Fama and French versus Behavioralists

Tests of the CAPM and the three-factor model for the Spanish stock market

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INTRODUCTION

Efficient markets, which Fama (1970) describes as markets where prices always “fully reflect” all the available information, has been discussed through a large number of empirical studies trying to determine whether particular markets are in fact efficient and if so to which extend.

Market efficiency cannot be tested by itself, it is necessary to use a model of equilibrium, an asset-pricing model (Fama 1991). The most widely accepted risk-return equilibrium model in the last forty years has been the Capital Assets Pricing Model (CAPM) of William Sharpe (1964) and John Lintner (1965). However, recent evidence has presented CAPM “anomalies”, that is, returns that cannot be explained by the market betas. These anomalies can be interpreted as an indication of market inefficiencies, or alternatively as shortcomings in the underlying asset pricing model. This controversy is the so-called joint hypothesis problem. (Dimson and Mussavian, 2000)

Several researches have tried to predict future prices using past prices. These studies are called “technical analysis” and are based in statistical methods. Most of these studies argue that any active trading strategy is profitable. Moreover, other studies have presented strategies based on stock market anomalies that could outperform passive or buy-and-hold strategies, but once that transactions cost are included, profits are eroded.

The Efficient Markets Hypothesis (EMH), formulated at the University of Chicago, has been one of the fundamental propositions in finance in last century. The term efficiency is used to describe a market where relevant information is reflected into the price of financial assets (Fama 1970), with the prerequisite that “information and trading cost, the cost of getting prices to reflect information, are always 0” (Fama 1991) Economists in the other hand, use this concept to denote operational efficiency, describing the way resources are employed to facilitate market operations. The EMH “rules out the possibility of any trading systems based only on currently available information that have expected profits or returns in excess of equilibrium expected profit or return” (Fama,
A different view of the efficiency hypothesis is given by Jensen (1978), he states that ‘prices reflect information to the point where the marginal benefits of acting on information do not exceed the marginal costs’. The extreme version of market efficiency as Fama (1991) suggests is probably false, but it provides a useful benchmark without problems of estimations and inclusion of information and trading costs.

However, the paradox of the EMH is that if all the investors think that the market is efficient, no one would study the behavior of the securities and the market would not be efficient. Actually, EMH relies on investors who try to outperform the market, thinking that they buy stocks that are worth more than the price and sell stocks that are worth less than the price. In fact, stock markets are neither entirely efficient nor completely inefficient.

In this paper it is presented a theoretical discussion of how CAPM “anomalies” both for serial and cross-section regressions are consistent with market efficiency from the point of view of rationalist like Fama against the behavioralists’ theories. Behavioral finance is the field which tries to explain how emotions and cognitive errors influence investors in their choices. Behavioral finance is based in two main ideas, limits to arbitrage and investor’s psychology.

In order to capture the anomalies not explained by the CAMP, Fama and French made an extension model of the CAMP. The so-called three-factor model based on Ross’s (1976) Arbitrage pricing theory (APT) and in Merton’s (1973) Intertemporal Capital Asset Pricing Model (ICAMP). They demonstrate that the three-factor risk-return relation has a higher explanatory power for the returns on portfolios formed on size and book-to-market equity. Moreover it captures the reversal of long-term returns presented by DeBondt and Thaler (1985).

One of the critics that Fama and French have received in some of their papers is that their findings are consequence of “data mining” as it is discussed afterwards. One simple way to verify this critic consists into replicate the model with different data set, that is, out-of-
the-sample tests. Fama and French (1998) perform an out-of-the-sample research for value and growth stocks in markets like Japan, France, Belgium, Sweden, etc. (Fama and French, 1998). They reach similar results in the value premium than in the US market. In this research Fama and French use the data of the non-US markets from the Morgan Stanley's Capital International Perspectives (MSCI) database, which contains only a data subset of the companies in some countries, mainly on those where Stanley and Morgan operates. The problem is that most of the MSCI data is for large companies, therefore the database does not allow for significant tests for the size effect, (Banz, 1981). This kind of data it is not easy to collect, therefore Fama and French did not make one out-of-the-sample test for the three-factor model and just realize a two-factor model, missing one of the variables of their original model.

In this paper it is tested the CAPM and the three-factor model of Fama and French for the Spanish continuous market (my home country). Concretely for the IBEX 35, the most representative stock index in Spain

This paper is organized as follows: In section I it is presented a literature review of the efficient market hypothesis both for theoretical and empirical fields, a review of the random-walk literature and of CAPM anomalies. Moreover it is presented an introduction to the behavioral theories and models and a confrontation between rational and irrational theories for CAMP anomalies. In section II it is presented a review of the main CAPM and ‘three-factor model” assumptions and hypotheses with a theoretical approximation of the models. In section III a derivation of both models is presented. In section IV it is explained the methodology used to test the models, using time-series and cross-section regressions, according to previous studies methodology. In section V the database is described and analyzed and a brief description of the IBEX 35 is done. In section VI, results are presented and discussed. Finally in section VII shortcomings of the models, methodology and database are discussed.

It will be seen in section V that there are a value and a size premium in the Spanish stock market of 7.52% and 3.26% respectively. Moreover the distribution of the returns is
studied. In section VI it is found that for time-series regressions the CAPM and the three-factor model have an acceptable performance, having the later a higher explanatory power. In the CAPM the market explains 50.55% of the variability of the returns with and equally market portfolio, while in the three-factor model 57.73% of the variability of the returns is explained by the explanatory variables.

It can be argued that this difference is not too big and that the costs associated to perform the Fama and French model are not worth. This reason could explain why the three factor model is not broadly used in practice. Moreover it will be seen in the cross-section regressions that the capacity of betas to explain the average returns is poor for both models. The CAPM betas are only able to explain 6.16% of the average returns, while the three-factor model betas explain 10.91% of the average returns using monthly returns. However it was estimated that using yearly returns in the three-factor model this value increases until 35.91%, but the confidence in this result is low due to the short sample length. This low relation between the average returns and the betas can be consequence of the difference between the betas that has been estimated in this paper and the betas that the investors really use. Furthermore, these two models assume strict assumptions like investors’ rationality, homogenous expectations, etc. Behavioral finance has demonstrated that investors are affected by several biases, so these results are not surprising.

Nevertheless any rational or irrational model has outperformed clearly the CAPM, and although several authors have documented anomalies it is still widely used.

The three-factor model applied for the Spanish stock market is able to explain average returns better that the CAPM, especially in the time-series regression the SMB factor seems to be significant. However in the cross-section regressions, for monthly returns they do not seem able to explain average returns. However with yearly returns the explanatory power of the SMB and HML factors seems to be better, however the inferences are not clear. A longer data set would be needed for stronger inferences.
Literature mainly suggests three forms of market efficiency: weak-form, semi-strong form, and strong form. Strong form will not be discussed in the theoretical part, evidence against it is large, and most of the tests of market efficiency are based on the weak and semi-strong form.

My interest for this area arises from 2 courses that I had in previous semesters, one at the ASB, “Advance Corporate Finance”, and other in my exchange period at the University of Maastricht, “Behavioral Finance”. In these courses different theories of market efficiency, event studies and behavioral models were studied. Diverse hypothesis of market efficiency/inefficiency were presented, starting from the Efficient Market Hypothesis (EMH) until behavioral theories.
Section I. Literature Survey

In the 60’s the EMH had a vast theoretical and empirical success. Academics developed theoretical reasons of why the hypothesis should hold mainly based on the empirical evidence. In 1978, Michael Jensen stated that ‘there is no other proposition in economics which has more solid empirical evidence supporting it than the Efficient Markets Hypothesis’.

However in the last 20-30 years both theoretical and empirical basis of the EMH have been challenged by rational and behavioral approaches. First, remark that traditional rational tests of market efficiency assume that expected returns are constant through time (Fama, 1991) and that prices changes are random and unpredictable (Fama and French 1970). Moreover models to test efficiency such the Sharpe-Lintner CAPM, assumes normally distributed returns. However, several studies suggest that returns do not follow a random walk (Lo and Mackinlay, 1988). Fama (1991) accepts that returns are to some extend predictable from past returns, and that the new tests reject the old market efficiency-constant expected returns, but argues that this is not a departure from the EMH. Fama and French (1989) argue that ‘if variation in expected returns is common to different securities, then it is probably a rational result in tasted for current versus future consumption or investment opportunities of firms”. However this controversy gives place again to the joint hypothesis problem. Researches discuss if this predictability is consequence of variations through time in expected returns or irrational deviations (bubbles) (Fama, 1991).

Second, from the behavioral point of view, an assumption such as arbitrage seems to be more limited than the EMH expects. With these new theories and evidence, behavioral finance has risen as a different view for financial markets, where significant and systematic deviations of prices from the fundamental value are expected to continue for long time intervals.
Theoretical framework of the EMH

The EMH is based on three arguments which depend on progressively weaker assumptions. First, investors are supposed to be rational, therefore they are expected to worth securities rationally. Second, in the case that some investors are not rational, their trades are independent so they cancel each other without influence the prices. Third, in the case that those irrational investors have correlated trades, they are eliminated by rational arbitrageurs who eliminate their influence on prices and return the stock prices to their fundamental value, that is, ‘the net present value of its future cash flows, discounted using their risk characteristics’. (Shleifer, 2000)

Investors quickly increase the prices when there are good news and vice versa. All the available information is incorporate in stock prices almost instantaneously and security prices reflect the new net present value of cash flows

It is important to point out that EMH does not depend if all investors are rational or not (Shleifer, 2000). If some investors are not fully rational and even if their trading strategies are correlated, markets are still expected to be efficient due to the arbitrageurs.

Arbitrage is ‘the simultaneous purchase and sale of the same, or essentially similar, security in two different markets at advantageously different prices” (Shleifer, 2000). Arbitrage also implies that irrational investors buy overpriced securities and sell underpriced securities. Therefore they earn lower returns than arbitrageurs and passive traders. EMH argues that they can not keep losing money forever and therefore they will disappear from the market. Competition between arbitrageurs guarantees the adjustment of prices to fundamental values very quickly.
Random Walk

The random walk hypothesis and the efficient market hypothesis have been largely linked in previous literature. A vast financial literature turned around the random walk hypothesis and the martingale model, two statistical theories of unpredictability stock price changes that were assumed to be a repercussion of efficient markets (Lo, 2000) which now it is demonstrated to be false.

In previous literature several researches have argued that returns have random fluctuations (Kendall, 1953). Kendall showed that the chart of the distribution of five years of the Standard and Poor’s Index and the chart of a coin flipped several times were indistinguishable (Brealey and Myers, 2000). This near-zero correlation in returns was an approach contradictory with the economists point of view. This empirical evidence was so-called “random walk theory”.

Samuelson (1965) presents evidence where stock prices follow a random walk in competitive markets with rational risk-neutral investors. However, in markets with risk-averse investors, with both varying levels of risk through the time and varying acceptance toward risk, security prices do not follow a random walk (Shleifer, 2000), but investors’ rationality still avoids the possibility of earn superior risk-adjusted returns (Fama, 1991). Independently from whether or not financial markets are efficient, one of the main discussion points of finance is whether financial asset price changes are predictable (Lo, 2000).

Even though evidence at the middle of the XX century of the randomness of stock price changes was large, there were some researches who presented anomalies in some price behavior, which seemed to follow predictable paths, (Cowels and Jones, 1937; and Kendall, 1953). However, Working (1960) suggests that the autocorrelation in return series was produced due to the use of time-averaged stock prices. By using returns series based on end-of-period prices, returns seem to follow a random walk.
Strong evidence supporting the random walk hypothesis is published by Fama (1965) who study the distribution and serial dependence of stock market returns. The random walk theory emphasizes that stock prices changes will not follow any pattern or trend and that past prices changes cannot be used to forecast future changes. Fama (1965) states that the logarithm of the stock prices do follow a random walk described as follows:

\[ \log P_t = \log P_{t-1} + \varepsilon_t \quad (1) \]

where \( \varepsilon_t \) is a random variable, Normally, Independent and Identically Distributed (i.i.d). Fama (1965) gives three reasons to use log prices rather than price changes:

1. “The change in log prices is the yield, with continuous compounding.”
2. “The variability of simple price changes for a given stock is an increasing function of the price level of the stock. Using log prices the price effect seems to be neutralized”.
3. “For change less that ±15% the change in log price is very close to the percentage price change, and for many purposes is it convenient to look at the data in terms of percentage price changes”.

Traditionally, for test of market efficiency, returns are used instead of stock prices. There are two alternative ways to calculate stock returns:

- Discrete returns, calculated applying the following equation,

\[ R_t = (P_t - P_{t-1}) / P_{t-1} \quad (2) \]
• Compounded return, define as the logarithm of the returns,

\[ R_t = \log(P_t / P_{t-1}) \] \hspace{1cm} (3)

so,

\[ R_t = \log P_t - \log P_{t-1} = \epsilon_t \] \hspace{1cm} (4)

Analogous to small changes in prices, when the returns are small the result between discrete and continuous returns is similar. Moreover, the advantage of calculating the returns in continuous time is that there will be a proportional distribution around 0, which is not the case using discrete time.

To test market efficiency, concretely the assumption of not predictability from past returns, it can be used a regression of the actual period against past periods as follows:

\[ R_t = \mu + \beta_1 R_{t-1} + \beta_2 R_{t-2} + \ldots + \epsilon_t \quad \text{with } \epsilon_t \sim \mathcal{N}(0, \sigma^2) \] \hspace{1cm} (5)

In regression (5), the error term is assumed to be normally distributed with zero mean and constant, finite variance, \( R_t \sim \mathcal{N}(\mu, \sigma^2) \). The null-hypothesis of market efficiency is described as: \( H_0 : \beta_i = 0 \)

If the samples are not normally distributed, rationalists argue that could be consequence of a change in investors’ expected returns or in the time varying variances and not a violation of the EMH (Cuthberson, 1997).

