggregate consumption, which is the resource balance condition of this economy.

3.B Search and the properties of the unemployment insurance scheme

The first order conditions determining search can conveniently be written

\[
\Lambda^U_s(s_U, b_U, \tau, \alpha, V_E - V_U) \equiv -\frac{\partial g(b_U + \Pi, 1-s_U)}{s_U} + \alpha [V_E - V_U] = 0
\]

\[
\Lambda^K_s(s_K, b_K, \tau, \alpha, V_N - V_K) \equiv -\frac{\partial g(b_K + \Pi, 1-s_K)}{s_K} + \alpha [V_N - V_K] = 0
\]
and the associated second order conditions read (sub-indices indicate derivatives wrt. to the variable stated)

\[ \Lambda^U_{ss} (\cdot) < 0 \]
\[ \Lambda^K_{ss} (\cdot) < 0. \]

In the following the benefits levels \((b_U, b_K)\) are given, as are all ”macro variables” \((\tau, \alpha, w_U, w_N)\), and we are interested in the role of \(p_{K,U}\) and \(p_{E,N}\). Totally differentiating we find

\[ \Lambda^U_{ss} ds_U + \Lambda^U_{sz} dz = 0 \text{ for } z = p_{K,U}, p_{E,N}. \]

It follows that

\[ \frac{ds_U}{dz} = -\frac{\Lambda^U_{sz}}{\Lambda^U_{ss}} \]

and hence

\[ \text{Sign} \left[ \frac{ds_U}{dz} \right] = \text{Sign} \left[ \Lambda^U_{sz} \right] \]

where

\[ \Lambda^U_{sz} = \alpha \frac{d[V_E - V_U]}{dz}, \text{ with } z = p_{K,U}, p_{E,N}. \]

Similar expressions apply for \(s_K\).

Hence, to clarify how \((p_{K,U}, p_{E,N})\) affect search for U- and K-types we need to know how \(V_E - V_U\) and \(V_N - V_K\) are affected. Defining the short-hands

\[ h_E (\cdot) \equiv h(w_E [1 - \tau] + \Pi, 1 - l) \]
\[ h_N (\cdot) \equiv h(w_N [1 - \tau] + \Pi, 1 - l) \]
\[ g_U (\cdot) \equiv g(b_U + \Pi, 1 - s_U) \]
\[ g_K (\cdot) \equiv g(b_K + \Pi, 1 - s_K) \]

and using the value functions, we have

\[ \rho [V_E - V_U] = h_E (\cdot) - g_U (\cdot) + p_{U,E} [V_U - V_E] - \alpha s_U [V_E - V_U] - p_{K,U} [V_K - V_U] \]
\[ \rho [V_N - V_K] = h_N (\cdot) - g_K (\cdot) + p_{U,E} [V_K - V_N] - \alpha s_K [V_N - V_K] + p_{E,N} [V_E - V_N] \]

implying

\[ \rho [V_E - V_U] = h_E (\cdot) - g_U (\cdot) + p_{U,E} [V_U - V_E] - \alpha s_U [V_E - V_U] - p_{K,U} [V_K - V_E + V_E - V_U] \]
\[ \rho [V_N - V_K] = h_N (\cdot) - g_K (\cdot) + p_{U,E} [V_K - V_N] - \alpha s_K [V_N - V_K] + p_{E,N} [V_E - V_K + V_K - V_N] \]

and thus

\[ [V_E - V_U] = \frac{h_E (\cdot) - g_U (\cdot) - p_{K,U} [V_K - V_E]}{\rho + p_{U,E} + \alpha s_U + p_{K,U}} \]  

(3.11)
Hence, \[ [V_N - V_K] = \frac{h_N(\cdot) - g_K(\cdot) - p_{E,N}[V_K - V_E]}{\rho + p_{U,E} + \alpha_s K + p_{E,N}}. \tag{3.12} \]

Using that \[ \rho [V_K - V_E] = g_K(\cdot) - h_E(\cdot) + \alpha_s K [V_N - V_K] - p_{U,E} [V_U - V_E] \]

(3.11) and (3.12) implies

\[
[\rho + p_{U,E} + \alpha s U + p_{K,U}] [V_E - V_U] = h_E(\cdot) - g_U(\cdot) - \frac{p_{K,U}}{\rho} [g_K(\cdot) - h_E(\cdot)]
+ \alpha s K [V_N - V_K] - p_{U,E} [V_U - V_E]
\]

\[
[\rho + p_{U,E} + \alpha s K + p_{E,N}] [V_N - V_K] = h_N(\cdot) - g_K(\cdot) - \frac{p_{E,N}}{\rho} [g_K(\cdot) - h_E(\cdot)]
+ \alpha s K [V_N - V_K] - p_{U,E} [V_U - V_E]
\]

which in turn can be written

\[
\begin{align*}
\left[ \rho + p_{U,E} + \alpha s U + p_{K,U} + \frac{p_{K,U}}{\rho} p_{U,E} \right] [V_E - V_U] &= h_E(\cdot) - g_U(\cdot) + \frac{p_{K,U}}{\rho} [h_E(\cdot) - g_K(\cdot)] \\
&\quad - \alpha s K [V_N - V_K]
\end{align*}
\]

\[
\begin{align*}
\left[ \rho + p_{U,E} + \alpha s K + p_{E,N} + \frac{p_{E,N}}{\rho} \alpha s K \right] [V_N - V_K] &= h_N(\cdot) - g_K(\cdot) + \frac{p_{E,N}}{\rho} [h_E(\cdot) - g_K(\cdot)] \\
&\quad - p_{U,E} [V_U - V_E].
\end{align*}
\]

Totally differentiating yields

\[
\begin{align*}
[V_E - V_U] (1 + \frac{p_{U,E}}{\rho}) dp_{K,U} + A_1 d [V_E - V_U] &= \frac{1}{\rho} A_2 dp_{K,U} - \frac{p_{K,U}}{\rho} \alpha s K d [V_N - V_K] \\
[V_N - V_K] \left[ 1 + \frac{1}{\rho} \alpha s K \right] dp_{E,N} + B_1 d [V_N - V_K] &= \frac{1}{\rho} B_2 dp_{E,N} - \frac{p_{E,N}}{\rho} p_{U,E} d [V_E - V_U]
\end{align*}
\]

where

\[
\begin{align*}
A_1 &\equiv \left[ \rho + p_{U,E} + \alpha s U + p_{K,U} + \frac{p_{K,U}}{\rho} p_{U,E} \right] > 0 \\
A_2 &\equiv [h_E(\cdot) - g_K(\cdot) - \alpha s K [V_N - V_K]] \lesssim 0 \\
B_1 &\equiv \rho + p_{U,E} + \alpha s K + p_{E,N} + \frac{p_{E,N}}{\rho} \alpha s K > 0 \\
B_2 &\equiv h_E(\cdot) - g_K(\cdot) - p_{U,E} [V_E - V_U] \lesssim 0.
\end{align*}
\]

Hence,

\[
\begin{align*}
A_1 d [V_E - V_U] &= \left[ \frac{1}{\rho} A_2 - [V_E - V_U] (1 + \frac{p_{U,E}}{\rho}) \right] dp_{K,U} - \frac{p_{K,U}}{\rho} \alpha s K d [V_N - V_K] \tag{3.13} \\
B_1 d [V_N - V_K] &= \left[ \frac{1}{\rho} B_2 - [V_N - V_K] \left[ 1 + \frac{1}{\rho} \alpha s K \right] \right] dp_{E,N} - \frac{p_{E,N}}{\rho} p_{U,E} d [V_E - V_U]. \tag{3.14}
\end{align*}
\]
Before proceeding we prove that
\[
\frac{1}{\rho} B_2 - [V_N - V_K] \left[ 1 + \frac{1}{\rho} \alpha s_K \right] > 0
\]
or
\[
h_E (\cdot) - g_K (\cdot) - p_{U,E} [V_E - V_U] > [V_N - V_K] [\rho + \alpha s_K].
\]

