Second Thoughts on Exporter Productivity

Philipp J.H. Schröder and Allan Sørensen
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Abstract

Empirical literature has established a positive link between firm productivity and export status, yet notable exceptions exist. The present paper shows that the underlying theory (Melitz, 2003) is in fact able to accommodate the rule as well as the exception. The fulcrum of the argument is the tension between empirical work measuring productivity based on average cost information, and theoretical work representing productivity by marginal cost. In a heterogeneous firms trade model, we compute productivity based on average cost and find that around the export-indifferent firm, exporters will be less productive than non-exporters. Furthermore, we show that this effect may feed through at the industry level.

JEL: F12, F13, F15
Key Words: Intra-industry trade, firm productivity, monopolistic competition, heterogeneous firms.
1 Introduction

Intra-industry trade models with heterogeneous firms have in recent years narrowed the gap between the theory and stylized facts of international trade. One of the central empirical findings is that export-active firms are more productive than their non-exporting counterparts, see the seminal contribution by Bernard and Jensen (1995); see Greenaway and Kneller (2007) and Wagner (2007) for comprehensive surveys of the literature that followed. In the breakthrough paper by Melitz (2003), the empirical regularity of productivity differences and heterogeneous firms is reconciled with the theory of international trade.¹

Despite its status as a stylized fact of modern trade, the finding of an exporter productivity premium is subject to some notable exceptions. To cite only a few examples, Bernard and Wagner (1997) and Wagner (2002) for Germany, Liu et al. (1999) for Taiwan, Aw et al. (2000) for Korea, Head and Ries (2003) for Japan, Damijan et al. (2004) for Slovenia, Hanson and Lundin (2004) for Sweden, Girma et al. (2004) for Ireland, Girma et al. (2005) for the UK, Castellani and Zanfei (2007) for Italy all report findings where the hypothesis of exporters having higher productivity than non-exporters is not supported for all the sectors, firm groupings, industries, years, productivity measures, or estimation specifications included.

The present paper argues that the existing theory following Melitz (2003) is in fact fully capable of capturing the rule as well as the exception. We investigate the concept of productivity in heterogeneous firms models. The underlying issue is that while the empirical work measures productivity based on average cost information, the theoretical work represents firm productivity by marginal cost.² The switch between the theoretical marginal cost concept and the empirical average cost data will most likely be unproblematic in the majority of applications, yet it contains room for a critical ambiguity. We show that once the theoretical framework is used to compute ‘observable’ productivity, as captured by empirical measures, the observable productivity of non-exporters may exceed that of exporters. The sign of the observable exporter productivity premium in an industry will ultimately depend on the specific distribution of marginal productivity draws. In particular, in proximity to the export-indifferent firm, exporters will have higher average costs and therefore lower observable productivities than non-exporters. Thus,

¹Simultaneously, Bernard et al. (2003) provide a Ricardian model with similar features and addressing the same issue. Earlier theoretical contributions on the issue were made by Schmitt and Yu (2001), Montagna (2001) and Jean (2002).
²More precisely the theoretical work ranks firms by marginal productivity, i.e. $\varphi$, in the Melitz (2003) notation, where accordingly marginal cost is $w/\varphi$. 

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if productivity is computed from average cost information, the theory does not in general predict that exporters are more productive than non-exporters.

We establish our result in a Melitz (2003)-type framework. Since a crucial driver of our findings is the decision of the export-indifferent firm, and is thus contingent on trade costs, we include additional trade costs, apart from the customary iceberg costs, to assess the robustness of our findings. Iceberg cost specifications in a marginal cost heterogeneity setting have the undesirable side effect that firms with lower marginal cost are not only more productive in producing goods, but also more productive in transporting goods. This biases the export market self-selection of firms. Accordingly, we include—in addition to iceberg costs—homogeneous unit trade costs and ad valorem trade costs. Our finding that non-exporters display higher observable productivity than exporters around the export-indifferent firm is established for all these cases.

Obviously, the effect highlighted here only matters in proximity to the export-indifferent firm, and will thus not feed through to the aggregate level for large enough marginal productivity heterogeneity. The actual distribution of marginal productivity and the resulting marginal cost distribution matters. For sufficiently narrow distributions of marginal costs in a given industry, the exporter productivity premium may become negative. In this case, the class of exporters will have higher average costs, and thus a lower observable productivity, than their non-exporting counterparts, even though they have lower marginal costs.