An alternative way to check the random walk hypothesis has been the autoregressive models, AR(1), where the current price of one asset depends of the previous one. This is described as follows,

\[ \log P_t = a \log P_{t-1} + \epsilon_t \] \hspace{1cm} (6)
This approach is used by Shiller (1981) or in similar models such the “fads” of Poterba and Summers (1988). This methodology consists into check if some statistical features of the series observed are consistent with the random walk hypothesis. For example, it is common to analyze if the autocorrelation coefficients are statistically different from 0 and if the variance has the expected growth as the time horizon increases (Lo, 2000). For example, Lo and MacKinlay (1988) using this approach reject the random walk hypothesis for a equal-weighted portfolio formed with weekly returns from 1962 to 1985 on the NYSE, showing an autocorrelation coefficient of first-order of 0.30, which means that around 9% of the variability of next week’s returns are predicted by this week’s return. Making a portfolio for “small” companies, the first order autocorrelation coefficient raises up to 0.42, which explain 18% of the variability. This evidence presents a violation of random fluctuation of stock prices showing returns predictability. However, there are still random fluctuations in the returns, so riskless profitable trading strategies are not available. In other words, the potential profitable arbitrage is likely to involve substantial risk. (Cuthberson, 1997)

Lo (2000) states that two other empirical facts are included in the random puzzle. Evidence presents that weekly portfolio returns have positive autocorrelation, but individual stocks generally are not correlated, actually the average of the autocorrelation across individual stocks is negative and insignificant. In the other hand, predictability of returns depends on the estimation period. Serial dependence for daily and weekly returns is significant and positive but for monthly or longer returns is close to zero.

For longer periods, Fama and French (1988) and Poterba and Summers (1988), using a period of three to five years, find negative serial correlation but not strong enough to reject the random walk hypothesis with statistical significance.

Lo (1991), based on the central limit theory makes a test for long-term memory compatible with the short-term correlation presented before (see, Lo and MacKinlay 1988) and suggests that there is little evidence for long-term memory in stock prices. Rejections of the random walk hypothesis can be explained by the short-term autocorrelation for time-series. However the new data and the features of time series are
unlikely to be stationary in the long term (Lo, 2000). This is a big area of research where different approaches and methodologies are used to explain the predictability of returns.

In the other hand, traditional techniques and studies which confirmed the efficiency hypothesis had a linear character, therefore unable to detect non-linear structures even if they exist.

However, contrary to the random walk literature, which is based on the conditional distribution of returns, a different trend has centered on the marginal distribution of returns, and the concept of stability, ‘the preservation of the parametric form of the marginal distribution under addition’(Lo, 2000). This represents an important feature. Lo (2000) states that “stocks returns are summed over various holding periods to yield cumulative investments returns”. That is, if $P_t$ is equal to the end-of-month stock price, $R_t$ it is the monthly compounded return defined as $\log \frac{P_t}{P_{t-1}}$, therefore its annual return is

\[
\log \frac{P_t}{P_{t-12}} = R_t + R_{t-1} + \ldots + R_{t-11}.
\]

The normal distribution belongs to the stable distributions, but non-normal stable distributions have different characteristics that the normal distribution does not have such leptokurtosis (fat tails), which is usually present in daily and weekly returns (Lo, 2000). Fama (1970) states that stock returns are better approximated by non-normal stable distribution. However, Fama and Macbeth (1973) show that assuming normality in the tests, the errors in the inferences are not serious if the values are not extreme, so normality it is quite often assumed for simplicity.

Remark that the Efficient Market Hypothesis and the Random Walk Hypothesis are two different concepts. LeRoy (1973) and Lucas (1978) state that ‘neither one necessary nor sufficient for the other, due to the necessity of some trade off between risk and expected return’. The random character of stock market prices based on LeRoy and Lucas agree with the idea that it is necessary some trade-off between risk and expected return. They suggest that if the stock’s expected price change is positive, it may be the premium
necessary for the investor to hold the security and bear the risk, if the investor is risk-averse, he/she would pay this premium cost if in this way he/she can avoid unpredictable returns. In this context, prices do not have to follow a random walk, even if the market is perfectly efficient and rational.

**Empirical evidence of EMH**

All these strong theoretical arguments have of course empirical evidence. In general terms, EMH empirical evidence can be classified in two broad groups, both of them created with the intention of testing whether the information is properly incorporated into stock prices using an asset-pricing model which defines clearly “properly” (Fama, 1991). First, when new news arises they have to be incorporated both quickly and correctly. “Quickly” means that those who receive the information “late” cannot profit from it, and “correctly” means that the adjustment has to be accurate. Security prices should neither underreact nor overreact to new information.

Second, since the prices must be the same that their fundamental value, prices should not change if there is not any news that affect to their fundamental value, i.e., non-reaction to non-information. The consequence of the evidence of these two groups is that stale information is of no value in making money (Fama, 1970). “Making money” can be interpreted like obtain a superior return after a risk-adjustment, showing therefore, evidence of market inefficiency.

There is a big difficulty by measuring the risk because it is necessary a model with a relationship between risk and return. There is a widely recognized and commonly used model for this complex task, the Capital Assets Pricing Model (Sharpe, 1964). Fama and French extended this model based on Ross's Arbitrage pricing theory (APT) and in Merton's (1973) Intertemporal Capital Asset Pricing Model (ICAMP) in order to capture the returns not explained by the market betas.
In general, when researches have presented any trading strategy on stale information which leads to an abnormal profit, critics quickly proposed a model which reduced the abnormal profit to a compensation for the risk involved.

From the term “stale information” there is a widely-accepted classification in three groups, which turns out three forms of EMH: weak, semi-strong and strong form. First, the “weak form” of the EMH claims that prices fully reflect the information implicit in the series of past prices and returns. In this way, it is not possible to earn superior risk-adjusted profits just following any trading strategy based on past prices and returns. Traditionally, expected returns were assumed constant through time and the best estimation of a return was its historical mean. It is important to remark that this form of EMH when it assumes that investors are risk-neutral supports the unpredictability of stock returns based on past returns into the random walk hypothesis (Fama, 1965).

Early tests suggested that daily, weekly and monthly returns are predictable, but with a low statistical significance, therefore market efficiency and constant expected returns were accepted. (Fama, 1991)

Fama (1991) in his review of efficient capital markets, rename these tests as ‘test for returns predictability”, which also include the predictability influence of variables such as dividend yields (D/P), earnings/price ratios (E/P), and term-structure variables. He also incorporates studies for long-term periods. In these new researches the predictable component of variances in daily, weekly and monthly returns is still small, but it even rise until 40% of the variance for periods of 2 to 10 years. Again the discussion is if these results have a rational explanation (large rational swings in expected returns) or an irrational one (irrational bubbles).

Recent studies reject the constant expected returns assumption, however confirm that “in individual stocks variation in daily and weekly expected returns is a small part of the variance of returns” Fama (1991, pp 1580).

More controversy is the predictability of long-horizon returns. In early tests, Fama (1991) states that the early literature found that the autocorrelation in daily and weekly
returns deviates from 0, nevertheless it was not economically significant. However, this argument is challenged by Shiller (1984) and Summers (1986). They show how stocks prices with low short-term autocorrelation have “large slowly decaying swings away from fundamental value (fads or irrational bubbles)” (Fama 1991). This evidence seemed prominent but the tests produced poor results. Fama and French (1988a) replicate the Shiller-Summers model for a sample between 1926-1985. They found autocorrelation close to 0 at short-term and negative and significant autocorrelation for 3 to 5 years, moving back toward 0 for longer return horizons. These results were coherent with the hypothesis that stock returns have a slowly decaying stationary component. As Fama and French point out “autocorrelation may reflect market inefficiency or time-varying equilibrium expected returns generated by rational investor behavior”. However when they excluded from the sample the 1926-1940 period the long-term autocorrelation disappear. They argue that the autocorrelation in the whole sample was a consequence of the Great Depression, or that perhaps “stock prices no longer have such temporary components.”

To conclude say that nowadays these attempts to forecast stock prices by using past prices and related statistics are so-called “Technical Analysis”. Most of them have showed that stock prices are adjusted quickly to new information and that active trading strategies do not provide any advantage while other researches argue that some technical strategies outperform the buy-and-hold strategy.

The semi-strong form states that prices reflect all relevant information that is publicly available (e.g. earnings announcements, stock splits, etc). Therefore investors cannot obtain a superior abnormal profit after a risk-adjustment based on publicly available information. This means that as soon as information is revealed, it is directly included into stock prices, so investors cannot obtain any advantage by using the information to predict returns.

Tests of the semi-strong form try to measure the speed which new information is reflected in stock prices. The methodology used consists into look at particular new
events, such as stock splits, earning and dividend announcements, takeovers, etc, that belong to a particular company and analyze whether prices are altered due to these news instantaneously or over a period of a few days, months, years etc.

This methodology, the so-called “event study”, is used by Fama, Fisher, Jensen and Roll (1969), but the first event study was made by Ball and Brown (1968). An event study makes an average of the cumulative performance of stocks over a particular period before and after the event. Then the performance for each is security is measured after realization and adjustment for market-wide movements in stock (Dimson and Mussavian, 2000)

Fama et al (1969) using the CAMP as benchmark, show that these event studies present evidence on the reaction of stocks prices for stock splits and earning announcements. In these two cases, the market seems to anticipate to the news, and the information is rapidly and accurately incorporated before the event is revealed to the market. Prices are adjusted “quickly” and “accurately” to new information.

Finally there is the possibility that investors could make a superior abnormal profit if they trade with information that is still not available for the rest of investors; this kind of information is called inside information. The strong form states that even with inside information it is impossible to make a superior abnormal profit, information that is known to any investor is reflected in market prices immediately. There is large evidence against the strong form. Remark that most of the literature deals with the weak and semi-strong form of EMH.

To sum up say that the EMH was a successful standard economic theory at the end of the 1970’s. Especially, with the theory of arbitrage it was ensured that financial markets were efficient. Evidence was so large, and the small anomalies founded were reduced with the explanation of the failure of properly risk adjustments.
Theoretical challenges to the EMH and a behavioral framework

A few years after of its origin, the EMH started to be confronted both on the theoretical and empirical ground. The initial challenges were empirical and then some potential anomalies in the theoretical field were converted in evidence.

Traditionally when it is said that investors are rational, means two things. First, it means that when investors receive new information they update the beliefs properly, following Bayes’ law. Second, given these new beliefs they make their decisions in a normally acceptable way consistent with Savage’s Subjective Expected Utility (SEU) when the probabilities of the outcomes are unknown (which is the case of the stock markets) or with the Expected Utility (Von Neumann and Morgenstern, 1944) when the probabilities are known a priori. These two ideas were widely accepted for a long time, but behavioralists started to present theories and evidence against them. They argue that the basic characteristics of the aggregate stock market, the cross-section of average returns and individual trading strategies cannot be easily explained with these approaches. Behavioral finance arises as an alternative field for the anomalies not explained by the traditional theories using models where investors are not fully rational, and where investors trade on noise rather than information. Behavioralists form these models ‘relaxing’ one or both of the two approaches stated above. Therefore in some models investors do not update their beliefs properly and in others their choices are incompatible with SEU.

As it is stated before rationalists that defend the EMH suggest that even though some of the investors are not fully rational, their trades will cancel each other or in the case that they trade in the same direction rational agents will cancel their influence into the stock prices through the arbitrage. One of the most important achievements of behavioral finance developed in several papers is the so-called “limits to arbitrage”, which show a market with rational and irrational investors, where the irrational investors can have a long-term influence in the stock prices. This is one of the main blocks in behavioral finance. The second block is based on models which explain investors’ irrationally,
showing how people deviate from Bayes’ law and from SEU. This block is so-called “Psychology”.

With the purpose of investigate more about the formation of these deviations, behavioralists presuppose a explicit form of irrationally, and use the empirical evidence gathered by physiologists on systematic biases that come up when people form beliefs, and on people preferences (Baberis and Thaler, 2002, BT henceforth).

The theory of limits to arbitrage states that when irrational investors cause deviations in the stock prices, arbitrageurs cannot always return the stock prices to its fundamental value. It therefore makes reference to the difficulties to return stock prices to their fundamental value. This process can be both risky and costly, thereby allowing the mispricing to continue in the long-term. These risks and costs that restrict the arbitrage can be gathered in three main groups.

- **Fundamental risk.** This problem arises due to the difficulty to find a perfect substitute security, i.e., a security with similar cash-flows. It is possible to find close substitutes in the same industry, removing to some extend the fundamental risk.

- **Noise trade risk** is the risk that stocks that are already mispriced do not return to the fundamental value and even the divergence can increases, rising a situation where arbitrageurs can be forced to liquidate their position with losses (Shleifer and Visny, 1997). Therefore arbitrageurs cannot return the stock price to its fundamental value, so the mispricing survives in the long-term. This situation is quite frequent when professional managers do not deal with their own money and they try to exploit a mispricing in the long-term but investors see a poor performance in the short term and withdraw their money.