We have from the value functions that
\[
[\rho + \alpha s_K] [V_N - V_K] = h_N (\cdot) - g_K (\cdot) + p_{U,E} [V_K - V_N] + p_{E,N} [V_E - V_N]
\]
and hence, the inequality can be rewritten
\[
h_E (\cdot) - h_N (\cdot) - p_{U,E} [V_E - V_U] > p_{U,E} [V_K - V_N] + p_{E,N} [V_E - V_N].
\]

Using that
\[
[\rho + A_2 - [V_E - V_U] (1 + \frac{p_{U,E}}{\rho}) > 0 or
\]
\[
h_E (\cdot) - g_K (\cdot) - \alpha s_K [V_N - V_K] > [V_E - V_U] (\rho + p_{U,E}).
\]

We have from the value functions that
\[
(\rho + p_{U,E}) [V_E - V_U] = h_E (\cdot) - g_U (\cdot) - \alpha s_U [V_E - V_U] - p_{K,U} [V_K - V_U]
\]
and hence, the inequality can be written
\[
g_U (\cdot) - g_K (\cdot) > \alpha s_K [V_N - V_K] - \alpha s_U [V_E - V_U] - p_{K,U} [V_K - V_U].
\]

Using that
\[
\rho [V_U - V_K] = g_U (\cdot) - g_K (\cdot) + \alpha s_U [V_E - V_U] + p_{K,U} [V_K - V_U] - \alpha s_K [V_N - V_K]
\]
the inequality reduces to
\[
\rho [V_U - V_K] > 0
\]
which is fulfilled.
Finally, also note that

$$B_1 - \frac{p_{E,N}}{\rho} p_{U,E} \frac{p_K U \alpha s_K}{A_1} = \frac{1}{A_1} \left[ A_1 B_1 - \frac{p_{E,N}}{\rho} p_{U,E} \frac{p_K U \alpha s_K}{A_1} \right] > 0.$$  

Returning to (3.13) and (3.14) we have for \( dp_{K,U} = 0 \) that

$$A_1 d[V_E - V_U] = -\frac{p_{K,U}}{\rho} \alpha s_K d[V_N - V_K]$$

$$\left[ B_1 - \frac{p_{E,N}}{\rho} p_{U,E} \frac{p_K U \alpha s_K}{A_1} \right] d[V_N - V_K] = \left[ \frac{1}{\rho} B_2 - [V_N - V_K] \left[ 1 + \frac{1}{\rho} \alpha s_K \right] \right] dp_{E,N}$$

$$d[V_N - V_K] = \left[ \frac{1}{\rho} B_2 - [V_N - V_K] \left[ 1 + \frac{1}{\rho} \alpha s_K \right] \right] dp_{E,N}$$

$$d[V_E - V_U] = -\frac{1}{\rho} \left[ \frac{1}{\rho} B_2 - [V_N - V_K] \left[ 1 + \frac{1}{\rho} \alpha s_K \right] \right] dp_{E,N}$$

$$\frac{d[V_N - V_K]}{dp_{E,N}} = \left[ \frac{1}{\rho} B_2 - [V_N - V_K] \left[ 1 + \frac{1}{\rho} \alpha s_K \right] \right]$$

$$\frac{d[V_E - V_U]}{dp_{E,N}} = -\frac{1}{\rho} \left[ \frac{1}{\rho} B_2 - [V_N - V_K] \left[ 1 + \frac{1}{\rho} \alpha s_K \right] \right]$$

and for \( dp_{E,N} = 0 \) that

$$B_1 d[V_N - V_K] = \frac{p_{E,N}}{\rho} p_{U,E} d[V_E - V_U]$$

$$\left[ A_1 - \frac{p_{E,N}}{\rho} p_{U,E} \frac{p_K U \alpha s_K}{B_1} \right] d[V_E - V_U] = \left[ \frac{1}{\rho} A_2 - [V_E - V_U] \left( 1 + \frac{p_{U,E}}{\rho} \right) \right] dp_{K,U}$$

$$d[V_E - V_U] = \left[ \frac{1}{\rho} A_2 - [V_E - V_U] \left( 1 + \frac{p_{U,E}}{\rho} \right) \right] dp_{K,U}$$

$$d[V_N - V_K] = -\frac{1}{\rho} \left[ \frac{1}{\rho} A_2 - [V_E - V_U] \left( 1 + \frac{p_{U,E}}{\rho} \right) \right] dp_{K,U}$$

$$\frac{d[V_N - V_K]}{d[V_E - V_U]} = -\frac{1}{\rho} \frac{p_{E,N}}{B_1} p_{U,E}$$
\[
\frac{d[V_E - V_U]}{dp_{K,U}} = \frac{\left[\frac{1}{\rho} A_2 - [V_E - V_U] (1 + \frac{p_{U,E}}{\rho})\right]}{\left[1 + \frac{p_{U,E}}{\rho} - \left(A_1 - \frac{p_{E,N}}{\rho} p_{U,E} \frac{p_{K,U} \alpha s_K}{B_1}\right)\right]}
\]

\[
\frac{d[V_N - V_K]}{dp_{K,U}} = -\frac{1}{B_1} \frac{p_{E,N}}{\rho} \frac{1}{p_{U,E}} \left[\frac{1}{\rho} A_2 - [V_E - V_U] (1 + \frac{p_{U,E}}{\rho})\right] \left[A_1 - \frac{p_{E,N}}{\rho} p_{U,E} \frac{p_{K,U} \alpha s_K}{B_1}\right].
\]

Hence, we have established the following signs

<table>
<thead>
<tr>
<th>( dp_{E,N} )</th>
<th>( dp_{K,U} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt; 0</td>
<td>&lt; 0</td>
</tr>
<tr>
<td>&lt; 0</td>
<td>&gt; 0</td>
</tr>
</tbody>
</table>

This implies that

\[
\frac{ds_U}{dp_{K,U}} > 0 ; \quad \frac{ds_K}{dp_{E,N}} < 0
\]

\[
\frac{ds_U}{dp_{E,N}} < 0 ; \quad \frac{ds_K}{dp_{E,N}} > 0.
\]

**Marginal rates of substitution**

Consider next the marginal rates of return, i.e., combinations of \( p_{K,U} \) and \( p_{E,N} \) leaving \( V_E - V_U \) and thus search \( s_U \) unchanged (and similarly for \( s_K \)). Using

\[
A_1 d[V_E - V_U] = \left[\frac{1}{\rho} A_2 - [V_E - V_U] (1 + \frac{p_{U,E}}{\rho})\right] dp_{K,U} - \frac{p_{K,U} \alpha s_K}{B_1} d[V_N - V_K]
\]

\[
B_1 d[V_N - V_K] = \left[\frac{1}{\rho} B_2 - [V_N - V_K] \left[1 + \frac{1}{\rho} \alpha s_K\right]\right] dp_{E,N} - \frac{p_{E,N}}{\rho} p_{U,E} d[V_E - V_U]
\]

and imposing \( d[V_E - V_U] = 0 \), we obtain

\[
0 = \left[\frac{1}{\rho} A_2 - [V_E - V_U] (1 + \frac{p_{U,E}}{\rho})\right] dp_{K,U} - \frac{p_{K,U} \alpha s_K}{B_1} \left[\frac{1}{\rho} B_2 - [V_N - V_K] \left[1 + \frac{1}{\rho} \alpha s_K\right]\right] dp_{E,N}
\]

and hence,

\[
\frac{dp_{K,U}}{dp_{E,N}} |_{[V_E - V_U]} = \text{const} = \frac{\frac{p_{K,U} \alpha s_K}{B_1} \left[\frac{1}{\rho} B_2 - [V_N - V_K] \left[1 + \frac{1}{\rho} \alpha s_K\right]\right]}{\left[\frac{1}{\rho} A_2 - [V_E - V_U] (1 + \frac{p_{U,E}}{\rho})\right]} > 0.
\]