The non-trivial relation between the underlying distribution of marginal productivity and the aggregate results may explain why previous theoretical work has not identified this issue. For example, Baldwin (2005) computes observable productivity in a similar fashion to the present paper, going beyond the weighted marginal cost-ranking contained in Melitz (2003). But since the measure is provided for a limited class of equilibria and a specific distribution function, the effect highlighted in the present paper does not show up. Moreover, the main focus in the theoretical literature has been on effects of trade liberalization on economy-wide productivity, i.e., the novel gains from trade, not the relative productivity of exporters and non-exporters.

The remainder of the paper is structured as follows. The next section

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3In fact, our finding is robust to changes in the source of heterogeneity. The inverse ranking occurs both for marginal cost heterogeneity but also for fixed export cost heterogeneity among firms (e.g., Schmitt and Yu, 2001, Jørgensen and Schröder, 2008). See the separate appendix (available upon request) for an illustration.

4If one computes the exporter productivity premium in Baldwin (2005), the large number of very unproductive non-exporters dominates the group of relatively unproductive exporters close to the export-indifferent firm.
establishes the ranking of exporters and non-exporters in terms of observable productivity in proximity to the export-indifferent firm. Section 3 provides results for a Melitz (2003) specification and shows that the effect may feed through to aggregated levels. Section 4 derives implications for empirical research strategies. Section 5 concludes.

2 Firm-level productivity

In this section, we consider the Melitz (2003) framework, augmented to include ad valorem and homogeneous unit trade costs in addition to the customary iceberg trade costs. Despite the extension, the analysis remains highly tractable since the central results rely solely on simple qualitative properties of firm-specific productivity measures for the export-indifferent firm.

Profit expressions

As in Melitz (2003), the demand for each variety is \( q = Q \left( \frac{p}{P} \right)^{-\sigma} \) where \( p \) is the price of the variety, \( Q \) the aggregate demand, \( P \) the price index and \( \sigma \) the elasticity of demand. Monopolistic firms have production technologies with increasing returns due to fixed costs of production \( (f) \) and a constant marginal productivity \( (\varphi) \). To enter the industry, firms invest \( (f_E) \) in developing a blue-print.\(^5\) Variation in blue-prints determines firm heterogeneity, i.e. variation in blue-prints is represented by differences in marginal productivity \( (\varphi) \). Firms entering the export market face fixed export market access costs \( (f_x) \), iceberg costs \( (\tau \geq 1) \), as well as ad valorem \( (t) \) and unit \( (T) \) trade costs.

Given constant elasticity of substitution, firms set prices as a constant markup \( \left( \frac{\sigma}{\sigma - 1} \right) \) on marginal costs. Profits on the domestic and foreign markets are

\[
\Pi_{\text{Dom}} = \left( p - \frac{w}{\varphi} \right) Q \left( \frac{p}{P} \right)^{-\sigma} - wf
\]

\[
= \frac{1}{\sigma - 1} \left( \frac{w}{\varphi} \right)^{1-\sigma} \left( \frac{\sigma}{\sigma - 1} \right)^{-\sigma} Q P^\sigma - wf \quad (1)
\]

\(^5\)Firms enter the industry until the expected value of profits equals the sunk investment cost, \( f_E \). Following the literature, we ignore time discounting; instead, firms face a constant probability of death \( (\delta) \).
\[ \Pi^\text{Exp} = \left( p^* (1 - t) - T - \frac{w}{\varphi} \right) Q^* \left( \frac{p^*}{P^*} \right)^{-\sigma} - w f_x \]
\[ = \frac{1}{\sigma - 1} \left( T + \frac{w}{\varphi} \right)^{1-\sigma} \left( \frac{\sigma}{\sigma - 11 - t} \right)^{-\sigma} Q^* P^\sigma - w f_x, \quad (2) \]
where * denotes foreign market variables and \( w \) is the wage rate.

The only sunk costs are those of developing a blue-print. Thus, export-indifferent firms are defined by \( \Pi^\text{Exp} = 0 \), and firms indifferent to leave the industry are defined by \( \Pi^\text{Dom} = 0 \).