- **Implementation costs**, such commission, bid-ask spread, etc. These costs can erode the profits of any trading strategy to exploit the mispricing based on arbitrage. Since shorting it is necessary for arbitrage, short-sale constrains are also included in this group. This makes reference to any factor that make less
profitable to hold a short position that a long one, for example the fee charged for borrowing a security. Sometimes another problem that arbitrageurs face is that they can not find stocks to borrow at any price. Finally, mention the cost of learning about how to exploit the mispricing.

Remark that if noise traders affect to the prices even in the long-term, there will be still little predictability in the returns (Shiller, 1984). It can also raise the situation where the arbitrageurs may speculate and trade intentionally in the same way as the noise traders, worsen the mispricing and leaving the market in a profitable situation for them afterwards. Finally mention the so-called “selective events” (Fama, 1998), that is ‘events that occur to take advantage of the mispricing of a firm’s stock’. For example, corporate managers tend to issue new shares when they think that their company’s stocks are overvalued, or to repurchase stocks when they think that they are undervalued.

In the Psychology block as it is stated before behavioralists try to create models which assume a specific form of irrationality in order to capture the deviations in stock prices not explained by rational approaches. They are based on the experimental evidence collected by psychologist on the systematic biases of people’s beliefs and preferences. Beliefs mean how investors form expectations. Now it is presented a brief summary of how people seem to form their expectations and beliefs. Remark that these biases are described in order to provide a framework to explain the behavioral approaches over CAPM anomalies

- **Overconfidence.** People use to be overconfident in their estimations. First, the confidence intervals that they give to their approximations of magnitudes. For example, the intervals for an index in a year. Second, people adjust deficiently their estimations about probabilities. Events that they think that will happen for sure only have 80%, and events that they judge impossible to happen have a 20% of probability (Fischhoff, Slovic and Lichtenstein, 1977).
- **Optimism,** people think that they have higher abilities or prospects over the average (Weinstein, 1980), such as intelligence, driving skills, etc.
• **Representativeness.** This phenomenon was presented by Kahneman and Tversky (KT, henceforth) in 1974. When people assign a probability to the fact that a data set A was produced by a model B, or that A belongs to a group B, they habitually use the representativeness heuristic, i.e., they assign a probability to A to the extend that A seems to have the attributes of B. Normally representativeness is a useful heuristic, but it has some pitfalls. The first consequence of representativeness is the ‘rate neglect’ effect. Remind that bayes laws states that:

\[
p(A | B) = \frac{p(B | A)p(A)}{p(B)}
\]

(7)

The rate neglect states that people put too much weight on \( p(B | A) \) which captures representativeness, and a small weight on the rate \( p(B) \).

Representativeness also conducts to another bias, the sample ‘size neglect’ effect. People use to do not take the size of the sample into account, and a small sample should be as representative as a large one. For example, 10 tosses of coins with 5 heads and 5 tails should be as representative that 1000 tosses with 500 heads and 500 tails. The problems arises when people see in small samples properties for the whole population, this is so-called ‘the law of small numbers’ (Rabin, 2002).

• **Conservatism.** In the other hand when the base rate, \( p(B) \), in bayes’ law is overemphasized leads to conservatism. If the data does not seem representative of any model, people respond too little to the data and depend too much on their past beliefs.

• **Belief perseverance.** When people form one opinion it is difficult to make them to change it (Lord, Ross and Lepper, 1979) First, people are averse to search for proofs that challenge their former beliefs, and even if they find them they still have doubts. For example, once that people started to believe in the EMH they treated with skepticism the new theories against it.
• **Anchoring.** When people make their estimations they normally use a referent point, an arbitrary value chosen by them. The following estimations are adjusted to it (KT, 1974). People normally give incorrectly too much weight to this referent point, they “anchor” to this initial value.

**Prospect Theory**

One of the bases of any asset pricing model or trading behavior model is to try to understand how investors assess risky gambles (stock investment it is considered as risky gamble) and set their preferences. Most of the models presume that investors base their preferences on the Expected Utility framework (EU, henceforth) of Von Neumann and Morgenstern (VNM, 1994). Behavioralists’ evidence presents that investors use to violate EU theory when forming their preferences. This raises non-EU theories, which try to explain people’s behavior. The most popular non-EU theory is the prospect theory of (KT, 1974; 1979).

If the stock market it is considered like a risky gamble all these theories can be helpful to explain some of the investor's choices. Evidence of behavioral finance suggests that some of the violations of the EU are important to understand some financial phenomena. The prospect theory of KT has been the most successful, and it captures most of the experimental results. Another reason of the success of the prospect theory is that it is not a normative theory, it is just a descriptive theory. KT design the prospect theory for gambles with at most two non-zero outcomes \((x,p;y,q)\) where outcome \(x\) has probability \(p\), and outcome probability \(q\), with \(x \leq 0 \leq y\) or \(y \leq 0 \leq x\). The value function (Figure 1) is described as follows:

\[
V(x, p; y, q) = \pi(p)v(x) + \pi(q)v(y)
\]  
(8)
One of the main differences respect to the expected utility theory is that the utility is defined over gains and losses rather than over final wealth situation. This theory is based on how gambles are presented everyday, and how people compute their position in relation to earlier levels rather than in absolute terms. One of the main characteristics of the value function in figure 1 is its concavity in the area of gains and convexity in the area of losses, this means that people are risk averse over gains, and risk-seeking over losses. As we can see in Figure 1, there is a kink in the V function at its origin. Moreover the line is stepper in the left side which reflects people’s ‘loss aversion’.

An important characteristic of the prospect theory is “the nonlinear probability transformation” (Figure 2). While in the gains area people are normally risk averse, if there is a small chance of a large gain then people are risk seeking (e.g. lotteries), overweighting the probabilities. In the other hand while in the losses area people are normally risk seeking, if there is a small chance of a large loss then people are risk-averse (e.g. insurances). Moreover people also place more weight on outcomes where they feel more confident than in those where are not certain, this is the so-called ‘certainty effect’.
KT (1979) make a generalization of the prospect theory for gambles with more than two outcomes.

![Graph](image)

Figure 2. - A hypothetical weighting function
Source: Kahneman, D., and A. Tversky (1979)

Prospect theory can explain why people take different choices when they have the same final wealth level, something that the EU does not explain properly. One of the properties of the prospect theory is the so-called ‘framing’. People vary their preferences according to how the situation is described or presented to them. Related to the framing phenomena and how people sum their gains and losses arises the so-called “mental accounting”, (BT, 2002). One important characteristics of the mental accounting is the phenomena known as narrow framing, which ‘is the tendency to treat individual gambles separately from other portions of wealth” (Barberis and Thaler, 2002).

These situations where probabilities are known before make the choices are rarely. As it is stated above, Savage (1964) develops the Subjective Expected Utility (SEU) in order to explain these situations contrary to the expected utility theory. However evidence presents situations where people do not know the distributions of the probabilities and their choices are incompatible with the SEU. In these situations of uncertainty people
show what is so-called “ambiguity aversion” (BT, 2002). SEU does not make people to expose their confidence about the distribution of the probability, so it does not reflect the ambiguity aversion.

To sum up, say that the Prospect Theory arose as a counterpart of the Expected Utility theory when the probabilities are known and Savage’s SEU as a counterpart to the Expected Utility when the probabilities are unknown. However, “ambiguity aversion” seems to capture some biases not captured by SEU.

**Empirical challenges to the EMH**

1. **Limits to arbitrage**

Clear evidence of limits to arbitrage is the presence of persistent mispricing in stock prices and the difficulties to return stock prices to their fundamental value. Although there are always problem when adjusting future cash flows, there are some cases where the mispricing is evident, and reflect the risks and costs outlined before. A clear example is the case of Royal Dutch and Shell Transport (Barberis and Thaler, 2002). These two companies decided to merge on a 60:40 basis while being independent companies. Royal Dutch stocks are listed mainly in U.S and the Netherlands, representing a 60% of the total cash-flows, while Shell stocks, are listed mainly in the UK, representing a 40% of the cash-flows. If the stock prices were equal to the fundamental value, the market value of both companies should have an equivalent ratio to the equity value, i.e., 1.5, because the stocks are trading the same thing at the same time. However, as we can see in Figure 3 this ratio varies strongly, showing evidence of market inefficiency. Royal Dutch is sometimes 35% underpriced and 15% overpriced. In this case, fundamental risk is clearly hedge, while the risk is produce by the noise trader risk. Arbitrage is limited if arbitrageurs are risk averse and have short horizon positions, and the noise trade risk is systematic.
Another example of limits to arbitrage is the jump in prices by an average of 3.5% (U.S. market) of the stocks of any company when it is included in the index (S&P 500) (Wurgler and Zhuravskaya, 2002). This is a clear example of reaction to non-information. Stock prices should remain the same after the inclusion in any index, because the fundamental value does not change. However arbitrageurs cannot easily profit from this situation because individual stocks do not have good substitutes, therefore the fundamental risk is not removed completely. Moreover there is also some noise trader risk involved after the inclusion, and the price rising may continue. Evidence presents that the jumps are larger for those securities that have the worst substitutes stocks (Wurgler and Zhuravskaya, 2002).
2. Cross-Section of average returns

There are several empirical studies of the behavior of individual stocks which have challenged the CAPM and therefore the market efficiency hypothesis. Most of these studies are done using a cross-section of average returns. These challenges as it is stated before are the so-called “anomalies”. These studies normally present groups of stocks that earn a higher return that other group sorted on price ratios.

De Bondt and Thaler (1985) are the first to document the long-term reversals, an anomaly of the weak-form. They analyzed the returns over long periods. They formed two portfolios, one with the stocks that had the best performance over the previous three years, the winner portfolio, and other with stocks that had the worst performance for the same period, the loser portfolio. After the portfolios formation they calculated the returns for both portfolios over the following five years (Figure 4). The results present that the loser portfolio outperforms the winner portfolio, especially in January. These results cannot be explained due to the risk of holding losers, at least using standard risk adjustments such the CAPM. DeBondt and Thaler interpret these results as a price overreaction where the losers stocks have become cheap and have higher returns afterwards while the winners stock have become too expensive and present lower returns afterwards.
This argument is coherent with psychological theories. There is an overreaction where investors extrapolate past results into the future. Loser portfolios are normally companies with poor earnings, so investors undervalue these companies, while winner portfolios normally present earnings and investors therefore overvalue these companies.

The behavioralists base the over and under-reaction behavior in the results and models proposed by Tversky and Kahneman (1974). They suggest that when investors are informed of earnings news about their stocks, they do not react as Bayesian statistics
predicts. Investors normally present two kinds of behaviors (Barberis, Thaler 2002). When investors present conservatism (Edwards, 1968) there is underreaction of stock prices to earnings announcements, and they do not think that there is any trend. In the other hand when investors receive series of news, such earnings announcement, they see an earnings trend and form a new model. As a consequence of the representativeness effect and concretely the law of small numbers, they form a new model.

However rationalist like Chan and Chen (1991) argue that these results are due to failure to risk adjustment returns. They suggest that there is a risk factor, the distress effect that is compensated in a rational equilibrium asset pricing model (Chan 1991, Fama 1991). Underreaction and overreaction will be discussed further at the end of this section.

Another anomaly not fully explained yet is documented by Jegadeesh and Titman (1993). With their theory of the “momentum effect” they proved that movements in individual securities prices over a interval of six to twelve use to indicate future changes in the stocks prices, suggesting a short-term continuation.

The semi-strong form has also empirical challenges. One of the most widely-know is that small stocks outperformed large stocks. Moreover, the higher return of small stocks has been accumulated in January. Banz (1981) is the first on certificate the size premium. He presents evidence of small stocks outperforming large stocks in the long term by an average of 1% per month on a risk-adjusted basis for the period 1931-75. However, recent evidence suggests that both effects seem to have disappeared in the last years (Barberis and Thaler 2002).

Basu (1977) demonstrates how using price/earnings ratios were possible to forecast stock returns to some extend. He found that low price/earnings securities outperformed high price/earnings by more than 7% per year for the period 1956-71. As we will see in more details, Fama and French (1992) using the book-to-market ratio, sort firms with the highest book-to-market ratio, the so-called “value” stocks, which
outperform the stocks of the firms with the lowest book-to-market ratio, the so-called called “growth” stocks.

All these studies present evidence that value stocks outperform growth stocks, and suggest that these returns cannot be explained by the traditional measures of risk, such the CAPM.

As it was outlined before, there are some other documented anomalies produced by corporate announcements, the so-called “event studies”. There are several event studies widely recognized. Some of them are:

**Earnings announcements.** Ball and Brown (1968) show evidence of post-earnings announcement “drift” in the same direction traced by an earning surprise. Bernard and Thomas (1989) sort US stocks by the size of the “surprise” in their recent earnings announcements. To measure the “surprise” they use the standardized unexpected earning (SUE), which is “the difference between a company’s earnings in a given quarter and its earnings during the quarter a year before scaled by the volatility of company’s earnings”. They find that stocks with surprisingly good earnings announcements outperform stocks with surprisingly bad earnings announcements showing also evidence of post-earnings announcement drift as it is showed in Figure 5.
Dividend initiations and omissions. Michaely, Thaler and Womack (1995) show how firms that initiated dividends outperform the market portfolio over the year of the announcement while firms that omitted dividends underperformed the market portfolio over the year of the announcement.