Similarly, for \( s_K \) where we have that \( d[V_N - V_K] = 0 \) implies

\[
0 = \left[\frac{1}{\rho} B_2 - [V_N - V_K] \left[1 + \frac{1}{\rho} \alpha s_K\right]\right] dp_{E,N} - \frac{p_{E,N} p_{U,E}}{A_1} \left[\frac{1}{\rho} A_2 - [V_E - V_U] (1 + \frac{p_{U,E}}{\rho})\right] dp_{K,U}
\]

and hence,

\[
\frac{dp_{E,N}}{dp_{K,U}} |_{[V_N - V_K]} = \text{const} = \frac{\frac{p_{E,N} p_{U,E}}{A_1} \left[\frac{1}{\rho} A_2 - [V_E - V_U] (1 + \frac{p_{U,E}}{\rho})\right]}{\left[\frac{1}{\rho} B_2 - [V_N - V_K] \left[1 + \frac{1}{\rho} \alpha s_K\right]\right]} > 0.
\]
Note that (recall that \( A_1 > 0 \) and \( B_1 > 0 \))
\[
\frac{dp_{K,U}}{dp_{E,N}} \mid [V_E-V_U] = \text{const} \quad \frac{dp_{E,N}}{dp_{K,U}} \mid [V_N-V_K] = \text{const} = \frac{p_{K,U}}{\rho} \frac{\alpha s_K}{p_{E,N}} \frac{p_{U,E}}{\rho} \frac{A_1}{B_1} \leq 1.
\]
From the envelope theorem we know that the utility effect of a given policy change is given by the direct utility effects (all indirect effects via behavior wash out via first order conditions).

\[
\begin{array}{c|cc}
& d[V_N-V_K] & d[V_E-V_U] \\ \hline
dp_{E,N} & > 0 & < 0 \\ dp_{K,U} & < 0 & > 0
\end{array}
\]

and
\[
\begin{align*}
\rho V_E &= h(w[1-\tau]+\Pi,1-l)+p_{U,E}[V_U-V_E] \\
\rho V_N &= h(w[1-\tau]+\Pi,1-l)+p_{U,E}[V_K-V_N]+p_{E,N}[V_E-V_N] \\
\rho V_U &= g(b_U+\Pi,1-s_U)+\alpha s_U[V_E-V_U]+p_{K,U}[V_K-V_U] \\
\rho V_K &= g(b_K+\Pi,1-s_K)+\alpha s_K[V_N-V_K]
\end{align*}
\]
we thus have: i) an increase in \( p_{K,U} \) leads to a decrease in \( V_E \) and a decrease in \( V_K \), while there is an ambiguous effect on \( V_N \) and \( V_U \), ii) an increase in \( p_{E,N} \) leads to an increase in \( V_E \) and an increase in \( V_K \), while there is an ambiguous effect on \( V_N \) and \( V_U \).

**Iso-gross unemployment loci**

Note first that we have
\[
u = \frac{p_{E,N}}{p_{K,U}} n
\]
implying
\[
k = \frac{p_{U,E}+\rho p_{E,N}}{\alpha s_K} n = \frac{p_{U,E}+\rho p_{E,N} p_{K,U}}{\alpha s_K} u
\]
and
\[
u = \frac{p_{U,E}}{\alpha s_U+p_{K,U}+p_{U,E}+p_{U,E} p_{U,E}+p_{U,E} p_{U,E}}
\]

\[
e = 1 - u - n - k
\]
\[
= 1 - (1 - \frac{p_{K,U}}{p_{E,N}} \frac{p_{U,E}+p_{E,N} p_{K,U}}{\alpha s_K} p_{E,N}) \frac{p_{U,E}}{\alpha s_U+p_{K,U}+p_{U,E}+p_{U,E} p_{K,U} p_{E,N}+p_{U,E} p_{E,N}}
\]
\[
= 1 - (\frac{p_{E,N} \alpha s_K}{\alpha s_K p_{E,N}} - (\alpha s_K + p_{U,E}+p_{E,N}) p_{K,U}) \frac{p_{U,E}}{\alpha s_U+p_{K,U}+p_{U,E}+p_{U,E} p_{E,N} p_{K,U} p_{E,N}+p_{U,E} p_{E,N}}.
\]
It follows that gross unemployment \((u+k)\) is given as
\[
u + k = u \left[ 1 + \frac{p_{U,E}+p_{E,N} p_{K,U}}{\alpha s_K} \right]
\]

101
\[
\frac{\partial}{\partial s_U} \left( \frac{1}{u + k} \right) = \frac{\alpha s_U + p_{K,U} + \frac{p_{U,E} p_{K,U}}{p_{E,N}}}{p_{U,E} + \frac{p_{U,E} + p_{K,U}}{\alpha s_K} p_{K,U} p_{U,E}} > 0
\]

or

\[
\frac{1}{u + k} = \frac{\alpha s_U + p_{K,U} + \frac{p_{U,E} p_{K,U}}{p_{E,N}}}{p_{U,E} + \frac{p_{U,E} + p_{K,U}}{\alpha s_K} p_{K,U} p_{U,E}} > 1
\]

Where

\[
\frac{\partial}{\partial s_K} \left( \frac{1}{u + k} \right) = \frac{\alpha s_K}{p_{U,E} + \frac{p_{U,E} + p_{K,U}}{\alpha s_K} p_{K,U} p_{U,E}} > 0
\]

\[
\frac{\partial}{\partial p_{EN}} \left( \frac{1}{u + k} \right) = \frac{p_{K,U} \left[ p_{U,E} + \frac{p_{U,E} + p_{K,U}}{\alpha s_K} p_{K,U} p_{U,E} \right]}{\left[ p_{U,E} + \frac{p_{U,E} + p_{K,U}}{\alpha s_K} p_{K,U} p_{U,E} \right]^2} \left( -\frac{p_{U,E}}{p_{E,N}^2} \right)
\]

\[
\frac{\partial}{\partial p_{KU}} \left( \frac{1}{u + k} \right) = \frac{\left[ 1 + \frac{p_{U,E}}{p_{E,N}} \right] \left[ p_{U,E} + \frac{p_{U,E} + p_{K,U}}{\alpha s_K} p_{K,U} p_{U,E} \right]}{\left[ p_{U,E} + \frac{p_{U,E} + p_{K,U}}{\alpha s_K} p_{K,U} p_{U,E} \right]^2} \left( -\frac{p_{U,E}}{p_{E,N}^2} \right) - \left[ \alpha s_U + p_{K,U} \left[ \frac{p_{K,U} p_{U,E}}{p_{E,N}^2} \right] \right] \left[ p_{U,E} + \frac{p_{U,E} + p_{K,U}}{\alpha s_K} p_{K,U} p_{U,E} \right]
\]

\[
\frac{\partial}{\partial p_{EN}} \left( \frac{1}{u + k} \right) = \frac{\left[ 1 + \frac{p_{U,E}}{p_{E,N}} \right] \left[ p_{U,E} + \frac{p_{U,E} + p_{K,U}}{\alpha s_K} p_{K,U} p_{U,E} \right]}{\left[ p_{U,E} + \frac{p_{U,E} + p_{K,U}}{\alpha s_K} p_{K,U} p_{U,E} \right]^2} \left( -\frac{p_{U,E}}{p_{E,N}^2} \right) - \left[ \alpha s_U + p_{K,U} \left[ \frac{p_{K,U} p_{U,E}}{p_{E,N}^2} \right] \right] \left[ p_{U,E} + \frac{p_{U,E} + p_{K,U}}{\alpha s_K} p_{K,U} p_{U,E} \right]
\]

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\[
\frac{p_{U,E} - \alpha s_U \frac{1}{\alpha s_K} p_{U,E}}{\left[ p_{U,E} + \frac{p_{U,E} + 1}{\alpha s_K} p_{K,U} p_{U,E} \right]^2 \left[ 1 + \frac{p_{U,E}}{p_{E,N}} \right]} < 0
\]

\[
\frac{p_{U,E} \left( 1 - \frac{p_{U,E}}{s_K} \right) \left[ 1 + \frac{p_{U,E}}{p_{E,N}} \right]^2 \left[ 1 + \frac{p_{U,E}}{p_{E,N}} \right] < 0 \text{ for } \frac{s_U}{s_K} \ll 1.
\]