**Observable productivity**

In the theoretical literature, productivity rankings of firms are provided in terms of marginal productivity, \( \varphi \). Yet in empirical work, marginal productivity is hardly an operational concept. Accordingly we introduce here – in line with empirical approaches – a productivity measure that is based on average cost information, more precisely, value added over factor use (see also Baldwin, 2005). In the specific model at hand, this reads value added per worker, since labour is the only factor of production. Moreover, under the above assumptions, value added equals revenue less trade costs. Thus observable productivity, depending on the firms market presence, reads

\[
\rho = \begin{cases} 
\frac{p Q(\frac{p}{P})^{-\sigma}}{\frac{\sigma}{\sigma - 1}} + f & \text{if not exporting} \\
\frac{p Q(\frac{p}{P})^{-\sigma}}{\frac{\sigma}{\sigma - 1}} + \frac{T}{\sigma - 1} Q^* \left( \frac{p^*}{P^*} \right)^{-\sigma} + f + f_x & \text{if exporting}
\end{cases}
\]

**Lemma 1.** Observable productivity (\( \rho \)), contingent on export status, is continuous and increasing in marginal productivity (\( \varphi \)).

**Proof.** Insert prices (e.g. \( p = \frac{w}{\varphi} \frac{\sigma}{\sigma - 1} \) on the home market) and differentiate wrt. \( \varphi \).

The new theoretical measure (observable productivity) – which is consistent with empirical work – is positively related to marginal productivity, the conventional theoretical productivity measure in the literature. However, this is only the case for a given export status (pure domestic or export-active). Indeed, for the export-indifferent firm, observable productivity drops when switching status from non-exporting to exporting.

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6Following the literature, we impose parameter restrictions such that there is partitioning into exporters and non-exporters and no firm chooses to operate on the export market alone.
Proposition 1. The export-indifferent firm’s observable productivity ($\rho$) over all units sold is lower when it is export-active than when it is a pure domestic firm.

Proof. Denote variables of the export-indifferent firm by $\tilde{\cdot}$. Using the productivity expressions and the fact that $\tilde{\Pi}_{\text{Exp}} = 0 \iff (p^* (1 - t) - T) Q^* (\frac{p^*}{P^*})^{-\sigma} = w f_x + \tau w Q^* (\frac{p^*}{P^*})^{-\sigma}$ from (2), it follows for the observable productivity of a pure domestic, $d$, and export-active, $x$, export-indifferent firm that

$$\tilde{\rho}^d > \tilde{\rho}^x \iff \frac{p Q \left( \frac{p}{P} \right)^{-\sigma}}{\frac{w}{\varphi} Q \left( \frac{p}{P} \right)^{-\sigma} + w f} > \frac{p Q \left( \frac{p}{P} \right)^{-\sigma} + w f + \tau w Q^* \left( \frac{p^*}{P^*} \right)^{-\sigma} + w f + w f_x}{\frac{w}{\varphi} Q \left( \frac{p}{P} \right)^{-\sigma} + \tau w Q^* \left( \frac{p^*}{P^*} \right)^{-\sigma} + w f + w f_x}$$

$$\iff \frac{1}{\sigma - 1} \left( \frac{w}{\varphi} \right)^{1-\sigma} \left( \frac{\sigma}{\sigma - 1} \right)^{-\sigma} Q P^\sigma - w f > 0$$

$$\iff \tilde{\Pi}_{\text{Dom}} (\varphi) > 0.$$ 

$\square$

Corollary 1. Ranked by marginal productivity, $\varphi$, there exists a cluster of firms around the export-indifferent firm such that all exporters in the cluster have lower observable productivity than the non-exporters included.

Proof. Follows from Lemma 1 and Proposition 1. $\square$

Crucial for the above results are the empirically relevant fixed costs of exporting, $f_x$, such as the administrative burdens or costs of maintaining a distribution network. In sum the above results imply that observable productivity increases with marginal productivity ($\varphi$) contingent on export status, but shows a discrete fall when firms start to export. Thus, the theory does not in general predict that exporters have higher observable productivity than non-exporters.

Figure 1 illustrates how the presence of fixed costs pushes a wedge into the average cost rankings of firms around the export-indifferent firm. While marginal costs fall continuously for larger marginal productivity, $\varphi$, the average costs feature a step-increase at $\varphi^*_x$. This step in the average costs results in turn in the discontinuity found in the observable productivity measures (Proposition 1).

