TWO ASPECTS OF ICT IN MATHEMATICAL ACTIVITY: TOOL AND MEDIA.

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This paper investigates two different approaches to artefacts that supports mathematical activity and leaning: the instrumental approach, that are concerned with the way that artefacts are made into personal instruments and the semiotic approach that is concerned with the way semiotic representations are crucial to mathematical activity. The motivation for considering these two views on artefacts that supports mathematical activity is that the computational technology used to support mathematical work is often dependent on various forms of representations, and some types of representations are often used in mathematical activities as a type of tool, that has to be learned and mastered, and which significantly affects solution techniques.

INTRODUCTION, SCOPE AND PURPOSE

In this paper I attempt to give a brief and general background for the importance of artefacts to mathematical/cognitive activities, describe why the semiotic aspects of such artefacts are important, and why the basic semiotic entities (such as sign and medium) are an important class of artefacts, in relation to mathematical activities. Furthermore I describe two theories from mathematics education: (1) Duval’s theory of the role of semiotic representations in mathematics education and (2) Truche’s instrumental approach. The purpose of comparing these two theories is to acknowledge the instrumental aspect of representations in mathematics education as well as the semiotic aspects of (computational) tools. Using these two frames I discuss two examples of mathematical activity, researchers mathematical writing and e-learning at university level.

A FUNDAMENTAL DIALECTICS

It is an important aspect of human actions that they rely on interplay between internal processes only perceivable by the individual and external actions that are also observable by others.

In a Piagetian psychological view the dialectic is expressed in an attempt to define cognition as an adaptive function developed from – and tested against – empirical reality through actions (Glasersfeld 1995, p. 59).

In a sociocultural tradition the concept of mediated activity designates how tools and signs influence, and support human activity (Vygotskij, 1978, p. 51). And using Actor Network Theory Shaffer and Clinton (2006) introduces the concept toolforthought to hieglight that “there are no tools without thinking, and there is no thinking without tools” (p. 291).
Semiotic artefacts such as text, diagrams, tables, etc. are important in a lot of knowledge work, also in mathematics. Signs and representations on paper can be used to support human thinking.

The study of human activity is deeply connected to the study of how humans interpret and manipulate their environment. In relation to mathematical activity there are at least two important types of toolforthoughts and two theories to describe each of their use in mathematical activity. The two issues that are concerned here are; semiotic representations (signs, drawings, diagrams etc.), and the use of tools (calculators, cas tools etc.).

The instrumental approach

The basic premise of the instrumental approach is that people use artefact for various purposes and hereby create personal instruments. More precisely Luc Trouche defines an artefact as a material or abstract object, aiming to sustain human activity. An instrument is what is what the subject builds from the artefact. (Trouche, 2005, p. 144). The process of building an instrument from an artefact is referred to as instrumental genesis and consists of two processes, instrumentation and instrumentalisation. Instrumentalisation is directed by the subject, towards the artefact. Stages describing this process are discovery, selection, personalisation and transformation. Where the first functions are a matter of getting to know the tool the latter tends more to be a matter of mastering the tool and applying it to ones own very specific needs. Instrumentation is directed by the artefact towards the subject. It is the process in which the subject adapts to the new possibilities and constraints the tools posses. In the instrumentation process it is the tool that shapes the behaviour of the subject confronted with a specific type of tasks.

The semiotic approach

Mathematical objects are typically not there to be pointed at as anything but representations, sometimes even representations of a very technical nature and furthermore mathematical objects have always more than one semiotic representation attached to it (Duval, 2006). These two facts leads to two fundamental issues in learning mathematics; (1) it is a common mistake to identify the mathematical object as being one of its representations, and (2) transformation of semiotic representations can be difficult but a lot of the creative potential in mathematics stem from transformations of semiotic representations (e.g. calculations) (Duval 2006).

Duval shows that changing type of representation can sometimes be very difficult for students, especially if this change of representational form does not have a recipe for translating parts of representation in the starting register to parts of the representation in the target register. One example of a problematic change in representational form is from a plot of a function to an algebraic formulae, whereas the other way (from formulae to graph is conceptually simple since creating a table of \((x,f(x))\) values in principle constitute a recipe.
The consequences of this framework is that conceptual understanding can be described as a person’s degree of freedom towards various semiotic representations of the same mathematical concept (Winsløw 2003), a number of learning difficulties and misconceptions in relation to mathematics is can be ascribed to the problem of misunderstanding one representation for the object, and that some semiotic transformations are very difficult for students.