Hence, the gross unemployment can be written in implicit form as

\[
u + k = F(s_U(p_{K,U}, p_{E,N}), s_K(p_{K,U}, p_{E,N}), p_{K,U}, p_{E,N})
\]

Note that

\[
\text{sign} \frac{\partial F(\cdot)}{\partial z} = -\text{sign} \left( \frac{1}{\nu + k} \right).
\]

It follows that

\[
\frac{\partial F(\cdot)}{\partial s_U} < 0 ; \frac{\partial F(\cdot)}{\partial s_K} < 0
\]

\[
\text{sign} \left( \frac{\partial F(\cdot)}{\partial p_{K,U}} \right) = \text{sign}(s_U - s_K)
\]

\[
\text{sign} \left( \frac{\partial F(\cdot)}{\partial p_{E,N}} \right) = -\text{sign} \left( \frac{\partial F(\cdot)}{\partial p_{K,U}} \right).
\]

We have that

\[
\frac{\partial F(\cdot)}{\partial s_U} ds_U + \frac{\partial F(\cdot)}{\partial s_K} ds_K + \frac{\partial F(\cdot)}{\partial p_{K,U}} dp_{K,U} + \frac{\partial F(\cdot)}{\partial p_{E,N}} dp_{E,N}
\]

\[
= \frac{\partial F(\cdot)}{\partial s_U} dp_{K,U} + \frac{\partial F(\cdot)}{\partial p_{E,N}} dp_{E,N} + \frac{\partial s_K}{\partial p_{K,U}} dp_{K,U} + \frac{\partial s_K}{\partial p_{E,N}} dp_{E,N}
\]

\[
+ \frac{\partial F(\cdot)}{\partial p_{K,U}} dp_{K,U} + \frac{\partial F(\cdot)}{\partial p_{E,N}} dp_{E,N}.
\]

For \( d(u + k) = 0 \) we have

\[
- \left[ \frac{\partial F(\cdot)}{\partial p_{E,N}} + \frac{\partial s_U}{\partial p_{E,N}} \frac{\partial F(\cdot)}{\partial s_U} + \frac{\partial s_K}{\partial p_{E,N}} \frac{\partial F(\cdot)}{\partial s_K} \right] dp_{E,N} = \left[ \frac{\partial F(\cdot)}{\partial p_{K,U}} + \frac{\partial s_K}{\partial p_{K,U}} \frac{\partial F(\cdot)}{\partial s_K} + \frac{\partial s_U}{\partial p_{K,U}} \frac{\partial F(\cdot)}{\partial s_U} \right] dp_{K,U}
\]

\[
= \frac{\partial F(\cdot)}{\partial p_{E,N}} + \frac{\partial s_K}{\partial p_{E,N}} \frac{\partial F(\cdot)}{\partial s_K} + \frac{\partial s_U}{\partial p_{E,N}} \frac{\partial F(\cdot)}{\partial s_U}.
\]

Notice that

\[
\frac{ds_U}{dp_{K,U}} > 0; \frac{ds_K}{dp_{K,U}} < 0
\]

\[
\frac{ds_U}{dp_{E,N}} < 0; \frac{ds_K}{dp_{E,N}} > 0.
\]

Hence, the numerator and denominator of (3.15) are in general ambiguously signed.
3.C Social welfare function

This appendix derives the social welfare function. From the value functions

\[ \rho V_E = h (w_E [1 - \tau] + \Pi, 1 - l_e) + p_{U,E} [V_U - V_E] \]

\[ \rho V_N = h (w_N [1 - \tau] + \Pi, 1 - l_e) + p_{U,E} [V_K - V_N] + p_{E,N} [V_E - V_N] \]

\[ \rho V_U = g (b_U + \Pi, 1 - s_U) + \alpha s_U [V_E - V_U] + p_{K,U} [V_K - V_U] \]

\[ \rho V_K = g (b_K + \Pi, 1 - s_K) + \alpha s_K [V_N - V_K] \]

we get

\[ \rho [eV_E + nV_N + uV_U + kV_K] = eh_E (\cdot) + nh_N (\cdot) + ug_U (\cdot) + kg (\cdot) + \Delta \]

where

\[ \Delta \equiv ep_{U,E} [V_U - V_E] + np_{U,E} [V_K - V_N] + np_{E,N} [V_E - V_N] + u\alpha s_U [V_E - V_U] \]

\[ + up_{K,U} [V_K - V_U] + k\alpha s_K [V_N - V_K] \]

\[ = [u\alpha s_U + np_{E,N} - ep_{U,E}] V_E + [ep_{U,E} - u\alpha s_U - up_{K,U}] V_U + [k\alpha s_K - np_{U,E} - np_{E,N}] V_N \]

\[ + [up_{K,U} - np_{U,E} - k\alpha s_K] V_K. \]

Using the flow equilibrium conditions

\[ (1 - u - k - n) p_{U,E} = \alpha s_U u + p_{K,U} u \]

\[ np_{U,E} + p_{K,U} u = \alpha s_K k \]

\[ \alpha s_K k = p_{U,E} u + p_{E,N} n \]

we arrive at \( \Delta = 0. \)

3.D Optimal social safety net

In this appendix we consider the problem of finding the optimal social safety net analytically. To simplify the calculations we consider the case where \( \rho \to 0 \) implying that profits are zero (see Appendix 3.E). The numerical analyses consider the more general case \( (\rho > 0; \Pi > 0) \).

Two-tier benefit scheme

Consider first the question whether the optimal benefit scheme has two tiers. We assume that employment automatically yields eligibility for unemployment benefits \( (p_{E,N} \to \infty, n \to 0) \). Hence, the following is basically asking the same question as in Fredriksson & Holmlund (2001) although in a somewhat more general setting.
The optimal policy satisfies (the associated second order conditions are assumed fulfilled)

$$\frac{\partial \Omega}{\partial b_U} = h_E(\cdot) \frac{\partial e(\cdot)}{\partial b_U} + g_U(\cdot) \frac{\partial u(\cdot)}{\partial b_U} + g_K(\cdot) \frac{\partial k(\cdot)}{\partial b_U} + e \frac{\partial h_E(\cdot)}{\partial b_U} + u \frac{\partial g_U(\cdot)}{\partial b_U} + k \frac{\partial g_K(\cdot)}{\partial b_U} = 0$$

$$\frac{\partial \Omega}{\partial b_K} = h_E(\cdot) \frac{\partial e(\cdot)}{\partial b_K} + g_U(\cdot) \frac{\partial u(\cdot)}{\partial b_K} + g_K(\cdot) \frac{\partial k(\cdot)}{\partial b_K} + e \frac{\partial h_E(\cdot)}{\partial b_K} + u \frac{\partial g_U(\cdot)}{\partial b_K} + k \frac{\partial g_K(\cdot)}{\partial b_K} = 0$$

$$\frac{\partial \Omega}{\partial p_{K,U}} = h_E(\cdot) \frac{\partial e(\cdot)}{\partial p_{K,U}} + g_U(\cdot) \frac{\partial u(\cdot)}{\partial p_{K,U}} + g_K(\cdot) \frac{\partial k(\cdot)}{\partial p_{K,U}} + e \frac{\partial h_E(\cdot)}{\partial p_{K,U}} + u \frac{\partial g_U(\cdot)}{\partial p_{K,U}} + k \frac{\partial g_K(\cdot)}{\partial p_{K,U}} = 0.$$  