The intuition for our finding is straightforward. First, the firm that is just indifferent towards starting to export makes positive profits on the home market; otherwise it would have exited to start with. The same is true for several of its neighbouring non-exporting firms that have higher marginal
costs but still make positive profits on the home market. They all have an operating surplus that more than covers their fixed costs of home production: i.e., observable productivity exceeds $w$. Second, consider a situation where the indifferent firm switches from non-exporting to exporting. Then we have added a zero-profit activity (namely exporting) to an otherwise profitable firm. In particular, the operating surplus that the export-indifferent firm can make on the foreign market suffices to exactly cover the fixed costs of exporting: i.e., observable productivity on the export activity equals $w$. Building the average cost across all sold units (i.e., the profitable domestic sales and the zero-profit foreign sales), the indifferent firm’s average costs must have gone up when switching status from non-exporter to exporter. Put differently, average profitability across all units produced must have gone down, and hence its observable productivity must now be lower than that of the neighbouring non-exporting firms.

3 Industry-wide exporter productivity premium

In the previous section, we examined rankings of firms’ observable productivity in proximity to the export-indifferent firm; we have shown that they go against the common empirical finding. This occurs when productivity is computed – in line with the empirical measures – from average cost information.
Obviously, this effect – since limited to the proximity to the export-indifferent firm – will typically be dominated in empirical work by the presence of very large and productive exporters, or very inefficient non-exporters. Hence the robustness of the positive exporter productivity premium found in empirical studies. We will now identify conditions under which observable productivity among all exporters may fall below the average observable productivity of all non-exporters within an industry, i.e., identify conditions compatible with the exceptions to the empirical consensus cited in the introduction.

Consider a two-country Melitz (2003) model taking account of general equilibrium effects, including endogenously determined cut-off values and firms’ entry decisions. In particular, assume that innovation processes fail with probability \( \Gamma \in [0, 1] \) and that after successful innovation, firms draw marginal productivities (\( \varphi \)) from a Pareto distribution \( G(\varphi) = 1 - \left( \frac{\varphi}{\varphi_0} \right)^{-k} \), where \( k \) is the shape parameter. In particular we consider sales weighted average observable productivities including sunk entry costs. The inclusion of sunk costs into the productivity measures biases the result against our finding, since exporters are able to spread the sunk costs over more units. We can state:

**Corollary 2.** If

\[
 f_x > (f + \delta f_E) \left[ (1 + \tau^{1-\sigma}) \frac{k}{k - (\sigma - 1)} - 1 \right],
\]

there exists \( \varepsilon > 0 \) and \( \hat{\Gamma} \in [0, 1] \) such that for all \( \Gamma \in [\hat{\Gamma} - \varepsilon, \hat{\Gamma}) \) average observable productivity of non-exporting firms, \( \rho^d \), exceeds the average observable productivity of exporters, \( \rho^e \).

**Proof:** See appendix.

In the class of equilibria depicted in Corollary 2, all firms with a successful innovation process stay in the industry, i.e., \( \varphi^* < \varphi_0 \) in the terminology of Melitz (2003). Importantly, Corollary 2 states that the exporter productivity may turn negative if the industry in question displays limited heterogeneity.

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7The literature on innovation pinpoints substantial failure rates in the innovation process, ranging from 30 to 90 percent, see Karakaya and Kobu (1994) or more recently Leenders and Voermans (2007). The inclusion of innovation failure allows us to identify negative exporter productivity premia even when including sunk entry costs in the productivity measure, since the risk of innovation failure decouples actual entry costs from the expected entry costs, which drive firm entry and flow profits of active firms. In the appendix we show analytical conditions for a negative exporter productivity premium to occur with zero innovation failure (\( \Gamma = 0 \)).
Namely, condition (3) is less strict when firms are more homogenous (higher $k$) ensuring a large fraction of firms inside the cluster identified in Corollary 1. This finding differs from Baldwin (2005), where the premium is positive for all analyzed equilibria. The reason is that Baldwin (2005) only considers equilibria with $\varphi_0 < \varphi^* < \varphi^*_x$. For such equilibria, the Pareto distribution gives substantial mass to inefficient non-exporting firms, thus making it impossible to identify the effect established in the present paper. It is straightforward to design distributions with limited but strictly positive mass around $\varphi^*$, generalizing our result for equilibria with $\varphi_0 < \varphi^* < \varphi^*_x$.

