ON THE RELATIONSHIP BETWEEN IMPLIED AND REALIZED VOLATILITY: EVIDENCE FROM THE NORWEGIAN OPTION AND EQUITY MARKETS
ABSTRACT:

The thesis presents evidence on the relationship between implied and realized volatility from the Norwegian option and equity markets. Studies of implied volatility in highly liquid markets have been numerous. However, much less is known about implied volatility’s behavior in markets with lower liquidity. The sampling procedure applied is in large inspired by Christensen and Prabhala (1998), who sample data on a monthly, non-overlapping basis, to avoid econometric problems due to “telescoping” data. The Black and Scholes option pricing formula, specifically taking dividends into account, is used when calculating implied volatilities. To test the information content of implied volatility, we apply ordinary least squares regression analysis, in addition to instrumental regression and root mean square deviation. This allows us to compare our results with a vast body of literature applying a similar way of calculating and testing implied volatility. We find that implied volatility can be argued to be an unbiased and efficient predictor of future realized volatility when we use the efficiency criteria put forward by Christensen and Prabhala (1998), Hansen (1999) and Christensen and Hansen (2002). They argue that implied volatility is efficient when it subsumes the information in lagged realized volatility. However, we extend the market information set in a natural way and calculate a GARCH(1,1) forecast. Thus, we apply a stricter, and more realistic, efficiency criteria. Implied volatility do not subsume the information in our GARCH(1,1) forecast and must therefore be considered inefficient. Our GARCH(1,1) forecast dominates implied volatility in an encompassing regression, however, both implied volatility and the GARCH(1,1) forecast are significant, making a combined forecast preferable in the Norwegian market.
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1 Introduction

The thesis’ main focus is to analyze the relationship between implied volatility and realized volatility in the Norwegian stock market. In common option pricing formulas, all parameters that are used to solve for an option’s price can be directly observed in the market, except volatility. Thus, if we observe the current market price of an option, it is possible to solve for volatility. This volatility is known as the implied volatility of an option and is widely regarded as the option market forecast of future return volatility over the remaining life of the relevant option. In the Norwegian market it is possible to compute the implied volatility for options using the OBX index as underlying. The OBX index is a tradable index with exchange traded futures and options available and hence a natural choice for the analysis. Data were provided by Oslo Børs, constituting the period February 1997 to February 2007.

In addition to a narrow analysis the Norwegian market, we will also investigate to which extent the empirical evidence from the US market can be transferred to the significantly smaller market at hand. Due to less frequent trading, some measurement errors are expected to be even more pronounced, compared to the US market. The sampling and analysis methodology is in large inspired by Christensen and Prabhala (1998). To the author’s knowledge, they are the first to apply a monthly sampling procedure for extracting implied volatility. This sampling technique enables us to avoid econometric problems in the regression analysis, e.g., reduce autocorrelation in the error terms.

To solve for implied volatility, we apply a version of the Black and Scholes (1973) option pricing formula, specifically taking dividends into account. Even though there are apparent arguments against the use of this formula, it gives a convenient starting point for analyzing the problem at hand. In addition, using the Black Scholes formula (1973) allows us to compare with a vast literature, applying the same formula to extract implied volatilities. Unlike Christensen and Prabhala (1998) we do not only analyze the near-the-money call implied volatilities, but broaden the analysis by extracting implied volatility for the two closest to at-the-money call options and the two closest to at-the-money put option in each time period. This gives us the benefit of being
able to create a weighted average of option implied volatilities, something which is suggested in the literature (e.g. Ederington and Guan 2002) being especially advantageous in less liquid markets. Thus, this paper analyze the single call and single put option implied volatility being closest to at-the-money, in addition to a weighted average of all our four option implied volatility measures.