Primary and secondary offerings. Another anomaly is the negative long-run performance of new issues, verified by Ritter (1991), over the period 1975-84. Loughran and Ritter (1995) study firms with primary and secondary offerings, they find that these companies had average returns in the following five years below the average returns compared to non-issuing companies.
There are statistical difficulties to measure these long-term events, papers of Fama (1998) and Brav (2000) discuss this topic. There are two general concerns with the event studies. First the cross-sectional correlation and overlapping between the events (dividends, IPOs, etc) which suggest that all the results are statistically weaker (Brav, 2000) even though it is difficult to say to which extend. Second, data-mining. Critics argue that when sorting stocks in different ways it is normal to find cross-sectional differences in averages returns (BT, 2002). The normal way to check this critic is to perform out-of-sample tests.

Based on these approaches and findings Fama and French (1992) try to capture these anomalies with a modification of the CAMP, adding two new variables, one related to Basu's earnings and Banz's size variables. Fama and French’s model explains much of the cross-sectional variation in stocks return for the period 1963-90 for the US market. The most important finding of Fama and French (1991) is that market capitalization and book-to-market equity includes not only the effect of these two variables but also the effect of price/earnings ratio and leverage.

Fama and French (1993, 1996) use both company’s market capitalization and its book-to-market ratio as variables to measure the fundamental risk of the stocks in the so-called “three factor model”. The main approach of this model is that companies with small stocks and/or value stocks must obtain a superior average returns because they are riskier due to their exposure to the size and book-to-market factors respectively. In the other hand large and growth stocks have lower returns because they are safer respect to size and book-to market factors respectively.

Fama and French obtained low intercepts in three-factor model which implies that the model is good. However, these results have to be taken with caution. As Roll (1977) points out: ‘in any specific sample, it is always possible to mechanically construct a one factor model that prices average returns exactly”.

One of the main characteristics of the rational approach is that betas explain average returns and not firm features. In a risk-adjusted basis, value stocks for example, will earn higher returns not as a consequence of having a high book-to-market ratio, but because
such securities have a higher beta on the book-to-market factor. Daniel and Titman (1997) disagree with this approach. They find that stocks with different betas but the same book-to-market ratios do not have different average returns. These results contradict the three-factor model approach. However, Fama, French and Davis (2000) performing the test with a longer data set and with a different methodology disagree with Daniel and Titman.

Fama and French (1993, 1995, 1996) suggest that the size and book-to-market ratios proxy for different facets of “distress risks”. Stocks with a higher book-to-market ratio and small stocks bear more risk than stocks with low book-to-market ratio. Fama (1998) states that “there is a common variation in the earnings of distressed companies that it is not explicated by market earnings and a common variation in the returns on distressed stocks that is not explicated by the market return”, showing that there are common size and book-to-market factors in earnings.

In the other hand as it is stated before, behavioralist like Lakonishok et al. (1994) suggest that the value premium arises because investors undervalue value stocks and overvalues growth stocks and once that the mispricing is corrected, value stocks obtain higher returns than growth stocks.

La Porta (1996) sorts U.S. stocks based on long-term earnings growth rate forecast realized by professional analyst. He found out that analysts are too bullish about the stocks that they think that will have a good performance and too bearish about the stocks they think that will have a poor performance. Therefore stocks with the highest growth forecast obtain a lower return than the stocks with the lowest growth forecast. This evidence presents overreaction in stock prices.

Another rational explanation for the value/growth premium is that investors of growth stocks who obtain a lower average returns in fact expect these low returns because growth stocks have attractive risk features for investors (BT, 2002).
Behavioralists argue that Fama’s critic has some shortcomings, especially due to the methodology that he uses. As Fama states:

“Event studies produce useful evidence on how stocks prices respond to information. Many studies focus on returns in a short window (a few days) around a cleanly date event. An advantage of this approach is that because daily expected return are close to zero, the model for expected returns does not have a big effect on inferences about abnormal returns” (Fama 1998)

A big dispute rises in interpreting these results, giving place to the joint hypothesis problem. Because the results are relative to the criteria of measurement, i.e. the benchmark, therefore there is room for different types of interpretations. The magnitude of over or under-performance depend on the benchmark, so we can see these anomalies like one signal of market inefficiency or in the other hand interpret the abnormal returns as an indication of the limitations in the underlying assets pricing model.

For the studies on long-term return anomalies which suggest market inefficiency Fama (1998a), supporting market efficiency, states that in an efficient market, underreaction and overreaction split randomly, therefore these phenomena are consistent with market efficiency. He also suggests that ‘long-term return anomalies are sensitive to the methodology ‘(Fama 1998a) and that they are likely to disappear or become insignificant when they are tested with different models or different statistical approaches. He also states that when they are analyzed individually they can be consequence of chance.

Fama (1998) claims that for long-term studies tests, the alternative hypothesis “market inefficiency”, is “vague”. Instead a better specific model of price formation should be tested.
3. The aggregate stock market:

There are several researches that present evidence of anomalies in the aggregate stock market. They can be classified in three groups.

**The Equity Premium.** The risk premium has been traditionally positive meaning that the stocks have earned a higher return than the T-bills. Campbell and Cochrane (1999) present evidence that in the US market the average log return on the S&P 500 index is around 3.9% higher than the risk-free rate average log return.

Although stocks seem an attractive investment because they have a high return and a low covariance with consumption growth investors will require a high risk premium to hold them (BT, 2002). Behavioral finance has two approaches for the equity puzzle, both based on preferences. One based on prospect theory and the other in ambiguity aversion.

Based on the prospect theory Benartzi and Thaler (1995) show how investors allocate their wealth between T-bills and stocks. Prospect theory suggests that people compute gains and losses for each gamble and then choose the one with the highest prospect utility. Therefore for the financial market, investors when forming a portfolio, compute for each allocation the expected gains or losses and then choose the one with the highest prospect utility. The try to maximize:

\[
E_x u [(1 - w)R_{f,t+1} + wR_{t+1} - 1]
\]  

(9)

where w is the fraction of financial wealth. From the prospect theory point of view it is important to calculate how often investors evaluate their portfolio, because it affects to the portfolio formation, if one investor compute his wealth every day and he is risk averse, he will find stocks risky and unattractive, while in one investor compute his wealth every 4 years will find his investment less risky. Benartzi and Thaler calculate the period where investors are indifferent between stocks and T-bill. They found out that with one-year of evaluation period, investors are indifferent between the equity premium of the stocks and the returns of the T-bills. Benartzi and Thaler justify these results as a
mix of loss aversion and frequent valuation, calling it “myopic loss aversion”. This is a clear example of how investors are affected by the way that results are presented.

Based on the ambiguity aversion approach it is demonstrated that when people face ambiguity, they assign a possible probability distribution and try to maximize the minimum expected utility under any distribution (BT, 2002). When investors think in a distribution they try to ensure their choices even if the reference model is wrong, that is why they “charge a substantially higher equity premium as compensation for the perceived ambiguity in the probability distribution”. (Maenhout, 1999). But still the 3.9% equity premium can not be fully explained by this approach, it would require a high concern about misspecification.

Volatility. One important challenge against EMH was presented by Shiller (1981) in his work over stock market volatility. He demonstrated that ‘stock market prices are far more volatile than could be justified by a simple model in which these prices are equal to the expected net present value of future dividends’. To calculate the net present value he used a constant discount rate and some concrete assumptions about dividends formation. He opened a new search area but he was also critiqued due to misspecifications of the fundamental value. He points out that it is difficult to justify the historical volatility of stock returns with the assumptions of rational investors and constant discount rates. A good approach for the volatility is the one based on changes in investor's risk-aversion (Campbell and Cochrane, 1999).

One of the main discussions in the literature over variance bound is the essential assumption that excess price volatility implies market inefficiency.

The behavioral explanations for the volatility puzzle are set in two groups, beliefs and preferences.

Beliefs. One possible explanation is that investors think that the mean dividend growth rate is more variable that in reality. When there is a flow of dividends they quickly think that the mean dividend growth rate has increased, driving the stock prices up, with a
consequent increase in the volatility of the returns. This could be justified with the representativeness approach, concretely with the law of small numbers (BT, 2002). Investors see a trend for the whole population in a short sample. Something similar happens when investors extrapolate past return to form expectation of future stock prices.

Another explanation is based on investors’ overconfidence. When they gather information by themselves about any company future cash-flows growth they overestimate its exactness and act giving too much confidence to their estimations. If they are positive, the will push stock prices up, adding volatility to the returns (BT, 2002).

Preferences. Thaler and Johnson (1990) based on the dynamic aspects of the loss aversion how investors make different choices depending on prior gains or losses. After gains investors take gambles that they normally do not accept, and after losses they reject gambles that they normally accept. This is known as the “house money effect”.

**Predictability**, early literature has presented evidence and demonstrated that stock returns are to some extend predictable. For example, Fama and French (1988) using dividend-price (D/P) ratio, demonstrated that 27% of the variability of cumulative stock returns over the following 4 years is explained by the D/P ratio. Recent literature debate the statistical significance of the time-series predictability as it was described before. Predictability puzzle is quite related to the volatility puzzle, all the arguments and approaches for it are valid for the predictability puzzle as well (BT, 2002).

**Underreaction and Overreaction**

Early literature about event studies have showed that stock prices react efficiently to new information, but it is possible that the stocks remain over or undervalued over long periods of time. It is quite complicated to test whether stock prices match to their fundamental values. There are two behavioral models, explained below, which argue how investors can overreact or underreact to some kind of events
Baberis, Shleifer and Vishny (1998) build a model based on the representativeness (KT, 1982) and conservatism (Edwards, 1968). They state that investors present these biases when forming expectations of future return with public information. They formed a model where earnings follow a random walk. However investors, when forming expectations, think that earnings follow two regimes: regime A: “mean-reverting” or regime B: “trending”. In regime A, investors think that earnings are mean-reverting, therefore they show conservatism and think that changes on earnings are temporary, so the stock prices underreacts. In regime B, investors think that earnings are trending, therefore they extrapolate the trend and the stock prices overreact.

Regime A is based on the evidence of the momentum effect (Jegadeesh and Titman, 1993) and the short-term responses of stock prices to earnings announcements (Ball and Brown, 1968; Bernard and Thomas, 1990), while regime B is based on the long-term reversals (DeBondt and Thaler, 1985). Their model captures post-earning drift, momentum, long-term reversal and cross-sectional predictability, but it does not cover other anomalies (Fama, 1998).

Fama (1998) suggests that the long-term reversal and long-term continuation are chance result and that they split randomly, therefore consistent with market efficiency.

Another behavioral model is proposed by Daniel, Hirshleifer and Subramanyam (DHS, 1997). They suggest that there are two kinds of investors: informed and uninformed. The former settle the price and they are affected by two biases: overconfidence and biased self-attribution. Overconfidence conduct them to magnify their own estimations about stock prices based on their private signals, while biased self-confidence leads them to underestimate public signals concerning stock prices, particularly when these signals challenge their own private signals. DHS’s model reaches similar result to BSV’s model “sharing the empirical success and failures” (Fama, 1998)

Remind that the joint hypothesis problem suggests that all the models for expected returns are not complete and have some shortcomings, therefore when testing market
efficiency with any model there will be always bad-model problems. There is also the possibility that there are sample specific patterns in the returns that are due to chance.

As it was seen before, these bad-model problems do not have serious repercussion in event studies because they use short return windows, where the expected returns are close to zero, therefore they do not affect too much to the estimations of abnormal returns. The problems arise when using longer horizon returns (Fama, 1998).

Fama argues that in long-term anomalies studies, Average monthly abnormal returns (AARs or CARs) can draw different conclusions than buy-and-hold abnormal returns (BHARs), he argues that when this happens, ‘the anomaly is not much evidence against the market efficiency’.

Equally-weight returns also produce different results than value-weight returns (Fama 1998). Most of the asset pricing models have difficulties to explain the average returns on small stocks. Especially when using equally-weight portfolios. As Fama (1998) points out ‘since equal-weight portfolio returns give more weight to small stocks, bad-model problems are more severe in inferences from equal-weight returns’.

However, behavioral finance models such as DHS’s model and BSV’s model does not predict the mispricing of small stocks, actually in DHS’s model, informed investors are more interested in large stocks, so the mispricing should affect more to this large stocks. In general terms, behavioral finance does not provide an explanation to the assumption that small stocks are more likely to be mispriced.

Fama (1998) with ‘reasonable’ changes in the estimations methods found that anomalies in abnormal returns disappear. This is the case for IPOs, SEOs, self-tenders, share repurchases, and dividend initiations. He also found that other anomalies are economically or statistically insignificant such as abnormal returns in acquiring firms in mergers. Other anomalies disappear in out-of-sample replications, such as splits and dividend omitting.
Moreover, when value-weight returns are used the anomalies become statistically less significant. Fama (1998) suggests that some anomalies are related to small stocks, which are ‘just a sure source of bad-model problems’.

There are some anomalies that Fama could not fully explain from a rational point of view. These are the post-earnings announcement drift (Ball and Brown, 1968) and its extension (Bernard and Thomas, 1990) and the short-term continuation of returns (Jegadeesh and Titman, 1993). These anomalies are still labeled by Fama as “open puzzles”.

The debate between EMH hypothesis and behavioral finance models has been a big dispute in recent years. Several authors have exposed this controversy in articles such as “A Random Walk Down Wall Street” (Malkiel's, 1973, 1999), “Efficient Capital Martingales” (Steven LeRoy) and “Market Efficiency, long-term returns, and behavioral finance”( Fama, 1998).

In the same way that behavioralist try to demonstrate the irrationally of financial markets, their own studies have been also contradicted, like DeBondt and Thaler findings. Conrad and Kaul (1993) demonstrated that their results were consequence of a measurement error.