![Figure 2: Exporter productivity premium in percent of pure domestic firms productivity.](image)

In the empirical work, the productivity premium is frequently investigated based on unweighted average productivity: for example, by regressing observed productivity on export status based on firm-level data. Even though one cannot make clear analytical statements about the unweighted averages, intuition suggests that the export productivity premium is likely to be lower using the unweighted than the sales-weighted average. In the latter case, the highly productive exporting firms with high sales only enter with the average weight. Figure 2 shows a numerical illustration for the above model with the unweighted exporter productivity premium. In line with Corollary 2, the premium decreases as firms become more homogenous ($k$ increases) and in
this case eventually turns negative. It is straightforward to extrapolate the above results to an augmented version of the standard Melitz (2003) framework including several industries with industry-specific parameters and industry-specific marginal productivity distributions. Such a model could immediately mirror the empirical picture of a positive exporter productivity premium for most but not all industries, countries, years, etc.

4 Implications for empirical research strategies

By examining observable productivity in a Melitz (2003) type setting, we have shown that the existing theory permits the productivity rankings between exporters and non-exporters to be inverted. The key point is that empirical measures of productivity must be computed from average cost information, while the theory ranks firms according to marginal costs. Thus, predictions derived from theory have to take account of fixed costs to become compatible with the empirical approach. Fixed costs – for example, the fixed costs of exporting – are a central element of the new theory. Such fixed costs vary in size, differ across firms, and are well established in the empirical literature, e.g., Roberts and Tybout (1997); Das et al. (2001).

The findings of the present paper add several novel perspectives to the empirical discussion. In particular, we have shown that the heterogeneous firms trade theory following Melitz (2003) contains additional rich results for predictions on exporter productivity. Firstly, bringing the theoretical productivity measure in line with empirical work, the theory suggests that for certain countries, industries, or periods, it might well be that groups of non-exporters in an industry display higher measured productivity than groups of exporters in the same industry. Secondly, the actual distribution of marginal productivity draws in the industry matters crucially for the theoretical prediction. For example, relatively more homogenous industries – i.e., those with little heterogeneity in the distribution of marginal productivity – will more likely display a negative exporter productivity premium. Thirdly, our results provide a new perspective on the empirical pre- and post-entry productivity differences, related to the causality between export status and firm productivity, i.e., learning from exporting. Our paper suggests that when

8The parameter values used in Figure 2 are $\sigma = 3, f = f_z = f_E = \delta = 0.1, \tau = 1.2$ and $\Gamma = 0.8$. Sunk costs are not included in the measures. When including sunk costs, the premium turns negative for $k = 24$. 
comparing observable and not marginal productivity, the theory predicts the existence of pre-entry productivity advantages of future exporters compared to future non-exporters, yet post-entry observable productivity – on the individual firm level – should, ceteris paribus, drop. Such an effect would distort measures of learning effects. The central reason is that exporting firms may face recurring and sunk fixed export costs, which by definition will depress their observable productivity compared to their previous non-exporting periods.

In this light, the exceptions to the empirical regularities on exporter productivity may deserve further examination. For example, Hanson and Lundin (2004) find for Sweden that exporters may be less productive than non-exporters, i.e. that there is a negative exporter productivity premium. Other examples of such exceptions are reported in Bernard and Wagner (1997), Liu et al. (1999), Aw et al. (2000), Head and Ries (2003), Damijan et al. (2004), Girma et al. (2004), Girma et al. (2005) and Castellani and Zanfei (2007). These results have previously been thought to go against the theoretical prediction. The current paper suggests that these types of ‘exceptions’ could, for example, be driven by a more narrow (more homogeneous) productivity distribution among manufacturers. Moreover, in terms of pre- and post-entry productivity differences, the empirical literature succeeds in identifying pre-entry differences, yet for post-entry differences (learning effects) the evidence is mixed, see, for example, Greenaway et al. (2005) or the surveys of Greenaway and Kneller (2007) and Wagner (2007). These difficulties in establishing post-entry differences are in line with our theoretical results.

Obviously, the present paper does not have the mission or the space to verify such alternative explanations empirically. Yet it is noteworthy that, for example, in Girma et al. (2004), an ambiguous finding when ranking pure domestic and export-active firms contrasts with a clear ranking vis-à-vis multinationals, and that multinationals feature a broader spread in the productivity distribution.