We apply a regression model to analyze the information content of implied volatility in the Norwegian stock market:

$$h_t = \alpha_0 + \alpha_1 i_{x,t} + e_t.$$ \hspace{1cm} (1.1)

Here, $h_t$ denotes the log-realized volatility for period $t$ and $i_{x,t}$ denotes the ex ante log-implied volatility forecast for implied volatility series $x$, in period $t$. In the literature it is usual to assess at least three hypotheses on the basis of this model:

1. If $\alpha_1$ is non zero and significant, this means that implied volatility contains some information about future realized volatility.

2. It can be said that the forecast is unbiased if $\alpha_0 = 0$ and $\alpha_1 = 1$, and these two restrictions are jointly significant

3. Finally, implied volatility is efficient if the residuals $e_t$ are white noise and uncorrelated with any variable in the market information set.

This will enable us to analyze the information content of implied volatility. Nevertheless, this is inherently a joint hypothesis of the forecasting ability of implied volatility and the efficient market hypothesis. However, the market efficiency is so fundamental when working with financial data, that we assume market efficiency also in the Norwegian option market. Next, we do an encompassing regression, comparing our implied volatility series with lagged realized volatility:
\[ h_t = \alpha_0 + \alpha_1 i_{x,t} + \alpha_2 h_{t-1} + \epsilon_t. \]  

(1.2)

Here, \( h_{t-1} \) denotes lagged log-realized volatility. Thus we can analyze if implied volatility subsumes the information in lagged realized volatility. If implied volatility subsumes all the information in lagged realized volatility, \( \alpha_2 \) should be insignificantly different from zero. We also estimate an encompassing regression which enables us to compare our different implied volatility forecasts:

\[ h_t = \alpha_0 + \alpha_1 i_{x,t} + \alpha_2 i_{y,t} + \epsilon_t, \]  

(1.3)

where

\[ x = c, p, a \quad \text{and} \quad y = c, p, a \quad \text{and} \quad x \neq y. \]

This enables us to analyze if any implied volatility forecast has a superior information content and subsumes the information in another implied volatility forecasts. In addition to this, we can see if a combined forecast is preferable, e.g., between put and call implied volatility.

When calculating implied volatility, the process is subject to sources of error. Some errors are due to how we choose to calculate implied volatility, e.g. by choosing a Black and Scholes framework, when we know that volatility is, de facto, time varying. Other errors are due to the nature of an option market, e.g. bid-ask spreads, non-simultaneous trading and infrequent trading. Since the number of option-contracts traded in the Norwegian market is significantly smaller than in the commonly analyzed US market, the issue of liquidity is of special interest. Mayhew and Stivers (2003) found a relationship between implied volatility’s ability to subsume the information in a GARCH forecast and the liquidity of the option. A comparison between the number of option contracts traded in Mayhew and Stivers’ (2003) study, the Norwegian market and the US market is carried out. However, it is out of the scope of the present thesis to measure the magnitude of the measurement errors - we simply want to argue for their existents.
To adjust for measurement errors, we estimate equation (1.2) and (1.3) with the help of *instrumental variables*. Instrumental variable regression is useful, e.g., when measurement errors in variables are present. To statistically test if any measurement errors are present, we apply the Hausman test.

A separate study of the relationship between implied and realized volatility is interesting in its own right. However, when analyzing the information content of implied volatility and its forecasting ability, it is natural to compare implied volatility with other frequently used volatility forecasting models. Thus, we introduce a GARCH(1,1) forecast and set up an encompassing regression in order to compare our GARCH(1,1) volatility forecast with our implied volatility forecasts:

\[ h_t = \alpha_0 + \alpha_1 i_{x,t} + \alpha_2 j_t + \alpha_3 h_{t-1} + e_t \]  

Here, \( j_t \) is the logarithmic transformed GARCH(1,1) forecast series. We will use equation (1.4) in mainly three ways; (1) to compare implied volatility and our GARCH(1,1) forecast, (2) to compare our GARCH(1,1) forecast with lagged realized volatility, and (3) analyze the information content in our GARCH(1,1) forecast itself. We can also see if a combined implied volatility/GARCH(1,1) forecast is preferable. In contradiction to Christensen and Prabhala (1998), Christensen and Hansen (2002) and Hansen (1999), which use lagged realized volatility as the market information set, we extend the information set to also include a GARCH(1,1) forecast. Thus, we apply a more realistic test with regard to if implied volatility is an efficient forecast or not. In addition to our regression analysis, root mean square error (RMSE) is applied to test for forecasting accuracy.