At the same time, other sources of market inefficiency, are recognized to be “purely historical or non greater than transactions costs” (Lo, 1999). However other anomalies persist to reviews, for example the difference in price between Dutch and Royal Shell at the same time. But even though the price does not match with its fundamental value, there are not riskless profit opportunities.

Facts such that fund managers do not outperform passive traders and the evaporation of CAMP anomalies support strongly the EMH.

Some anomalies such as the January effect, Monday effect, end-of-month effect seem to have disappeared, while others like small effect and value effect are still controversy. While Fama and French built a model based in this two “anomalies”, recent evidence suggests that these effects has been disappear (Falkenstein, 2001). Maybe this is due to
that once that some effects are widely-known they disappear because the market already take them into consideration.

Behavioralist anomalies should be viewed as “exceptions” of the EMH, otherwise we should replace the EMH by Kahneman and Tversky’s model. It can be also said that there is not a complete true and that extreme hypotheses are probably false. Behavioral finance should be a tool to understand how the stock markets work and to understand investors’ preferences and behavior. And it is necessary to be aware that models such the CAPM and the three-factor model do not fully explain average returns.

**Section II.**

**CAPM fundamentals**

The Capital Asset Pricing Model (CAPM) of Sharpe (1964) and Lintner (1965) is the origin of the asset pricing theory. Before it, there were no models with a clear relation between risk and return. The CAPM is still widely used, for example to calculate the cost of equity and to evaluate the performance of managed portfolios. The triumph of the CAPM is its simplicity and logic predictions about risk and its relation with expected return. Perhaps due to this simplicity and like almost all the models the empirical results of the model are poor. But these results may be the consequence of shortcomings of the empirical tests, especially due to the bad proxies used, which are critical for the model predictions.

The fundamental hypotheses behind the CAPM are:

- All the investors have the same future return expectations over all the assets. Moreover investors agree on the joint distribution of asset returns from $t$ to $t-1$.
- There are not borrowing and lending costs.
• There are not transaction costs and information is publicly available for all the investors.
• Investors are risk averse
• All the investors have the same investment horizon length

The main fundamental consequences of CAPM are:

• Any combination of risk-free assets and the market portfolio has higher returns than any portfolio of only stocks or risk-free assets.
• All the investors will have a portfolio formed by a combination of stocks and risk-free assets. The proportion will depend on the utility function.
• The market portfolio is formed by all the assets that exist and the quantity of each is proportional to its market value.

Therefore the portfolios are mean-variance efficient. They minimize the portfolio return variance, given the expected return and maximize the expected return given the variance. The CAPM explains how the assets are combined given these assumptions.

**Relationship between average returns and betas**

The CAPM was created in the 60’s and several papers started to challenge it soon. The main idea of the model is that the systematic risk of an asset is measured by its beta and that the expected returns of the securities have a linear dependence with their betas. Black, Jensen and Scholes (1972) and Fama and Macbeth (1973) present evidence of a good performance of the model. They found out that the expected returns on a security and its risk in any efficient portfolio is linear and that beta is a complete measure of the risk. However, Roll (1977) states that the CAPM cannot be tested while other papers found variables which explain the average returns and also show that the relation between beta and expected returns was low.
It was in 1992, when Fama and French show that the relationship between the betas and the expected returns was low but that other variables such the BE/ME and size ratios were able to explain average returns.

Roll and Ross (1994) state that there is a positive relationship between the expected returns and the betas if the market portfolio used to estimate the betas is on the ex ante mean-variance efficient frontier. As Roll states in 1977, the market portfolio is not observable, so it is needed an approximation, therefore when there is a low relationship between average returns and betas it means that the market portfolio used is not efficient ex ante and when this happens it is not strange that other variables complete the explanation given by the betas.

Ross and Roll (1977) state that previous researchers when testing efficiency of different market index proxies have been often rejected. They conclude that a market portfolio can be close to the efficient frontier, without been efficient, and when it is used as a market proxy to estimate the betas it provides non relationship between the average returns and the betas, therefore Fama and French findings are not surprising.

Given this low relationship between average returns and betas Fama and French propose a group of variables which explain the average returns in a multifactor model. In 1993 they found five factors which explain the average returns of stock and bonds. They use the time-series methodology proposed by Black, Jensen and Scholes, which will be explained later. They conclude that average returns are explained by their model.

In 1995 they present a research where they link average returns with accounting data, concretely with earnings, in order to find a justification for the BE/ME and size factors.

They conclude that:

“There are market, size, and BE/ME factors in earnings like those in returns. The market and size factors in earnings help explain those in returns, but we find no link between BE/ME factors in earnings and returns”
CAMP anomalies

Since the 70s there are several papers that challenge the CAPM and the hypothesis that the market portfolio is efficient. One of the implications of the CAPM is that the expected returns on all assets have a linearly dependence with their market betas, and that there are not other variables with explanatory power. One of the challenges identified against the CAPM is the existence of variables with significant descriptive power.

Empirical studies have presented a set of anomalies based in a group of stocks that earn higher average returns than another, normally sorted by price ratios. As we saw before, Basu (1977) shows evidence of how the stocks when are sorted on earning-price (E/P) ratios present different average returns. Stocks with higher E/P ratios have higher future return and stocks with lower E/P ratios have lower future returns, an anomaly that cannot be explained by the CAPM betas. Banz (1981) shows that when stocks are sorted on market capitalization, average return on small stocks are higher that those forecasted by the CAPM. Statman (1981) presents evidence where high book-to-market stocks have higher average returns that the returns explained by the betas, and low book-to-market stocks have lower returns that the returns explained by their betas.

There are some other studies which have presented evidence of CAPM anomalies; the conclusion is that ratios which involve information of stock prices have explanatory power miss by the market betas. This is due to the fact that the stocks prices depend both on the expected cash-flows that the security will offer and on the expected return that discount the cash-flows. The CAPM anomalies saw above suggest that ratios like earning-prices, book-to-market ratios have explanatory power in the CAMP.

Two different views arise from this point. First, the behavioralists argue that stocks with high book-to-market or earning-price ratios are normally companies which had a recent poor performance, while low book-to-market ratios or earnings-price ratios belong to companies with a recent good performance (Lakonishok, Shleifer and Vishny, 1994). Behavioralist argue that investors suffer of overreaction and extrapolate past results
which increase the stock prices for companies with good past performance and reduce the
stock prices for companies with poor past performance. Once that the overreaction is
corrected, it produces higher returns for high B/M and E/P stocks and lower returns for
low B/M and E/P stocks. DeBondt and Thaler (1987) also provides evidence of this
approach.

Second, the view that suggests that such anomalies point out the need of a new asset
pricing model. These two approaches raise the joint hypothesis problem.

Following the arguments of the joint hypothesis theory it is possible that the CAPM holds
and that the market portfolio is efficient, and that the empirical rejections of the CAPM
are due to bad proxies for the market portfolio, which use to be restricted to common
stocks.
Fama and French (1992) also present the same evidence that previous researches
(Stambaugh, 1982), where the relation between average returns and the betas is flatter
than the expected. But this has not significant consequences due to the strong evidence
that several price ratios have explanatory power. Remark that if betas are not able to
explain the expected returns the market portfolio is not efficient.

The repercussion of this flatter relation between beta and average returns than the
estimated by Sharpe-Lintner CAMP is that cost of equity estimates for high-betas stocks
are too high (relative to historical returns) and estimates for low-betas are two low
(Friend and Blume, 1970), similarly CAMP cost of equity estimated for high B/M (value)
stocks is too low and for low B/M (growth) stocks is too high.

Stambaugh (1982) shows that the CAPM tests do not change almost the results adding to
the market proxy new assets. Mainly because the volatility is still dominated by stock
returns. And it is unlikely that price ratios are a bad proxy raising CAPM problems.
Moreover, sorting stocks on price ratios does not produce a big variation on the betas in
relation to the market portfolio. (Lakonishhok, Sheilfer, and Vishny, 1994).
Three-factor model fundamentals

The Intertemporal Capital Asset Pricing Model of Merton (1973) is the logical extension of the CAPM. The ICAMP assumes different approaches about investor’s preferences. In the CAPM, investors only care about the wealth that their portfolio produces at the end of the current time (t). In the ICAPM, investors not only care about the end-of-period wealth, but also with how they will consume or invest their wealth, therefore when they form they portfolio at time t-1 they care how their wealth might diverge with future state variable. These variables included by Merton are:

- “The prices of consumption goods and the nature of portfolio opportunities at t.”
- “Expectation about the consumption and investment opportunities to be available after t.”

Analogous to the CAPM, investors have a preference for high expected returns and low return variance, i.e., they are mean-variance efficient. But since their utility relies on state variables, investors also worry about the covariance of portfolio returns with the state variables. Therefore optimal portfolios are multifactor efficient (Fama and French 1992, 1995). They are a subset of multifactor minimum-variance (MMV), i.e., portfolios with the smallest return variances, given their expected returns and the covariance of their returns with the state variables, and the largest expected returns, given their variance and the covariance of their returns with the state variables.

Fama and French (1992) with their three-factor model, capture all the evidence over CAPM anomalies, and using the cross-section regressions corroborate that size and book-to-market ratios provide a higher explanatory power to the CAPM. Fama and French (1996) reach the same conclusion by using time-series regression with portfolios sorted on the same ratios outlined above.
Fama and French (1998) present evidence where value stocks have higher returns than growth stocks in twelve non-US markets, and emerging markets. These results show that CAPM anomalies are not sample specific. Moreover, it presents evidence against the argument which states that the anomalies are produced due to data dredging.

Behavioralists accept that the three-factor model captures much of the size and value effects in average, but they suggest that the return premium associated with these new variables is produced by investors overreaction, which is shaped in a way which seems correlated across firms and gives the impression to be produced by the risk. They argue that the investors try to “set the CAPM prices” therefore the anomalies are produced due to mispricing.

Section III.

CAPM derivation

The CAPM is based on Markowitz’ (1959) mean-variance portfolio model, where the investors select the portfolio at time $t-1$ which gives an unpredictable return at time $t$. One of the assumptions of the model is that investors are risk averse, and when forming the portfolio they only care about the mean and variance of the return in the subsequent period. The distributions of one-period percentage returns are assumed to follow a normal distribution or to match to other two-parameter stable distribution.

The Sharpe-Lintner CAPM is described as follows:

$$E(R_i) = R_f + \beta_{iM}(E(R_M) - R_f) \quad i = 1, ..., N \quad (10)$$

where $E(R_i)$ is the expected return on assets $i$, $R_f$ is the return on the risk-free rate asset.
and \( \text{E}(R_M) \) is the expected return on the market portfolio. \( \beta_{iM} \) is the systematic risk on the market portfolio calculated as follows:

\[
\beta_{iM} = \frac{\text{Cov}(r_i, r_M)}{\sigma_M^2}
\]  

(11)

Beta is interpreted as the sensitivity of the asset’s return to the variation in the market return, moreover is the slope in the regression of \( R_i \) on \( R_M \). Remark that when beta is estimated in this paper other risk factors that could affect to the expected returns are not included in the models, moreover risk loadings are not time-varying in the time-series regressions.

One of the limitations to perform empirical test in (10) is that the model is based on expectations, both for the returns and the market risk, so it is necessary to assume rational and homogenous expectations.

If investors do not have the same expectations of the distributions of the returns the market portfolio will not be efficient for all the investors. Investors with different expectation will hold different portfolios, and there would be also investors who hold only stocks or only risk-free assets.

Unrestricted risk-free borrowing and lending is quite unrealistic, but if this assumption is not allowed, then it is necessary to allow unrestricted short sales of risky assets (Black, 1972). If it is not allowed any of the assumptions then, market portfolio would be probably not efficient.

There are several researches which analyze CAPM predictions. Roll (1977) suggests that these tests cannot be made because two things are analyzed simultaneously, first that the market portfolio is efficient a priori, and second, the CAPM expression.
Normally the methodology used for empirical test in time-series is the following:

\[ R_{it} - R_{ft} = \alpha_i + \beta_i M (R_{Mt} - R_{ft}) + \epsilon_{it} \quad t = 1, \ldots, N \]  

(12)

where \( \alpha_i \) has to be equal to zero. Most of the literature finds that \( \alpha_i \neq 0 \) and that the risk premium \( R_p \), is lower than \( R_M - R_f \). Moreover evidence suggests that the relation between average return and beta is flatter than predicted by CAPM.

**Three- factor model derivation**

As we saw before Fama and French (1996) state that the ICAPM is the natural extension of the CAPM. Moreover, without short-selling constrains of risky assets, market clearing prices involve that the market portfolio is multifactor efficient. If there is riskfree borrowing and lending, the relation between expected return and beta risks is,

\[ E(R_i) - R_f = \beta_{iM} [E(R_M) - R_f] + \sum_{s=1}^{S} \beta_s [E(R_s) - R_f] \]  

(13)

where \( R_s \), \( s = 1, \ldots, K \) are returns on state variable mimicking portfolios and the betas are slopes from the regressions: \( R_i - R_f, R_M - R_f \) and \( R_s - R_f \)

Fama and French (1993, 1996) demonstrate that the CAPM anomalies uncovered by sorts of stock by price ratios determine the need for a multifactor ICAPM. Therefore they proposed the following model:
\[ E(R_i) - R_f = \beta_{iM} [E(R_M) - R_f] + \beta_{iS} E(SMB) + \beta_{ih} E(HML) \]  
(14)

where,

- \( E(R_i) \) is the expected return on assets \( i \)
- \( R_f \) is the return on the risk-free rate asset
- \( E(R_M) \) is the expected return on the market portfolio
- \( E(SMB) \) is the expected return on the portfolio for the ‘small minus big’ size factor
- \( E(HML) \) is the expected return on the portfolio for the ‘high minus low’ book-to-market factor.