Overall our results indicate that an explicit and careful treatment of fixed and variable costs is needed when examining firm productivity. Recent approaches that apply such explicit treatments include the works by Aw et al. (2008) and Lawless and Whelan (2008).

5 Conclusion

In recent years, the empirical international economics literature has established a positive link between firm productivity and export status. Yet, some notable exceptions exist. The present paper shows that the workhorse model
of heterogeneous firms trade, Melitz (2003), does in fact accommodate the rule as well as the exception. This prospect of a negative exporter productivity premium is possible at the firm level and the industry level, and is robust to changes in the specification of trade cost and changes in the source of firm heterogeneity. Thus, heterogeneous firms trade theory does not, in general, predict that exporters have higher observable productivity than non-exporters.

The key driver is that empirically observable productivity is computed from average cost information, while the theory ranks firms according to marginal costs. Thus, predictions derived from theory must take account of fixed costs in order to become compatible with the empirical approach.

The findings of this paper have important implications for future empirical work. Firstly, in trade models with heterogeneous firms, it will not generally be true that exporters are more productive than non-exporters, since empirically relevant productivity measures include fixed costs of production and market access, i.e., compute observable productivity. Secondly, the actual predictions of the theory for the sign of the exporter productivity premium in an industry – measuring observable productivity – will depend on the distribution of marginal productivity in the industry. Thirdly, possible firm-level learning effects from engaging with foreign markets (post-entry productivity developments) will be blurred to some extent by a drop in observable productivity that stems from the fixed market access costs incurred by exporters.
Appendix: Proof of Corollary 2

Consider a two-country symmetric equilibrium in which we allow for innovation failures. Innovation fails with probability $\Gamma \in (0, 1)$. Conditional on successful innovation, firms draw $\varphi$ from a Pareto distribution with scale parameter $\varphi_0$ and shape parameter $k$.\(^9\) We focus on equilibria in which no firms with a successful innovation exit the market endogenously but some choose not to export, i.e. $\varphi^* < \varphi_0 < \varphi^*_x$ in the terminology of Melitz (2003).

The export productivity threshold $\varphi^*_x$ is defined by $\Pi^{Exp}(\varphi^*_x) = 0$ implying for $T = t = 0$ that
\[
\varphi^*_x = \frac{\tau}{\sigma - 1} \left( f_x \sigma^\sigma \right)^{\frac{1}{\sigma - 1}} \left( Q P^\sigma \right)^{\frac{1}{1 - \sigma}}
\]

The free entry condition reads (expected value of flow profits equal to sunk entry costs)
\[
(1 - \Gamma) \left[ \int_{\varphi_0}^{\infty} \Pi^{Dom}(\varphi) dG(\varphi) + \int_{\varphi_0}^{\infty} \max \{ \Pi^{Exp}(\varphi), 0 \} dG(\varphi) \right] = \delta f_E
\]

Inserting profit expressions and the Pareto distribution, the condition can be written as\(^{10}\)
\[
\frac{1}{\tau^{1 - \sigma} (\sigma - 1) k} \left( \frac{\varphi_0}{\varphi^*_x} \right)^{\sigma - 1} + \frac{(\sigma - 1)}{k - (\sigma - 1)} \left( \frac{\varphi_0}{\varphi^*_x} \right)^k = \frac{\delta f_E}{1 - \Gamma} + \frac{f}{f_x} \tag{4}
\]

Defining group-level productivity as total value added divided by total labour costs (including innovation costs of the active firms), we have that the groups of pure domestic firms ($d$) and exporters ($x$) have

\(^9\)Cumulative density is given by $G(\varphi) = 1 - \left( \frac{\varphi}{\varphi_0} \right)^k$ for $\varphi \geq \varphi_0$. Furthermore, it is assumed that $k > \sigma - 1$ to ensure that expected profit is bounded.

\(^{10}\)Despite using the Pareto distribution, there is no closed-form solution to the model, as we consider an equilibrium with $\varphi^* < \varphi_0$. 