The key question here is whether $p_{K,U} > 0$; i.e., is it optimal to have a finite duration of unemployment benefits after which unemployed are offered a lower social assistance? A sufficient condition for this to be the case is

$$\lim_{p_{K,U} \to 0} \frac{\partial \Omega}{\partial p_{K,U}} > 0$$

i.e., the introduction of a second tier should improve welfare starting out from a one-tier scheme. We have that (since $k \to 0$ for $p_{K,U} \to 0$)

$$\lim_{p_{K,U} \to 0} \frac{\partial \Omega}{\partial p_{K,U}} = [g_U(\cdot) - h_E(\cdot)] \frac{\partial u(\cdot)}{\partial p_{K,U}} + [g_K(\cdot) - h_E(\cdot)] \frac{\partial k(\cdot)}{\partial p_{K,U}} + e \frac{\partial h_E(\cdot)}{\partial p_{K,U}} + u \frac{\partial g_U(\cdot)}{\partial p_{K,U}}.$$  

The welfare effect thus has four terms. Hence, welfare improves (deteriorates) if $i$) unemployment decreases (increases) since $g_U(\cdot) - h_E(\cdot) < 0$, $ii$) the number of social assistance recipients decreases (increases) since $g_K(\cdot) - h_E(\cdot) < 0$, $i$) the instantaneous utility for those employed and eligible for benefits increases (decreases), $iv$) the instantaneous utility for those receiving UIB increases (decreases).

Using that $(1 - u - k) p_{U,E} = \alpha s_U u + p_{K,U} u$ we have

$$- \frac{\partial u}{\partial p_{K,U}} p_{U,E} - \frac{\partial k}{\partial p_{K,U}} p_{U,E} = \frac{\partial (\alpha s_U)}{\partial p_{K,U}} u + \alpha s_U \frac{\partial u}{\partial p_{K,U}} + u + p_{K,U} \frac{\partial u}{\partial p_{K,U}}$$

and using $p_{K,U} u = \alpha s_K k$ we have

$$u + p_{K,U} \frac{\partial u}{\partial p_{K,U}} = \frac{\partial (\alpha s_K)}{\partial p_{K,U}} k + \alpha s_K \frac{\partial k}{\partial p_{K,U}}.$$  

In the limit for $p_{K,U} \to 0$ it follows that

$$u = \frac{\alpha s_K}{\partial p_{K,U}} \frac{\partial k}{\partial p_{K,U}} = \frac{u}{\alpha s_K}.$$
and hence
\[- \frac{\partial u}{\partial p_{K,U}} p_{U,E} - \frac{\partial k}{\partial p_{K,U}} p_{U,E} = \frac{\partial (\alpha s_U)}{\partial p_{K,U}} u + \alpha s_U \frac{\partial u}{\partial p_{K,U}} + u - \frac{u}{\alpha s_K} p_{U,E} = \frac{\partial (\alpha s_U)}{\partial p_{K,U}} u + [\alpha s_U + p_{U,E}] \frac{\partial u}{\partial p_{K,U}} + u - \frac{u}{\alpha s_K} p_{U,E} - \frac{\partial (\alpha s_U)}{\partial p_{K,U}} u - u \]
\[
\left[ \alpha s_U + p_{U,E} \right] \frac{\partial u}{\partial p_{K,U}} = - \frac{u}{\alpha s_K} p_{U,E} - \frac{\partial (\alpha s_U)}{\partial p_{K,U}} u - u \frac{u}{\alpha s_K} p_{U,E} - \frac{\partial (\alpha s_U)}{\partial p_{K,U}} u - u \frac{\partial u}{\partial p_{K,U}} \frac{u}{\alpha s_U + p_{U,E}}.
\]

A sufficient condition for \( \frac{\partial u}{\partial p_{K,U}} < 0 \) is \( \frac{\partial (\alpha s_U)}{\partial p_{K,U}} > 0 \). Note that

\[
\frac{\partial e}{\partial p_{K,U}} = - \left[ \frac{\partial u}{\partial p_{K,U}} + \frac{\partial k}{\partial p_{K,U}} \right] \alpha s_K
\]
\[
= \left[ \frac{p_{U,E}}{\alpha s_U + p_{U,E}} \frac{\partial k}{\partial p_{K,U}} - 1 \right] \frac{\partial k}{\partial p_{K,U}} + \frac{\partial (\alpha s_U)}{\partial p_{K,U}} u + u
\]
\[
= - \frac{\alpha s_U}{\alpha s_U + p_{U,E}} \frac{\partial k}{\partial p_{K,U}} + \frac{\partial (\alpha s_U)}{\partial p_{K,U}} u + u
\]
\[
= - \frac{\alpha s_U}{\alpha s_U + p_{U,E}} \frac{\partial k}{\partial p_{K,U}} + \frac{\partial (\alpha s_U)}{\partial p_{K,U}} u + u
\]
\[
= 1 - \frac{s_U}{s_K} u + \frac{\partial (\alpha s_U)}{\partial p_{K,U}} u + \frac{\partial (\alpha s_U)}{\partial p_{K,U}} u
\]

and sufficient conditions for \( \frac{\partial e}{\partial p_{K,U}} > 0 \) are \( s_U < 1 \) or \( s_K > s_U \) and \( \frac{\partial (\alpha s_U)}{\partial p_{K,U}} > 0 \).

To sum up, we have shown that for \( p_{K,U} \to 0 \)

\[
\frac{\partial k}{\partial p_{K,U}} > 0
\]
\[
\frac{\partial u}{\partial p_{K,U}} < 0 \text{ under the sufficient condition that } \frac{\partial (\alpha s_U)}{\partial p_{K,U}} > 0
\]
\[
\frac{\partial e}{\partial p_{K,U}} > 0 \text{ under the sufficient conditions that } \frac{\partial (\alpha s_U)}{\partial p_{K,U}} > 0 \text{ and } s_K > s_U.
\]

Note also that

\[
\frac{\partial h_E (\cdot)}{\partial p_{K,U}} = \frac{\partial h_E (\cdot)}{\partial I_E} \left[ (1 - \tau) \frac{\partial w_E}{\partial p_{K,U}} - w_E \frac{\partial \tau}{\partial p_{K,U}} \right]
\]

i.e., for the employed eligible for UIB to be better off requires an increase in the disposable income. In the case where the wage is independent of \( p_{K,U} \), this only requires that the tax falls, which follows from an increase in employment, cf. above.

Furthermore, we have that

\[
\frac{\partial g_U (\cdot)}{\partial p_{K,U}} = \frac{\partial g_U (\cdot)}{\partial s_U} \frac{\partial s_U}{\partial p_{K,U}}
\]
which is negative for $\frac{\partial s_U}{\partial p_{K,U}} > 0$.

Hence, the sign of $\frac{\partial \Omega}{\partial p_{K,U}} |_{p_{K,U}=0}$ is ambiguous, and under the sufficient conditions listed above only the first and the third welfare effects work in favor of a two-tier benefit scheme. Therefore, it is not a general result that the optimal UI scheme exhibits two tiers.

**Employment as an eligibility condition**

Consider next whether it is optimal to include an employment condition as part of the eligibility conditions in the benefit scheme; that is, is it optimal to have $0 < p_{E,N} < \infty$?

Notice first that $p_{E,N} \to 0$ effectively implies that the system becomes a one-tier system, cf. above. Hence, if the condition ensuring that the optimality of a two-tier scheme is satisfied, it follows that $p_{E,N} > 0$. The question is whether employment automatically implies entitlement for unemployment benefits, or whether there should be certain employment requirements for such eligibility ($p_{E,N} < \infty$).