The independent variables in the ICAMP are the expected return on MMV portfolios in excess of the risk-free rate. However SMB and HML are the difference between two portfolio returns. The three factor model is still valid if the components of SMB (S and B) and HML (H and L) are MMV. Fama and French (1996) demonstrate that \( R_b - R_f \) and \( R_L - R_f \) are perfect linear combinations of \( R_M - R_f , R_s - R_f \), and \( R_H - R_f \), so when they form SMB subtracting \( R_b \) from \( R_s \) and HML subtracting \( R_L \) from \( R_H \) the intercepts and the explanatory power of the model are not affected.

The model outlined above is based on expected returns, to make estimations with observed returns the model is transformed as follows,

\[ R_{it} - R_{ft} = \alpha_i + \beta_{iM} (R_{Mt} - R_{ft}) + \beta_{iS} SMB_t + \beta_{ih} HML_t + \epsilon_{it} \]  
(15)

where,

- \( R_{it} \) is the realized return on asset \( i \) at time \( t \)
- \( R_{ft} \) is the realized return on the risk-free asset at time \( t \)
- \( R_{Mt} \) is the realized return on the market portfolio at time \( t \)
- \( SMB_t \) is the realized return on the portfolio for the size factor at time \( t \)
- \( HML_t \) is the realized return on the portfolio for the book-to-market factor at time \( t \)
the null hypothesis constitutes that the intercept $\alpha_i$ is equal to zero in all assets $i$. Fama and French (1993, 1996) show how for portfolios sorted on size and B/M have values for the intercepts close to zero. As they state ‘the intercept in regression of excess returns on the excess returns on any three MMV portfolio are equal to 0’. Moreover the R-squared should be close or equal to 1.

One of the limitations of the three-factor model is its empirical inspiration, contrary to the the ICAPM, the SMB and HML are not state variables of concern to the investors. Instead they are used to capture the anomalies not explained by the CAPM.

The three-factor model also faces another limitation. The model is not able to capture the momentum effect proposed by Jegadeesh and Titman (1993) outlined before, where the stock with relative good performance in the last months (3-12) continue to have a good performance for a few months and vice versa. The momentum effect is neither captured by the three-factor model nor the CAPM.

One solution proposed by Following Carhart (1997) is to add a momentum factor to the three-factor model, which would be the difference between diversified portfolios of short-term winner minus a diversified portfolio of short-term losers. This will not be worth if the goal is to estimate the cost of equity, which is calculated for long-term periods.

**Shortcomings to test the CAMP and the three-factor model**

There are some problems when testing the CAPM and the three-factor model.

- The choice of the market portfolio $R_M$, i.e., benchmark problems. Roll (1977) suggests that the CAPM cannot be tested because it is impossible to know the true market portfolio which should include all the assets which provide some value to the economy. However, Stambaugh (1982)
argues that the tests for the CAPM are not too sensible to the market portfolio proxy.

- Distributions of the returns. This is an important issue because CAMP assumes that investors decisions are based on the mean and the volatility of the returns, which are suppose to follow a normal distribution. Miller and Scholes (1972) suggest that there are some problems of right-asymmetry and leptokurtosis (Fama, 1976) on the distributions of the returns. Fama (1976) realizes a deep study about this topic, where he finally accepts the normal distribution for the returns based on the Central Limit Theorem.

- Interval period. There is a big discussion in the literature about if the returns should be calculated over daily, weekly, monthly, etc. data. Reilly and Wright (1988) show that the weekly betas are lower than the monthly betas. This problem could be linked with the company size, especially for short intervals, related with asynchrony problems because these small companies may need more time to reflect on their stocks the information about the market. Normally, weekly and monthly or even annual data is used. In this paper monthly data is used like in most of the literature of this topic.

- Length period. For monthly returns, it is recommendable to use period of 5-7 years, due to that for longer periods, betas seems to change (Fama 1976)

- Stability of the betas. When betas are estimated with historical data to make forecast, it is important that they present some stability. Blume (1975) detects a problem which is called “regression tendency” which implies that the portfolio betas tend to the unit.
- Econometric problems. In the time-series regressions, it can show up problems with sector relations due to the correlation between the contemporaneous random error terms of the individual stocks. While in the cross-section regression there can be problems of heteroscedasticity, autocorrelation and errors in the variables.

- Errors in the specification of the model. This issue makes reference to the possible existence of other common variables, different of the systematic risk measured by the market beta, which could affect to the expected returns. There are two groups:
  ♦ Variables such as squared-beta, used to test lineality (Fama and Macbeth, 1973)
  ♦ Fundamental variables: size, leverage, B/M, P/E, etc

Section IV.

Early Tests

An early test for the relationship between average return and risk is performed by Fama and Macbeth (1973). They suggest that the expected return on a security is equal to the expected return on a riskless security plus the risk premium times the security’s betas. This can be written as follows:

\[
E(R_i) = E(R_0) + \beta_i [E(R_m) - E(R_0)]
\]  

(16)

notice that the returns are random variables (~ is omitted). This two-parameter model has three testable implications also valid for the CAPM. If the market portfolio is efficient, then:
C1: The expected return on all securities and their market betas is linear
C2: There are not other variables with explanatory power
C3: If investors are risk-averse, the risk premium should be positive.

To test equation (16) Fama and MacBeth construct a model of period-by-period returns that permits to use observed returns to test the conditions C1, C2 and C3. They suggest the following stochastic model:

\[ R_{it} = \gamma_{0t} + \gamma_{1t}\beta_{1t} + \gamma_{2t}\beta_{2t}^2 + \gamma_{3t}s_{it} + \eta_{it} \]  

(17)

The testable hypothesis of the two-parameter model for expected returns are:

C1: Linearity \( E(\gamma_{2t}) = 0 \)
C2: No others variables have systematic effects \( E(\gamma_{3t}) = 0 \)
C3: The expected value of the risk premium is positive, \( E(\gamma_{1t}) = E(R_{Mt}) - E(R_{0t}) > 0 \)

\( \eta_{it} \) is assumed to have zero mean and to be independent of the rest of variables

C1-C3 hypotheses imply that investors hold efficient portfolios. If it is included the hypothesis \( E(\gamma_{0t}) = R_f \) and \( \beta_f = 0 \) it is obtained the Sharpe-Lintner model. So if this hypothesis is rejected, it is only rejected this specific two-parameter model of market equilibrium.

Normally cross-section tests concentrate on C3 and on the Sharpe-Lintner hypotheses. Using an approach based on (17 and 18), they regress average stock returns on their estimated market betas, and then test if the intercept is equal to the riskfree rate and if the slopes are significant and positive. The R-squared average is also used to measure the performance of the model. There are two problems in this method. First, it could be a
common variation in the regression residuals. Second, the betas estimated for individual
stocks are inaccurate. Blume (1971) suggests to use a stock portfolios based on sorted
beta estimates for securities instead individual stocks, but in this paper due to the small
number of securities individual securities are used

Fama and Macbeth (1973) to avoid problems originated by the correlation of the residuals
used monthly cross regressions rather than as single regression of average returns on
betas. In this paper this regression will be done for individual stocks. The regression is
described as follows:

\[ R_{it} = \gamma_{0i} + \gamma_{1i} \beta_{it} + \epsilon_{it} \quad i = 1, ..., N \quad t = 1, ..., T \quad (18) \]

where \( \beta_{it} \) is the beta estimated for each stock \( i \) and \( t \) is the number of monthly cross
regressions.

Fama and Macbeth (1973) and Miller and Scholes (1972) rejected the Sharpe-Lintner
hypothesis. They found that the average value of \( \gamma_{0i} \) in (18) is superior to the riskfree rate
and \( \gamma_{1i} \) average values are lower than the observed risk premium. Fama and French
(1992) confirm these results. Black, Jensen and Scholes (1972) using time-series
regression also found positive intercepts for low \( \beta_{it} \) portfolios and negative for high
\( \beta_{it} \) portfolios in (12).

In the other hand, average returns and their market beta relation appears to be linear. This
suggests that the Black’s model which only suggests that the beta premium is positive
seems to be a better description of the risk-return equilibrium. Moreover Black’s model is
based on the evidence that suggest that the relation between average return and beta is
flatter than estimated by Sharpe-Lintner CAPM.

Tests for condition C2 has been discussed in the previous section.
Three-factor model’s methodology

In an analogous way to Fama and French’s methodology at the end of each year from 1991-1998, stocks of the 23 companies which belonged to the IBEX 35 during this period were distributed in two groups, small or big (S or B), based on whether their market equity (ME) was below or above the median of the IBEX 35. Moreover the sample was distributed in an independent sort to three book-to-market equity (BE/ME) groups (low, medium, high; L, M or H) based on the breakpoints for the bottom 30 percent, middle 40 percent, and top 30 percent of the values of BE/ME for IBEX 35 stocks.

Six size-BE/ME portfolios (S/L, S/M, S/H, B/L, B/M, B/H) are described as the intersections of the two ME and the three BE/ME groups as in Figure 6. The benchmark factors, R_M, SMB, and HML, are built from six size-BE/ME benchmark portfolios where:

<table>
<thead>
<tr>
<th>Median ME</th>
<th>70th BE/ME percentile</th>
<th>30th BE/ME percentile</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Small Value</td>
<td>Big Value</td>
</tr>
<tr>
<td></td>
<td>Small Neutral</td>
<td>Big Neutral</td>
</tr>
<tr>
<td></td>
<td>Small Growth</td>
<td>Big Growth</td>
</tr>
</tbody>
</table>

Source: Kenneth French’s web-page

Figure 6

- R_M is the excess return on the market minus the one-month Treasury bill rate

- SMB (Small Minus Big) is the average return on three small portfolios minus the average return on three big portfolios
\( SMB = \frac{1}{3} (\text{Small Value} + \text{Small Neutral} + \text{Small Growth}) - \frac{1}{3} (\text{Big Value} + \text{Big Neutral} + \text{Big Growth}) \)

- HML (High Minus Low) is the average return on two value portfolios minus the average return on two growth portfolios.

\[ HML = \frac{1}{2} (\text{Small Value} + \text{Big Value}) - \frac{1}{2} (\text{Small Growth} + \text{Big Growth}) \]

The market return \( R_M \) can be calculated as the value or equally-weight returns on all all stocks of the size-BE/ME portfolio. The value-weight \( R_M \) is calculated as follows:

\[
R_M = \sum_{i=1}^{N} R_{it} \frac{ME_i}{\sum_{i=1}^{N} ME_i} \quad (19)
\]

while the equally-weight \( R_M \) is calculated as follows:

\[
R_M = \frac{\sum_{i=1}^{N} R_{it}}{N} \quad (20)
\]

BE is the book value of stockholders’ equity at the end of the year.

In the US studies, with a large number of stocks in the indexes, stock portfolios are used to perform the tests. In this paper due to the small number of stocks which form the IBEX 35, individual stocks are used, being the total number 23.

The excess returns on the 23 stocks, formed on size and book-to-market equity, are the dependent variables in the time-series regression.
Two different approaches are used, time-series regression like Black, Fisher and Jensen (1972) and Fama and French (1993, 1996, 1998), and cross-section regression like Fama and Macbeth (1973) and Fama and French (1992).

Remark that when the CAPM or the three factor model are tested, both are a models of expectations, therefore observed returns are used.

**Time-Series:**

**CAPM:**

This methodology is first used by Black, Fisher and Jensen (1972). The model is described as follows:

\[
    r_{it} - r_{ft} = \alpha_i + \beta_{iM} (r_{Mt} - r_{ft}) + \epsilon_{it} \quad (21)
\]

where \( \epsilon_{it} \) follows a normal distribution with zero mean and constant variance and it is independent of the rest of variables. Moreover it is implicit the linearity hypothesis between the dependent and independent variable and that \( \alpha_i \) and \( \beta_{iM} \) are constant through the whole period.

In equation (21) \( \alpha_i \) has to be equal to zero. Miller and Scholes (1972) demonstrate that there are some advantages by using excess returns. However the results using returns and excess returns are similar. Once that equation (21) is estimated, the test consists into analyze the estimation of the \( \alpha_i \) called since now \( a_i \) for each stock, because \( \alpha_i \) is not the real value and it is just and estimation. The test, that is, if it is accepted that if the real value is equal zero, is done with the following t-statistic:
\[
\frac{a_i - H_0}{S_{ai}}
\] (22)

where \( a_i \) is the estimation of the intercept, \( H_0 \) is the value of the null hypothesis (in this case it is equal to zero) and \( S_{ai} \) it is the estimation of the standard deviation of \( a_i \). The R-squared is used to interpret the model as well.

This test can be done individually for each stock or for a portfolio as Black, Jensen and Scholes suggest. Because the data sample is has only 23 stocks and it does not meet the necessary conditions to use portfolios they are not formed, and the tests are done for individual securities. Remark that it is assumed that there is not autocorrelation between the \( \varepsilon_u \) on individuals stocks for time-series regression. Otherwise it could be performed a multivariable test such the Gibbons, Ross and Shanken (GRS's) F-statistic (Fama and French 1992) used to test the multivariable significance of the intercepts using Seemingly Unrelated Regression methodology and where (21) is estimated using Generalized Least Squares. Another alternative is the Generalized Method of Moments. But these methodologies are omitted in this paper.