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that the free entry condition becomes

\( \rho^d = \frac{\binom{\sigma}{\sigma-1}^{1-\sigma} Q P^\sigma \varphi_0 \frac{k}{(\sigma-1)} \left( \varphi_0^{\sigma-1-k} - \left( \varphi_x^* \right)^{\sigma-1-k} \right)}{\binom{\sigma}{\sigma-1}^{1-\sigma} Q P^\sigma \varphi_0 \frac{k}{(\sigma-1)} \left( \varphi_0^{\sigma-1-k} - \left( \varphi_x^* \right)^{\sigma-1-k} \right) + (f + \delta f_E) \left[ 1 - \frac{\varphi_0}{\varphi_x^*} \right]^k} \)

\( = \frac{\sigma - 1}{\sigma} + \frac{(f + \delta f_E)}{\sigma \tau^{1-\sigma} f_x \frac{k}{(\sigma-1)}} \left(\frac{\varphi_0}{\varphi_x^*}\right)^{\sigma-1} - \left(\frac{\varphi_0}{\varphi_x^*}\right)^k \right]^{-1}\)

\( \rho^x = \frac{\binom{\sigma}{\sigma-1}^{1-\sigma} Q P^\sigma \varphi_0 \frac{k}{(\sigma-1)} \left( \varphi_0^{\sigma-1-k} - \left( \varphi_x^* \right)^{\sigma-1-k} \right)}{\binom{\sigma}{\sigma-1}^{1-\sigma} Q P^\sigma \varphi_0 \frac{k}{(\sigma-1)} \left( \varphi_0^{\sigma-1-k} - \left( \varphi_x^* \right)^{\sigma-1-k} \right) + (f + \delta f_E + f_x) \left(\frac{\varphi_0}{\varphi_x^*}\right)^k} \)

Comparing the groups, we have

\( \rho^d > \rho^x \iff \frac{(f + \delta f_E)}{(f + \delta f_E + f_x)} \left(\frac{\varphi_0}{\varphi_x^*}\right)^{\sigma-1} - \left(\frac{\varphi_0}{\varphi_x^*}\right)^k \right] \equiv g \left(\frac{\varphi_0}{\varphi_x^*}\right) \) (5)

where \( \lim_{x \to 1} g \left(\frac{\varphi_0}{\varphi_x^*}\right) = \frac{k-(\sigma-1)}{k} \).

Next we show that this inequality may hold in equilibrium. From (4) we have that \( \frac{\varphi_0}{\varphi_x^*} \to 1 \) as \( \Gamma \to 1 \). From (4) we have that \( \frac{\varphi_0}{\varphi_x^*} \to 1 \) as \( \Gamma \to 1 \). From (4) we have that \( \frac{\varphi_0}{\varphi_x^*} \to 1 \) as \( \Gamma \to 1 \). Inequality (5) for \( \Gamma \to \hat{\Gamma} \) becomes

\( f_x > \frac{(f + \delta f_E)}{\left(1 + \tau^{1-\sigma} \frac{k}{(\sigma-1)} - 1\right) f_x} \) (6)

which holds for \( f_x \) sufficiently large. Thus provided (6) holds then for \( \Gamma \) sufficiently close to \( \hat{\Gamma} \) there exists a set of equilibria with \( \varphi_x^* < \varphi_0 < \varphi_x^* \) and \( \rho^d > \rho^x \).

If we exclude the sunk entry costs \( f_E \) from the productivity measures, we have an equilibrium with the above properties for \( \Gamma = 0 \). To see this note that the free entry condition becomes

\( \frac{1}{\tau^{1-\sigma} k^{\sigma-1} \left(\frac{\varphi_0}{\varphi_x^*}\right)^{\sigma-1} + \frac{\sigma-1}{k^{\sigma-1} \left(\frac{\varphi_0}{\varphi_x^*}\right)^k}} = \frac{\delta f_E + f}{f_x} \)
and $\delta f_E \rightarrow \frac{1}{\tau} \frac{\kappa^{(\sigma-1)} + \kappa^{(\sigma-1)}}{\tau_f} - f \Rightarrow \frac{\bar{\omega}}{\bar{\omega}^*} \rightarrow 1 \Rightarrow g \left( \frac{\bar{\omega}}{\bar{\omega}^*} \right) \rightarrow \frac{k - (\sigma - 1)}{k}$ and inequality (5) in this limit reads

$$f_x > f \left( \frac{\sigma - 1}{k} + \tau^{1-\sigma} \right) \frac{k}{k - (\sigma - 1)} > f \tau^{1-\sigma}$$

which is again satisfied for sufficiently large $f_x$. 
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