The optimal policy in this case is characterized by

$$\frac{\partial \Omega}{\partial b_U} = h_E(\cdot) \frac{\partial e}{\partial b_U} + h_N(\cdot) \frac{\partial n}{\partial b_U} + g_U(\cdot) \frac{\partial u}{\partial b_U} + g_K(\cdot) \frac{\partial k}{\partial b_U} + e \frac{\partial h_E(\cdot)}{\partial b_U} + n \frac{\partial h_N(\cdot)}{\partial b_U} + u \frac{\partial g_U(\cdot)}{\partial b_U} + k \frac{\partial g_K(\cdot)}{\partial b_U} = 0$$

$$\frac{\partial \Omega}{\partial b_K} = h_E(\cdot) \frac{\partial e}{\partial b_K} + h_N(\cdot) \frac{\partial n}{\partial b_K} + g_U(\cdot) \frac{\partial u}{\partial b_K} + g_K(\cdot) \frac{\partial k}{\partial b_K} + e \frac{\partial h_E(\cdot)}{\partial b_K} + n \frac{\partial h_N(\cdot)}{\partial b_K} + u \frac{\partial g_U(\cdot)}{\partial b_K} + k \frac{\partial g_K(\cdot)}{\partial b_K} = 0$$

$$\frac{\partial \Omega}{\partial p_{K,U}} = h_E(\cdot) \frac{\partial e}{\partial p_{K,U}} + h_N(\cdot) \frac{\partial n}{\partial p_{K,U}} + g_U(\cdot) \frac{\partial u}{\partial p_{K,U}} + g_K(\cdot) \frac{\partial k}{\partial p_{K,U}} + e \frac{\partial h_E(\cdot)}{\partial p_{K,U}} + n \frac{\partial h_N(\cdot)}{\partial p_{K,U}} + u \frac{\partial g_U(\cdot)}{\partial p_{K,U}} + k \frac{\partial g_K(\cdot)}{\partial p_{K,U}} = 0$$

$$\frac{\partial \Omega}{\partial p_{E,N}} = h_E(\cdot) \frac{\partial e}{\partial p_{E,N}} + h_N(\cdot) \frac{\partial n}{\partial p_{E,N}} + g_U(\cdot) \frac{\partial u}{\partial p_{E,N}} + g_K(\cdot) \frac{\partial k}{\partial p_{E,N}} + e \frac{\partial h_E(\cdot)}{\partial p_{E,N}} + n \frac{\partial h_N(\cdot)}{\partial p_{E,N}} + u \frac{\partial g_U(\cdot)}{\partial p_{E,N}} + k \frac{\partial g_K(\cdot)}{\partial p_{E,N}} = 0.$$  

For $p_{E,N} < \infty$ to be optimal we require

$$\lim_{p_{E,N} \to \infty} \frac{\partial \Omega}{\partial p_{E,N}} < 0.$$

The complexity of the model makes it difficult analytically to say something meaningful about this condition. It is however clear that there are effects working both for and against $p_{E,N} < \infty$, for example $\frac{\partial g_K}{\partial p_{E,N}} \frac{\partial p_{E,N}}{\partial p_{E,N}} < 0$ for $\frac{\partial g_K}{\partial p_{E,N}} > 0$ and $\frac{\partial g_U}{\partial p_{E,N}} = \frac{\partial g_U}{\partial p_{E,N}} \frac{\partial p_{E,N}}{\partial p_{E,N}} > 0$ for $\frac{\partial g_U}{\partial p_{E,N}} < 0$.  

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3.E Profit

In this appendix we show that profits are zero for \( \rho = 0 \). We have that

\[
\Pi = [y - w_E] e + [y - w_N] n - vk.
\]

Furthermore,

\[
J_{\text{EXP}} = \frac{\alpha s U u J_E + \alpha s K k J_N}{\alpha s U u + \alpha s K k} = \frac{s U u J_E + s K k J_N}{s U u + s K k}
\]

and

\[
\frac{\kappa}{q} = \frac{\kappa}{s} \frac{v}{\alpha}
\]

where it has been used that \( m(\theta \theta^{-1}, \theta) = \theta m(\theta^{-1}, 1) = m(1, \theta) \) and hence

\[
\frac{\alpha}{q} = \frac{m(1, \theta)}{m(\theta^{-1}, 1)} = \theta = \frac{v}{s}.
\]

It follows that

\[
\frac{\kappa}{q} = \frac{\alpha s U u J_E + \alpha s K k J_N}{\alpha s U u + \alpha s K k}
\]

and

\[
\frac{v}{s} \frac{1}{\alpha} = \frac{\alpha s U u J_E + \alpha s K k J_N}{\alpha s}
\]

\[
\kappa v = \alpha s U u J_E + \alpha s K k J_N.
\]

Using that

\[
J_E = \frac{1}{\rho + p u, E} [y - w_E]
\]

and

\[
[\rho + p u, E + p E, N] J_N = y - w_N + p E, N J_E
\]

\[
J_N = \frac{1}{\rho + p u, E + p E, N} [y - w_N + p E, N J_E]
\]

\[
J_N = \frac{1}{\rho + p u, E + p E, N} \left[ y - w_N + \frac{p E, N}{\rho + p u, E} [y - w_E] \right]
\]

we get

\[
\kappa v = \alpha s U u \left[ \frac{1}{\rho + p u, E} [y - w_E] \right] + \alpha s K k \left[ \frac{1}{\rho + p u, E + p E, N} \left[ y - w_N + \frac{p E, N}{\rho + p u, E} [y - w_E] \right] \right]
\]

\[
= \left[ \frac{\alpha s U u}{\rho + p u, E} + \frac{\alpha s K k}{\rho + p u, E + p E, N} \right] [y - w_E] + \alpha s K k \frac{1}{\rho + p u, E + p E, N} [y - w_N].
\]

From the flow equations we have

\[
e p u, E = \alpha s U u + p k, u u
\]

\[
n p u, E + p k, u u = \alpha s K k
\]
\[ \kappa v = \left[ \left( e_{\theta,\xi} - p_{K,U}u \right) \frac{1}{\rho + p_{u,e}} + \frac{\alpha_{K} k \rho_{E,N}}{\rho + p_{u,e} + p_{E,N} \rho + p_{u,e}} \right] [y - w_E] \]
\[ + \frac{p_{u,e} + p_{E,N}}{\rho + p_{u,e} + p_{E,N}} n [y - w_N] \]
\[ = \left[ e_{\theta,\xi} \frac{1}{\rho + p_{u,e}} - p_{K,U}u \frac{1}{\rho + p_{u,e}} + \frac{\alpha_{K} k \rho_{E,N}}{\rho + p_{u,e} + p_{E,N} \rho + p_{u,e}} \right] [y - w_E] \]
\[ + \frac{p_{u,e} + p_{E,N}}{\rho + p_{u,e} + p_{E,N}} n [y - w_N] \]
\[ = \left[ e_{\theta,\xi} \frac{1}{\rho + p_{u,e}} + \left[ \frac{\alpha_{K} k}{\rho + p_{u,e} + p_{E,N}} - n \right] \frac{p_{E,N}}{\rho + p_{u,e}} \right] [y - w_E] \]
\[ + \frac{p_{u,e} + p_{E,N}}{\rho + p_{u,e} + p_{E,N}} n [y - w_N] \]
\[ = \left[ e_{\theta,\xi} \frac{1}{\rho + p_{u,e}} + \left[ \frac{-\rho n}{\rho + p_{u,e} + p_{E,N}} \right] \frac{p_{E,N}}{\rho + p_{u,e}} \right] [y - w_E] + \frac{p_{u,e} + p_{E,N}}{\rho + p_{u,e} + p_{E,N}} n [y - w_N] . \]

Hence,

\[ \kappa v = e [y - w_E] + n [y - w_N] \quad \text{for} \quad \rho = 0. \]

3.F Business cycle version

This appendix describes the business cycle version of our model. The economy is assumed to shift between a good (G) and a bad (B) state, where a good state is characterized by a higher productivity level, i.e., \( y^G > y^B \). The transition rate from the good to the bad state is denoted by \( \pi^G \), whereas \( \pi^B \) denotes the transition rate from the bad to the good state. Hence, the expected duration of a good (bad) state is \( 1/\pi^G \) (\( 1/\pi^B \)). For simplicity we follow Ek (2012) and focus solely on steady states; i.e., the economy is assumed to jump directly between the two states.