**TWO EMPIRICAL EXAMPLES**

Representations and media can be viewed as a type of tools in relation to mathematical activity. Below I show two empirical examples, research mathematicians use of various media to support their writing process and elearning based, teaching of undergraduate mathematics.

**Mathematical Writing**

I have conducted an interview study among professional mathematicians (Misfeldt, 2006). The objective of the investigation was to understand the writing process of professional mathematicians from early idea to finished paper. In particular, I compared the purposes that writing served to the mathematician to the types of representations he/she used and the media (computer or pen and paper) he/she chooses to use.

The result of the investigation suggests that it makes sense to consider the following five different functions of writing in mathematics: Heuristic treatment, control treatment, information storage, communication, and production of a paper (Misfeldt 2006).

Two observations are important: The respondents showed a strong tendency to use visual and diagrammatic types of semiotic representations to support the functions of heuristic treatment, and to some extent control treatment.

Pen and paper was dominant in the respondents’ choice of medium to support heuristic and control treatment, and for some of the respondents also to support information storage.

**E-learning in mathematics: remediation of a flexible medium.**

The initiative Delta, matematik på nett, at the Norwegian technical university, consists of eight courses in undergraduate mathematics, taught fully virtual. The topics includes: Calculus, Linear algebra, geometry, number theory, probability and statistics. The main platform for the teaching is the learning management system “it’s learning”. The course material consists of syllabus, short video lectures and mandatory exercises.

The communication between teacher and learners on the written assignments are mediated by handwriting. The students are required to buy a scanner and scan their handwritten manuscripts in order to upload them to the learning management system.
where it is reviewed and commented by the teacher, using an electronic pen or printout (Misfeldt & Sanne 2007). From the teachers experience this way of communication best meet their needs. The learning management system provides a discussion forum, for the student, here normal keyboard generated text is used to discuss mathematics. The support for mathematical notation in the discussion forum is by the students considered insufficient so they either use basic keyboard notation (such as \( f(x) = (3+x)/(2-x) \)) or tries to cut and paste formulas from other applications (Misfeldt and Sanne 2007).

**DISCUSSION**

This discussion is organised around three themes (1) instrumental aspects of semiotic representations, and (2) instrumental aspects of media. Semiotic aspects of mathematical instruments will, despite of its importance, not be treated in this paper.

**Instrumental aspects of semiotic representations**

We can view semiotic representations as a specific class of artefacts that can be turned into instruments by a process of instrumental genesis. The framework of Duval (2006) highlights that transformation of semiotic representations is an example of an instrumented technique applied to solve a mathematical problem. In this case the instrument is the semiotic representations, and the development of such instrumented techniques can potentially lead to learning problems due to mistake one of the representations of as a mathematical object as the object itself. Furthermore if the formation of viable mathematical concepts are connected to the ability to coordinate various representations of the same concept without favouring one of them, it is a relevant hypothesis that broadening out the number of representations of a concept acts as a way of improving conceptual understanding.

In the example with mathematicians it is clear that some of the more visual and diagrammatic uses of representations belongs more in the creative phase of mathematical work than in the communicating phase. This can be seen as an indication that these visual representations are used as mathematical instruments in a problem solving process.

**Instrumental aspects of media**

In the mathematical e-learning project described we see how the dependence on various forms of semiotic representations affects the choice of media. They use handwritten drafts scanned to pdf files even though this is an unusual way to communicate electronically. This remediation can be seen as a pragmatic attempt to make the computer support handwriting, and in that sense it is a process of instrumentalisation, where the computer medium is controlled by the mathematical user and changed in order to fit his/hers needs. The cognitive instrument that is
developed through the instrumental genesis is in many ways similar to handwriting. This result is interesting from an instrumental point of view, because one plausible reason for developing this instrument consisting of remediated handwriting, is that handwriting has some properties that allows it to serve as a mathematical instrument. That handwriting has some properties that allows it to serve as a mathematical instrument, is also backed up by the interviews with mathematicians. The mathematicians show a clear movement from handwriting in the early/creative part of the work, towards the use of computer for information storage and communication. Again this implies that there are some aspects with pen and paper that allows it to develop into a mathematical instrument.

REFERENCES