Three-factor model

In the three-factor model, monthly returns are regressed on the returns of a market portfolio and on the mimicking portfolios for size and BE/ME. The time-series regressions slopes are factor loadings which have an easy interpretation of risk-factor sensitivities for stocks. Time-regression regressions are suitable to analyze two important concerns of asset-pricing:

- One of the main goals is to test if assets are priced rationally, variables that are linked to average returns, such price ratios (size, BE/ME, etc) must be a good proxy to measure the sensitivity to common risk factors in stocks returns. The slopes and the R\(^2\) values are used to interpret if the mimicking
portfolios for size and BE/ME risk factors capture the variation in returns not explained by the market betas.

- In the time-series regressions, excess returns, i.e., monthly returns minus one-month T-bill rate, are the dependent variables. If the asset pricing model is well-defined the intercepts must be equal to zero.

For the three factor model, the following regression as it was seen in previous section is used for the period 1992-1998,

\[ R_{it} - R_{ft} = \alpha_i + \beta_{it}^M (R_{Mt} - R_{ft}) + \beta_{it}^{SMB} SMB_t + \beta_{it}^{HML} HML_t + \epsilon_{it} \]  \hspace{1cm} (23)

where the intercept has to be equal to zero. The significance of the betas and the R-squared are also used to interpret the model as well. To estimate equation (23) it has been used Least Squared (LS) like in traditional papers. Notice that it is assumed that there is not contemporaneous correlation among error terms across equations otherwise Seemingly Unrelated Regression (SUR) methodology should be used instead.

**Cross-section methodology.**

Cross sectional regressions are done for a specific period of time using a group of stocks. They are done in two stages: first, betas are estimated, and second regressions are done between the betas and the returns of the stocks. There are two kinds of test in this method, first, using cross sectional regression with averages, and cross sectional regression without averages, which is used by Fama and Macbeth (1973).
Cross section without averages

CAPM

In this paper it is used the cross-section without averages proposed by Fama and MacBeth (1973). In Figure 7 in the appendix it is described the methodology with a graph for a better description. First, to estimate the CAPM, they suggest that for each stock in the sample, an estimation using least squares for \( \beta_{iM} \) is obtained from the following formula:

\[
 r_{it} - r_{ft} = \alpha_i + \beta_{iM} (r_{Mt} - r_{ft}) + \varepsilon_{it} \quad t = 1, \ldots, N 
\]

which it was estimated for the periods 1991-95, 1992-96 and 1993-97 with monthly returns and equally-weighted market returns. Then, for all the securities, estimation for the risk premium \( \gamma_{t+1} \) in 1996, 1997 and 1998 is calculated using \( \beta_{iM} \) as the explanatory variable as follows:

\[
 r_{it+1} - r_{ft+1} = \gamma_{0t+1} + \gamma_{1t+1} \beta_{iM} + \varepsilon_{it+1} \quad i = 1, \ldots, N 
\]

Where \( \gamma_{0t+1} \) should be equal to the risk-free rate in t+1 and \( \gamma_{1t+1} \) to the risk premium in t+1. This methodology has the inconvenient that the estimations present a high variability between the different months, so it is necessary to calculate an average for each period. This calculation could raise problems of heteroscedasticity (due to the variability of the estimations) and autocorrelation. The heteroscedasticity is produced to the different specific risk for each security. \( \varepsilon_{it+1} \) in equation (25) captures the return
component not explained by the systematic risk. Fama and Macbeth do not take into consideration the heteroscedasticity problems in their estimations. In this paper for simplicity heteroscedasticity and autocorrelation are not considered even if they exist. And the covariance between \( \varepsilon_{i,t+1} \) and \( \beta_{iM_t} \) is assumed to be equal to zero. Although this methodology avoid the problem caused by the correlation of the residual in the cross-section regressions it is assumed that it does not exit.

The main advantage of this methodology is that in equation (25) is that \( \beta_{iM_t} \) is allowed to vary through the time. However in equation (24) remains the assumption that in the time-series methodology.

Two criteria are used to measure the performance of this process. First, the significance of the average estimated value for the risk premium, \( \gamma_1 \), calculated as follows:

\[
\bar{\gamma} = \frac{1}{N} \sum_{t=1}^{N} \gamma_{1t+1} \quad (26)
\]

This value must be significant and positive. The significance of the coefficient estimates were calculated by averaging \( \gamma_{j,t} \), \( j=1996, 1997, 1998 \). The significance of \( \gamma_k \) is measured by:

\[
t(\gamma_j) = \frac{\gamma_j}{s(\gamma_j) / \sqrt{n}}, \quad (27)
\]

where \( n \) is the number of months in the period and at the same time the number of estimates of \( \gamma_{j,t} \). In this way it is measured how much beta can explain of the variation in the following year stock returns. This method is then repeated to find estimations for the expected returns in the following years for the whole sample.
Second, the average of R-squared which explains the capacity of the beta estimation to explain differences in the returns on the individual securities in the subsequent years to the estimation.

This method was also calculated for yearly returns. Remark that the left hand side of equation (25) is independent of the data frequency used in equation (24). Using yearly returns it is avoided to some extend the high variability in the estimations, however the short length of the data period does not allow to perform the t-statistic in equation (27). Therefore the t-statistics of each year and the R-squared will be used to draw the inferences.

Again Least Squared has been used to make the estimations. Some solutions to face the econometric problems are now suggested. First to avoid heteroscedasticity problems, weighted least squared methodology can be used. Second, to consider heteroscedasticity and autocorrelation problems at the same time Generalized Method of Moments can be used. Third, to avoid heteroscedasticity and errors in the estimations maximum likelihood methodology can be used. (Gómez-Bezares, 1993).

One of the traditional problems is that estimates of beta for individual securities are imprecise. Therefore they produce measurement problems when they are employed to explain average returns. As it is stated before, portfolios are usually formed to avoid this problem. However, in this paper, they have been not formed. Therefore it is necessary to be aware that using Least Squares the estimations are not as precise as one would like. Averaging monthly returns and using yearly returns are measures to try to reduce this problem. Furthermore using individual stocks would have the advantage that allows to calculate expected return or cost of equity for an individual company if this was the goal.
Three-factor model

For the three-factor model, to make an estimation of excess return for each stock, first and estimation of the beta for each factor is calculated in a time-series regression as follows:

\[ r_{it} - r_{ft} = \alpha_i + \beta_{iM} (r_{Mt} - r_{ft}) + \beta_{is} SMB_i + \beta_{ih} HML_i + \epsilon_{it} \quad t = 1, \ldots, N \]  

(28)

The time period is the same than in the CAMP, the periods 1991-95, 1992-96 and 1993-97 with monthly data. Then estimation for the risk premium for the following years is calculated for each stock using the estimation of the betas, \( \beta_{iM}, \beta_{is}, \beta_{ih} \) as explanatory variable as follows for 1996, 1997 and 1998.

\[ r_{it+1} - r_{ft+1} = \gamma_0 + \gamma_{Mt+1} \beta_{iM} + \gamma_{st+1} \beta_{is} + \gamma_{h3t+1} \beta_{ih} + \epsilon_{it} \quad i = 1, \ldots, N \]  

(29)

or

\[ r_{it+1} = \gamma_0 + \gamma_{Mt+1} \beta_{iM} + \gamma_{st+1} \beta_{is} + \gamma_{h3t+1} \beta_{ih} + \epsilon_{it} \quad i = 1, \ldots, N \]  

(29b)

Again the criterion used to measure the performance of this process is the significance of the average estimated value for the coefficients of the betas, calculated as follows:

\[ T_j = \frac{1}{T} \sum_{t} \gamma_{jt+1} \quad j = M, s, h \quad (t = 1 \text{ o } t+1)? \]  

(30)

Similarly to the estimations done in the CAPM, yearly returns are used in equation (29) and the t-statistic and the R-squared are used to interpret equation (29).
Section V

Data

The data used to test the models belongs to the Spanish stock market, where there are 155 companies listed in the continuous market, which was in created in 1989. The continuous market is divided in fours main markets, Madrid, Barcelona, Bilbao and Valencia. The IBEX 35 has been an accurate representation of the evolution of the most liquid securities. It is a weighted index by market capitalization of the 35 most liquid companies with the highest volume traded of the 4 markets and it is a precise indicator due to the high correlation with them, close to 100%

The Spanish stock market was first formed by ‘Mercado de Corros” (1959 -1988). The continuous market started on the 24th of April of 1989 with the CATS (Computer Assisted Trading System); imported from Toronto. In that time only five companies were included in the system. On the 14th of February of 1992 the IBEX 35 was formed after a joint of other two indexes (FIEX 35 y MEFF 30). On the 4th of January of 1999, the stocks started to be trade in Euros.

The data of the Spanish stock market, concretely for the IBEX 35 is available in several web-pages (www.ibex35.es, www.megabolsa.com), moreover each index corresponding to the main four markets has its own web-page. Information of the companies is available on www.cnmv.es (national commission of market securities, it is like the SEC) and their own web-pages. Some of the data necessary to calculate the BE/ME and size ratios were extracted from the book ‘Value Creation for shareholders” (Pablo Fernandez, 2000). This data it is only available until 1998, therefore it is one of the reasons to restrict the sample period until this year.
In the Spanish market 155 companies are public listed, and the IBEX 35 is formed by 35 companies, which normally amount 85% of the total value. The IBEX 35 was formed in 1992, so the data available in the sample will amount 7-8 years. One of the characteristics of the IBEX 35 is that the companies are included on it change quite frequently and it is revised every 6 months by a commission. Therefore only the companies that were included most of the time during the sample period were included. No newly added firms where included avoiding in this way backfilling problems. However it does not included firms that have disappeared therefore the data is subject of survivor bias.

The IBEX 35 is it not adjusted for dividends. This means that an investor who holds a portfolio which replicates the IBEX 35, with the same shares and the same proportion, would obtain a higher return that the IBEX 35. This higher performance before taxes is approximately 3-4% per year.

With these requirements, the companies that were included most of the time during the period 1991-1998, and it was possible to gather information about their size and BE/ME ratios amount a total number of 23 securities.

The estimation period for the time-series regression begins with the formation of the IBEX 35 in 1992, the reason is that the formation of the interconnected stock market system could affect to the efficiency of the markets, and it is not mixed the data of the ‘mercado de corros’(1959 -1988) and the ‘continuous market’. The data of the IBEX 35 was not mixed with the data of the continuous market, even though since the formation of the IBEX 35 they have a extreme high correlation, but the intention of this paper is to measure the efficiency of the IBEX 35, so in this way data problems are avoided. Another reason to do not include the period 1988-1991 is that companies were not included simultaneously in the continuous market, they were introduced progressively. The models were estimated for the period 1991-1998 as well, drawing similar results.
One-month risk-free Treasury bills (letras del tesoro) are used as the assets with zero correlation with the stocks. The information about the Treasure bills was collected from the Spanish Bank’s web-page (Banco de España, www.bde.es)

The stocks data has the following adjustments:

- In stock splits, the stock price is multiplied or divided by the factor.
- In new issues the value of the right is discounted.
- In devolutions, the amount is discounted.
- In dividend payments, it is discounted the gross amount.

**Size and Value Premium in the Spanish Stock Market**

To study the value and size premium in the data it is used a similar methodology employed by Lakonishok, Shleifer, and Vishny (1994). First, the sample data is divided in the same way the six size-BE/ME portfolio for each year. Then, first, to study the value premium, the returns of the three companies with the highest BE/ME ratios and the three lowest BE/ME ratios are tracked through the next five years, using Cumulative Average Returns (CAR). Then, for the remaining years the same step is repeated and the returns for all the years are averaged. In this way it is possible to contrast value returns against growth returns.

The same process is done for largest and smallest stocks to verify the size premium. This time the returns of the three companies with the highest size ratio and the three companies with the lowest size ratios are tracked through the next five years.

The results show that there is a size premium of 3.26% and a value premium of 7.52% for the period 1991-1998. Due to the short length of the sample, these results cannot draw strong inferences.
Returns distribution

The returns have been calculated both for discrete and continuous time. Normally the total return price differs from the current price because it reflects the value in the selected stock, i.e. adjusted for dividends, stock split, etc. The advantage, as is stated before, of using continuous time is that there will be a proportional distribution around 0. This can be seen in tables 5 and 6 where in the continuous time, more returns distributions of the individual stocks follow a normal distribution and with higher confidence (a necessary assumption for the models tested). The results using both continuous and discrete time are similar in the tests, but the confidence in interpreting the results is different.

There are two reasons why these results are similar. First, stock prices are already adjusted for dividends, splits, repurchases, etc in the database. Second, as it is stated before, when the returns are small the results are similar between continuous and discrete time.

Returns are calculated from end-of-the-month prices and not as an accumulation of daily or weekly returns. This presents an advantage because “natural” months are used and not four-weeks-months.

One of the main assumptions in the CAMP is that investors are risk averse and that they are concerned with the mean-variance of the returns. Therefore the distributions should be defined by two numbers (Brealey and Myers, 2002): the average or expected return and the variance or standard deviation. If the returns are normally distributed investors only have to regard these two numbers. The lognormal distribution is also defined by its mean and standard deviation.

The first normality test was made using Jarque-Bera test, results are showed in tables 3 and 6, rejecting the normality for 18/25 of the stocks returns for discrete time and for 13/25 for continuous time respectively.

In a normal distribution skewness should be equal to zero. A positive value means that the distribution is skewed right while a negative value means that the distribution is
skewed left. An approximate measure of the standard error of this distribution is \((6/n)^{\frac{1}{2}}\). The sample size is 95 (months), so the standard error would be 0.2513. If a 95% confidence interval is used it would be obtained an interval of 0±0.502. If the value lies within that interval the null hypothesis is not rejected and the distribution is not skewed. Results are showed in table 1 and table 4 for discrete and continuous time respectively.