The value functions in each state are

\[ \rho V_E = h \left( w_E [1 - \tau] + \Pi^i, 1 - l \right) + p_{u,e} \left[ V_E^0 - V_E^r \right] + \pi^i \left[ V_E^i - V_E^r \right] \]
\[ \rho V_N = h \left( w_N [1 - \tau] + \Pi^i, 1 - l \right) + p_{u,e} \left[ V_N^0 - V_N^r \right] + \pi^i \left[ V_N^i - V_N^r \right] \]
\[ \rho V_i^i = g \left( b_i^i + \Pi^i, 1 - s_i^i \right) + \alpha^i s_i^i \left[ V_E^0 - V_E^r \right] + p_{K,U} \left[ V_K^0 - V_K^r \right] + \pi^i \left[ V_i^r - V_i^r \right] \]
\[ \rho V_i^j = g \left( b_i^j + \Pi^i, 1 - s_i^j \right) + \alpha^i s_i^j \left[ V_N^0 - V_N^r \right] + \pi^i \left[ V_i^r - V_i^r \right] \]
with \( i, j = G, B \) and \( i \neq j \). Thus, the optimal search decisions imply
\[
\frac{\partial g (b_U^i + \Pi^i, 1 - s_U^i)}{\partial (1 - s_U^i)} = \alpha^i [V_E^i - V_U^i],
\]
\[
\frac{\partial g (b_K^i + \Pi^i, 1 - s_K^i)}{\partial (1 - s_K^i)} = \alpha^i [V_N^i - V_K^i],
\]
and aggregate search is given by
\[
s^i = s_U^i u^i + s_K^i k^i.
\]
The matching function is \( m (s^i, v^i) \), and therefore we get the following job finding rate and job filling rate
\[
\alpha^i = \frac{m (s^i, v^i)}{s^i} = m \left( 1, \theta^i \right),
\]
\[
q^i = \frac{m (s^i, v^i)}{v^i} = m \left( \left( \theta^i \right)^{-1}, 1 \right)
\]
where \( \theta^i \equiv \frac{v^i}{s^i} \) and hence \( \alpha^i = \alpha (\theta^i), \alpha' (\theta^i) > 0, q^i = q (\theta^i), q' (\theta^i) < 0 \).

Since we focus on steady states, the flow equilibrium conditions in each aggregate state are
\[
(1 - u^i - k^i - n^i) p_{U,E} = \alpha^i s_U^i u^i + p_{K,U}^i u^i
\]
\[
n^i p_{U,E} + p_{K,U}^i u^i = \alpha^i s_K^i k^i
\]
\[
\alpha^i s_K^i k^i = p_{U,E} n^i + p_{E,N}^i n^i
\]
for unemployment, social assistance and non-eligible jobs, respectively.

Firms post vacancies to find vacant workers, and they are not allowed to post different types of vacancies for UIB recipients and SA recipients. Thus, the value of a vacancy is expressed in terms of the expected value of a filled job
\[
\rho J_V^i = -\kappa + q^i \left[ J_{EXP}^i - J_V^i \right] + \pi^i \left[ J_V^i - J_V^i \right]
\]
where \( J_{EXP}^i \) is the expected value of a filled job. The free-entry-condition, \( J_V^i = J_V^i = 0 \), then implies
\[
J_{EXP}^i = \frac{\kappa}{q^i}.
\]
After a firm and a worker are matched, the firm knows whether the candidate is entitled to UIB, and therefore the wage will depend on the worker’s UIB eligibility.

The value of a job filled with an UIB eligible worker is
\[
\rho J_E^i = y^i - w_U^i + p_{U,E} \left[ J_V^i - J_E^i \right] + \pi^i \left[ J_E^i - J_E^i \right],
\]
whereas the value of a job filled with a non-eligible worker is
\[
\rho J_N^i = y^i - w_N^i + p_{U,E} \left[ J_V^i - J_N^i \right] + p_{E,N}^i \left[ J_E^i - J_N^i \right] + \pi^i \left[ J_N^i - J_N^i \right].
\]
The expected value of a filled job is then
\[ J_{i,\text{EXP}} = \frac{\alpha_i s_i^E u_i^E + \alpha_i^s s_i^K k_i^i J_{i,\text{EXP}}^i}{\alpha_i s_i^u u_i^u + \alpha_i^s s_i^K k_i^i} = \frac{s_i^E u_i^E + s_i^K k_i^i J_{i,\text{EXP}}^i}{s_i^u u_i^u + s_i^K k_i^i}, \]
and aggregate firm profits are
\[ \Pi_i^i = [y^i - w_{E}^i] \epsilon_i^i + [y^i - w_{N}^i] n^i - v^i \kappa. \]

Wages are determined through Nash bargaining, i.e.,
\[ w_{E}^i = \arg \max_{w_{E}^i} \left( V_i^E - V_i^U \right) \beta \left( J_i^E - J_i^V \right) \]
\[ w_{N}^i = \arg \max_{w_{N}^i} \left( V_i^N - V_i^K \right) \beta \left( J_i^N - J_i^V \right) \]
with the first-order conditions
\[ \beta \frac{\partial V_i^E}{\partial w_{E}^i} + (1 - \beta) \frac{\partial J_i^E}{\partial w_{E}^i} = 0 \]
\[ \beta \frac{\partial V_i^N}{\partial w_{N}^i} + (1 - \beta) \frac{\partial J_i^N}{\partial w_{N}^i} = 0. \]

The public budget is required to balance in expectation across states, i.e.,
\[ \pi_B^B \pi_B^G + \pi_B^G \pi_B^G \tau \left( w_{E}^G e^G + w_{N}^G n^G \right) - \left( b_{U}^G u^G + b_{K}^G k^G \right) \]
\[ \pi_B^B \pi_B^G \tau \left( w_{E}^B e^B + w_{N}^B n^B \right) - \left( b_{U}^B u^B + b_{K}^B k^B \right) = 0. \]

Finally, we define welfare as
\[ W = \frac{\pi_B^B}{\pi_B^G + \pi_B^G} W^G + \frac{\pi_B^G}{\pi_B^G + \pi_B^G} W^B \]
where \( W^i = e_i^i \rho V_{E}^i + n_i^i \rho V_{N}^i + u_i^i \rho V_{U}^i + k_i^i \rho V_{K}^i \) for \( i = G, B \).
3.G Tables