In the other hand, in a normal distribution kurtosis should be equal to 3. If the value is higher than 3, the distribution is peaked and has relatively small tails. If the value is lower than 3, the distribution is flat and has larger tails than expected in a normal distribution. The standard error of kurtosis is \((24/n)^{\frac{1}{2}}\). Again the sample size is 95, so the standard error would be 0.5026. Applying the 95% confidence the interval obtained would be 3±1.0052.

Results are showed in table 2 and table 5 for discrete and continuous time respectively. Results for tables 1-5 are resumed in the following tables:

<table>
<thead>
<tr>
<th>Test</th>
<th>Discrete</th>
<th>Null Hypothesis:Normality</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>right</td>
<td>left</td>
</tr>
<tr>
<td>Skewness test</td>
<td>23/23</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>leptokurtosis</td>
<td>platikurtosis</td>
</tr>
<tr>
<td>Kurtosis test</td>
<td>19/23</td>
<td>4/23</td>
</tr>
<tr>
<td></td>
<td>Normal</td>
<td>Non-normal</td>
</tr>
<tr>
<td>Normality test Jarque-Bera</td>
<td>8/23</td>
<td>15/23</td>
</tr>
</tbody>
</table>
There is asymmetry to the right and leptokurtosis for most of the securities. These results are similar to classic studies (Miller and Scholes 1972, Fama 1996). This is the reason to reject normality in some of the stocks. However, the results will be interpreted following the approach suggested by Fama and Macbeth (1973).

Fama (1965) suggests that returns distributions are ‘thick-tailed” compared to a normal distribution and that they are better approximated by non-normal symmetric stable distributions. This implies that when interpreting large t-statistics assuming normality, the significance levels tend to be overestimated. Therefore if the hypotheses are still not rejected, the results are even stronger.

Further evidence to use t-statistics with monthly returns is presented by Officer (1971). Under the assumption that monthly returns follow a symmetric stable distribution, he estimated that the critical value is approximately 1.8 rather than 2.

This means that interpreting the results in the usual way does not conduce to wrong inferences except for extreme values (Fama and Macbeth, 1973). Moreover they argue that monthly returns ‘have finite variances and converge toward the normal as one takes sums or averages of individual returns”. Given these arguments and the results, normality is accepted with reservations.
Finally say that Fama (1976) suggests that with more than 20 stocks chosen randomly the unsystematic risk is eliminated. In the sample, there are 23 stocks, so we can assume that this risk is eliminated.

Section VI

Empirical Analyses

The goal of this paper is to analyze if all the CAMP and the three-factor hypothesis and predictions hold for the IBEX 35 after its formation in 1992. There is a large literature of US-studies for markets such New York, London, Tokio, etc. where due to the large number of stocks included in the indexes it is possible to form portfolios which has statistical advantages. However for small markets, such Spain, it is necessary to perform a different methodology.

Remind that the equally-weighted market portfolio was used due to its theoretical advantages and the minimum differences compare to the value-weighted market portfolio. Moreover it is easier to calculate. Although the estimations are slightly different, the conclusions are similar using equally or value-weighted market portfolio. The returns were also calculated for discrete and continuous time. Due to that the differences between the returns are small, the result are similar between them. Previous literature shows that using returns and excess returns the results are also similar. To sum up, in this paper, returns are calculated in continuous time using lognormal returns, with an equally-weighted portfolio and using excess returns.

The period chosen starts in 1992, the formation year of the IBEX 35, and concludes in 1998, the year when the euro was introduced in the IBEX 35. With the introduction of the euro there were several economic changes in Spain and in the stocks markets in general, so it is convenient to do not mix the data for the inferences. Moreover in 1998-1999 several companies changed their stock nominal value to make round numbers to facilitate the market operations. For example, stocks with a nominal
value of 500 ptas, changed to 1 euros (=166.386 pts). This could affect the results, so this paper focuses only in the period 1991-1998. However, is a good estimation period, because for longer periods of 5-7 years, Fama suggests that the betas vary.

The 23 companies included in the sample are those which were traded frequently in the IBEX 35, with adjustments for some market operations (dividend, splits, repurchases, etc). These companies amount the 70-80% of the total value of the IBEX 35, so they are representative of the total population.

CAMP

Time-series:

Using (21) $\alpha_i$ and $\beta_i$ were estimated using least squares for each stock, both for the period 91-98 and 92-98 for a robustness check. Tables (8) show the beta estimations and their significance test (t-student), moreover the stability test (chow-test) is presented in table 7

All the betas are significant and all the intercepts are equal to zero with a great confidence. Intercepts are showed in table (8). The average R-squared for the whole period is 50.55%. In table 10 is presented the number of betas which are higher and lower than 1. The R-squared average is slightly higher for the period 1992-98 than for the period 1991-1998 (not showed). This result could be interpreted as a better efficiency of the continuous market once that the IBEX 35 was created.

<table>
<thead>
<tr>
<th>Table 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beta&lt;1</td>
</tr>
<tr>
<td>13/23</td>
</tr>
</tbody>
</table>
Resume of the results:

- There are not companies with betas higher than 2 or with negative values, moreover the number of companies with betas above and below to 1 is equilibrated with a slightly bigger percentage for defensive betas.
- The explanatory power of the model is quite relevant: the market explains around 50% of the variability of the returns both with equally and value market portfolios, with a higher power for the former.
- For alpha equal to 1% and 5% all the betas result significant both for equally and value-weighted market portfolios.
- For alpha equal to 5% 20 of the 23 securities present stability. Chow-test was performed, setting the breakpoint in January of 1996.

Cross-section:

There are several ways to interpret the results obtained in the cross-section regressions. To accept the CAMP following the classic version of Sharpe-Lintner, it is necessary the acceptance of the significance of the parameters $\gamma_0$ and $\gamma_1$ in (25 or 25b) simultaneously, and that their values are equal to their theoretical values, i.e., the averaged risk-free rate and risk premium respectively. The Black version is less exigent and accepts the model if the risk premium is significant. There is also the possibility, where it is accepted that the parameters are equal to their theoretical values but at the same time the significant test are not accepted. In this case the power of the test is low, so CAPM is it neither clearly accepted nor rejected. Therefore the cross section was done both with regression (25) and the following equation:

$$r_{it+1} = \gamma_0 + \gamma_1 \beta_{it} + \varepsilon_{it+1}$$

(25b)

In the cross-section regression with monthly returns, in (25b) $\gamma_0$ is significant and equal to the risk-free rate. However, remark that the average value of $\gamma_0$ is higher than the
observed risk-free rate. $\gamma_1$ is not significant, but equal to the risk premium. The average value of $\gamma_1$ is lower than the observed risk premium. The average R-squared is 6.16%.

These results present results in a similar line than previous cross-section studies; Black, Jensen and Scholes (1972), Miller and Scholes (1972), Fama and Macbeth (1973) reject that the expected return of a portfolio with zero covariance with the market portfolios is equal to the risk-free rate of the Sharpe-Lintner model. Particularly, the estimate average value of $\gamma_0$ in (25b) is higher than the average risk-free rate and the estimate average value of $\gamma_1$ is lower than the observed average risk premium.

In the cross-section regression with yearly returns, the inferences are drawn looking to the t-statistics of each year. Using (25) $\gamma_0$ is equal to zero in 1996 and 1997 but not in 1998. Using (25b) $\gamma_0$ is significant in 1996 and 1998. Moreover it is equal to the risk-free rate in 1997, but not in 1996 and 1998. The average value of $\gamma_0$ is higher than the risk-free rate.

$\gamma_1$ is significant in 1998 but not in 1996 and 1997. Moreover it is equal to the risk premium in 1996 but not in 1997 and 1998. The average R-squared is 13.23%, so it gives better results than with monthly returns. This can be due to the high variability in the beta estimations with monthly returns. However due to short length of the database, the results cannot provide clear inferences.
Three-Factor Model

Time-series:

In table 11 is reflected the number of betas that were significant for regression (23)

<table>
<thead>
<tr>
<th>Factor</th>
<th>significant</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market</td>
<td>23/23</td>
</tr>
<tr>
<td>SMB</td>
<td>16/23</td>
</tr>
<tr>
<td>HML</td>
<td>7/23</td>
</tr>
</tbody>
</table>

The average R-squared in regression (23) for the period 1992-98 is equal to 57.73%. This value is higher that the R-squared obtained in the CAPM. This means that the model has a higher explanatory power. The significant test reflects that the SMB factor has a higher influence in the model than the HML, around 70% of the individual test are significant for the SMB factor while only 30% of the individual test are significant for the HML factor.

Cross-Section

In the cross-section regressions using monthly returns a similar approach was used than in the CAPM interpretation. In equation (29b) $\gamma_0$ is significant and equal to the risk-free rate. Like in the CAPM $\gamma_1$ is not significant but equal to the risk-premium. Even though it seems that there is a value and size premium in the sample, and that in the time-series regression SMB and HML factors are significant for 70% and 30% of the securities respectively, in the cross-section regressions, $\gamma_s$ and $\gamma_h$ are not significant. However they cannot be compared to any theoretical value. The R-squared is equal to 10.91%, which is higher than the value obtained in the CAPM.
This means that the returns are better explained by the betas of the three-factor model but still with poor results. However, these results do not provide clear inferences.

With yearly returns, using (29) $\gamma_0$ is it accepted to be equal to zero in all the years. Using (29b) $\gamma_0$ is significant in 1996. $\gamma_1$ is not significant but equal to the risk premium in 1997 and 1998. The SMB betas are significant for 1997 and 1998 but not for 1996. The HML betas are insignificant for 1997 and 1998 but significant for 1996. These results cannot draw a clear inference, however, remark that the R-squared raises up to 35.91\% which shows an important improvement compared to the CAPM. This, again, can be consequence of the lower variability in the estimation when using yearly returns. A longer data set would be required for stronger conclusions.

Section VII.

Shortcomings and suggestions for future research

The results of this paper are tied to the short length of the database. Fama and French’s studies are based in a large sample. All the studies based in the U.S market have the advantaged that the data available is large. However for smaller markets, such the Spanish market, this data is not so accessible. Remark also that the methodology has been adapted to the small sample characteristics, drawing probably not such strong inferences like in the U.S studies. Moreover the goal of this paper was not to perform a deep econometric study. Ordinary Least Squares have been used in all the estimation. First, because they have been used in most of the traditional papers in this topic. Second, for simplicity. However, all the econometric problems that could arise have been addressed with possible solutions. Further research with a longer data set and more precise and advance econometric methods would give stronger inferences.
As it was seen in section I The EHM ‘rules out the possibility of any trading systems based only on currently available information that have expected profits or returns in excess of equilibrium expected profit or return” (Fama, 1970). The interrelation between national and transnational securities forces to reformulate the efficiency since another point of view, because it is possible that one market can be efficient when it is analyzed individually, but that is not efficient in a global context. It is widely known that the strong stock markets (such as New York or Tokyo) behave as a ‘guide’ for smaller market (as the Spanish market for example). Hence past information in the small markets is ‘irrelevant’ and cannot predict the evolution of the index while the information of the strong markets has predictable power.

If stock markets are efficient, the question for investment managers is what role do they play, and are paid for. Those that defend the EMH argue that their role consist mainly into match the investment with investor’s tax considerations and risk profile. Optimal portfolios will depend of factors such age, risk aversion, tax bracket, etc. So, in practice the role of the investment managers is to form a portfolio given these factors, rather than outperform the market.
Bibliography


CONTENTS

Introduction .............................................................................................................................................1

Section I.
  - Literature survey .............................................................................................................................6
  - Theoretical framework of the EMH .................................................................................................7
  - Random Walk ..................................................................................................................................8
  - Empirical evidence of EMH ............................................................................................................13
  - Theoretical challenges to the EMH and a behavioral framework ..............................................17
  - Empirical challenges to the EMH
    Limits to arbitrage ............................................................................................................................24
    Cross-Section of average returns ....................................................................................................25
    The aggregate stock market
      The Equity Premium ......................................................................................................................34
      Volatility .......................................................................................................................................35
      Predictability ..................................................................................................................................36
  - Underreaction and Overreaction ....................................................................................................36

Section II:
  - CAMP fundamentals .......................................................................................................................40
  - Relationship between average returns and betas .........................................................................41
  - CAMP anomalies .............................................................................................................................42
  - Three-factor model fundaments .....................................................................................................44

Section III.
  - CAPM derivation .............................................................................................................................46
  - Three-factor model derivation .........................................................................................................48
  - Shortcomings to test the CAMP and the three-factor model .......................................................50
Section IV.
- Early Tests……………………………………………………………..52
- Three-factor model’s methodology………………………………....54
- Time-Series
  o CAPM………………………………………………………..57
  o Three-factor model…………………………………………………58
- Cross-Section
  o CAMP………………………………………………………..59
  o Three-factor model…………………………………………………62

Section V
- Data……………………………………………………………….....63
- Size and Value Premium in the Spanish Stock Market………..65
- Returns distribution…………………………………………………..66

Section VI.
- Empirical analyses…………………………………………………..69
  o CAPM
    ♦ Time-series……………………………………………………..70
    ♦ Cross-section………………………………………………….71
  o Three-factor model
    ♦ Time-series……………………………………………………..73
    ♦ Cross-section………………………………………………….74

Section VII.
- Shortcomings and suggestions for future research……………..74

Bibliography…………………………………………………………..75

Figures and Tables……………………………………………………..81