Table 3.16: Distortionary effects of the eight different policy instruments

<table>
<thead>
<tr>
<th>Instrument</th>
<th>$b_U^G$</th>
<th>$b_U^B$</th>
<th>$b_K^G$</th>
<th>$b_K^B$</th>
<th>$p_{K,U}^G$</th>
<th>$p_{K,U}^B$</th>
<th>$p_{E,N}^G$</th>
<th>$p_{E,N}^B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u^G$</td>
<td>0.125</td>
<td>0.005</td>
<td>0.131</td>
<td>0.009</td>
<td>-0.679</td>
<td>-0.003</td>
<td>0.197</td>
<td>0.001</td>
</tr>
<tr>
<td>$u^B$</td>
<td>0.005</td>
<td>0.122</td>
<td>0.007</td>
<td>0.139</td>
<td>-0.002</td>
<td>-0.745</td>
<td>0.001</td>
<td>0.215</td>
</tr>
<tr>
<td>$u^{MEAN}$</td>
<td>0.063</td>
<td>0.066</td>
<td>0.067</td>
<td>0.076</td>
<td>-0.329</td>
<td>-0.386</td>
<td>0.096</td>
<td>0.112</td>
</tr>
<tr>
<td>$k^G$</td>
<td>0.132</td>
<td>0.017</td>
<td>0.573</td>
<td>0.029</td>
<td>0.291</td>
<td>-0.008</td>
<td>-0.090</td>
<td>0.004</td>
</tr>
<tr>
<td>$k^B$</td>
<td>0.019</td>
<td>0.117</td>
<td>0.024</td>
<td>0.723</td>
<td>-0.008</td>
<td>0.233</td>
<td>0.003</td>
<td>-0.074</td>
</tr>
<tr>
<td>$k^{MEAN}$</td>
<td>0.067</td>
<td>0.074</td>
<td>0.259</td>
<td>0.426</td>
<td>0.120</td>
<td>0.130</td>
<td>-0.037</td>
<td>-0.041</td>
</tr>
<tr>
<td>$(u + k)^G$</td>
<td>0.129</td>
<td>0.012</td>
<td>0.372</td>
<td>0.020</td>
<td>-0.149</td>
<td>-0.006</td>
<td>0.040</td>
<td>0.003</td>
</tr>
<tr>
<td>$(u + k)^B$</td>
<td>0.013</td>
<td>0.119</td>
<td>0.017</td>
<td>0.490</td>
<td>-0.006</td>
<td>-0.156</td>
<td>0.002</td>
<td>0.041</td>
</tr>
<tr>
<td>$(u + k)^{MEAN}$</td>
<td>0.065</td>
<td>0.070</td>
<td>0.178</td>
<td>0.278</td>
<td>-0.070</td>
<td>-0.088</td>
<td>0.019</td>
<td>0.024</td>
</tr>
<tr>
<td>$s_U^G$</td>
<td>-0.327</td>
<td>-0.012</td>
<td>-0.365</td>
<td>-0.022</td>
<td>0.195</td>
<td>0.006</td>
<td>-0.035</td>
<td>-0.003</td>
</tr>
<tr>
<td>$s_U^B$</td>
<td>-0.018</td>
<td>-0.413</td>
<td>-0.023</td>
<td>-0.588</td>
<td>0.008</td>
<td>0.272</td>
<td>-0.004</td>
<td>-0.047</td>
</tr>
<tr>
<td>$s_K^{MEAN}$</td>
<td>-0.181</td>
<td>-0.201</td>
<td>-0.204</td>
<td>-0.289</td>
<td>0.107</td>
<td>0.131</td>
<td>-0.020</td>
<td>-0.024</td>
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<tr>
<td>$s_K^G$</td>
<td>0.045</td>
<td>-0.004</td>
<td>-0.305</td>
<td>-0.009</td>
<td>-0.027</td>
<td>0.002</td>
<td>0.017</td>
<td>-0.001</td>
</tr>
<tr>
<td>$s_K^B$</td>
<td>-0.006</td>
<td>0.053</td>
<td>-0.009</td>
<td>-0.408</td>
<td>0.002</td>
<td>-0.035</td>
<td>-0.001</td>
<td>0.022</td>
</tr>
<tr>
<td>$s^{MEAN}$</td>
<td>0.020</td>
<td>0.024</td>
<td>-0.160</td>
<td>-0.204</td>
<td>-0.012</td>
<td>-0.016</td>
<td>0.008</td>
<td>0.010</td>
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<tr>
<td>$s^G$</td>
<td>0.031</td>
<td>0.005</td>
<td>0.073</td>
<td>0.007</td>
<td>-0.024</td>
<td>-0.003</td>
<td>0.017</td>
<td>0.001</td>
</tr>
<tr>
<td>$s^B$</td>
<td>0.005</td>
<td>0.023</td>
<td>0.005</td>
<td>0.069</td>
<td>-0.002</td>
<td>-0.016</td>
<td>0.001</td>
<td>0.018</td>
</tr>
<tr>
<td>$s^{MEAN}$</td>
<td>0.017</td>
<td>0.015</td>
<td>0.037</td>
<td>0.041</td>
<td>-0.013</td>
<td>-0.010</td>
<td>0.008</td>
<td>0.010</td>
</tr>
</tbody>
</table>

Note: $x^{MEAN}$ denotes the unconditional mean of $x$, i.e., $\frac{x^H}{\pi^H}x^G + \frac{x^G}{\pi^G}x^B$.

Table 3.17: Optimal business cycle dependent UI scheme for different levels of risk aversion (the optimal uniform policy for each $\eta$ is index 100)

<table>
<thead>
<tr>
<th>$\eta$</th>
<th>-3.00</th>
<th>-0.75</th>
<th>-0.50</th>
<th>-0.25</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_U^G$</td>
<td>92.2</td>
<td>96.2</td>
<td>96.8</td>
<td>97.5</td>
</tr>
<tr>
<td>$b_U^B$</td>
<td>105.8</td>
<td>103.5</td>
<td>103.1</td>
<td>102.6</td>
</tr>
<tr>
<td>$b_K^G$</td>
<td>91.1</td>
<td>99.3</td>
<td>101.4</td>
<td>104.3</td>
</tr>
<tr>
<td>$b_K^B$</td>
<td>105.2</td>
<td>100.5</td>
<td>99.4</td>
<td>97.9</td>
</tr>
<tr>
<td>$p_{K,U}^G$</td>
<td>112.8</td>
<td>106.7</td>
<td>105.9</td>
<td>105.0</td>
</tr>
<tr>
<td>$p_{K,U}^B$</td>
<td>85.8</td>
<td>96.0</td>
<td>97.0</td>
<td>97.8</td>
</tr>
<tr>
<td>$p_{E,N}^G$</td>
<td>115.9</td>
<td>107.5</td>
<td>107.0</td>
<td>106.6</td>
</tr>
<tr>
<td>$p_{E,N}^B$</td>
<td>91.4</td>
<td>95.5</td>
<td>95.6</td>
<td>95.7</td>
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Table 3.18: Optimal business cycle dependent UIB duration and entitlement requirement when UIB and SA levels are fixed for different levels of risk aversion (the optimal uniform policy for each $\eta$ is index 100)

<table>
<thead>
<tr>
<th>$\eta$</th>
<th>$p_{K,U}^G$</th>
<th>$p_{K,U}^B$</th>
<th>$p_{E,N}^G$</th>
<th>$p_{E,N}^B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3.00</td>
<td>148.2</td>
<td>71.6</td>
<td>123.2</td>
<td>94.0</td>
</tr>
<tr>
<td>-0.75</td>
<td>111.8</td>
<td>92.8</td>
<td>106.4</td>
<td>96.4</td>
</tr>
<tr>
<td>-0.50</td>
<td>107.8</td>
<td>95.1</td>
<td>105.6</td>
<td>96.5</td>
</tr>
<tr>
<td>-0.25</td>
<td>104.1</td>
<td>97.3</td>
<td>104.9</td>
<td>96.7</td>
</tr>
</tbody>
</table>

Table 3.19: Optimal business cycle dependent UIB duration when the other three instruments are invariant for different levels of risk aversion (the optimal uniform policy for each $\eta$ is index 100)

<table>
<thead>
<tr>
<th>$\eta$</th>
<th>$p_{K,U}^G$</th>
<th>$p_{K,U}^B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3.00</td>
<td>142.0</td>
<td>72.6</td>
</tr>
<tr>
<td>-0.75</td>
<td>110.1</td>
<td>93.6</td>
</tr>
<tr>
<td>-0.50</td>
<td>106.4</td>
<td>95.9</td>
</tr>
<tr>
<td>-0.25</td>
<td>103.0</td>
<td>98.1</td>
</tr>
</tbody>
</table>

Table 3.20: Optimal business cycle dependent entitlement requirement when the other three instruments are invariant for different levels of risk aversion (the optimal uniform policy for each $\eta$ is index 100)

<table>
<thead>
<tr>
<th>$\eta$</th>
<th>$p_{E,N}^G$</th>
<th>$p_{E,N}^B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3.00</td>
<td>71.6</td>
<td>131.4</td>
</tr>
<tr>
<td>-0.75</td>
<td>94.8</td>
<td>104.0</td>
</tr>
<tr>
<td>-0.50</td>
<td>97.7</td>
<td>101.7</td>
</tr>
<tr>
<td>-0.25</td>
<td>100.6</td>
<td>99.5</td>
</tr>
</tbody>
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<th>Title</th>
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<td>Essays on Economic Policies over the Business Cycle</td>
<td>Mark Strøm Kristoffersen</td>
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