In the next chapter, a review of the literature concerning implied volatility is offered. In addition to the literature which solely studies implied volatility, it is natural to mention studies which compare implied volatility with other volatility forecasting models utilizing past time-series, e.g., GARCH-type forecasts. In chapter three, a qualitative introduction to the Norwegian stock and option market is given, in addition to a discussion of OBX index historical data descriptive statistics. In chapter four, we formally go through our sampling and calculation processes, which leads us to our
implied volatility, realized volatility and GARCH(1,1) time-series. Various sources and explanations for errors in variables are offered, in addition to the calculation and discussion of descriptive statistics for our time-series. Time-series properties are also investigated. The models used to analyze our time-series are presented in chapter five, and the results, as well as relevant comments and discussions, are given in the subsequent chapter. In chapter seven we summarize the most important findings in the thesis, and in chapter eight areas for further research are suggested and the thesis’ limitations are commented upon.
2 Literature review

Attempts to understand and explain the true nature of option-implied volatility has been vast in numbers. The research area was initiated much due to the famous work of Black and Scholes (1973) and their option pricing formula. In this formula, all parameters needed to solve for an option’s price, except volatility, can be directly observed in the market. Thus, it is possible to solve for volatility, if we observe the currently traded price for a certain option. With the use of the Black and Scholes (B-S) formula, we find the implied volatility, which is the non-stochastic volatility over the remaining lifetime of the option.

Option weighting schemes were central in the first branch of the implied volatility literature. In essence, these weighting schemes put different weights on the different option implied volatilities, often with regards to how close or far away the option is from being at-the-money. Thus, the implied volatility from a deep-in-the-money option will be given less weight than the implied volatility from an at-the-money option. The weighting schemes were introduced to reduce noise in the implied volatility estimates. The noise, in turn, stems from the fact that different implied volatilities are observed when the option has different strike prices and time to maturity. The idea behind the weighting schemes is that if the B-S pricing model is correct, then deviations from this true value are noise and can be reduced by introducing more observations.¹

Latané and Rendleman (1976) were the first to investigate the relationship between implied and realized volatility. They find it unreasonable to give equal weight to all implied volatilities, as some options are less sensitive to a precise specification of the standard deviation, thus likely to be unrepresentative to the market’s underlying expectation. Their solution to this problem was to weight the implied standard deviation by the partial derivative of the B-S equation,

\[ WISD_{it} = \left[ \sum_{j=1}^{N} ISD_{ijt}^2 \times d_{ijt}^2 \right]^{0.5} \times \left[ \sum_{j=1}^{N} d_{ijt} \right]^{-1}, \] (2.1)

where WISD denotes the weighted implied standard deviation\(^2\) and \(d_{ijt}\) denotes the partial derivation of the price of option \(j\) of company \(i\) in period \(t\) with respect to its implied standard derivation using the B-S model - referred to as an option’s vega in the option literature. However, the weighting scheme has been subject to criticism, as the weights do not sum to one. As for their findings, they concluded that weighted implied standard deviation (WISD) is generally a better predictor of future variability than standard deviation predictors based on historical data. In the cross sectional analysis, 24 companies over a 38 week sample period were considered, and a correlation of 0.82 was found between implied standard deviations and realized volatility. Another weighting scheme, used by Trippi (1977) and by Schmalensee and Trippi (1978), was inherently simple and placed equal weights on all \(N\) implied volatilities:

\[ \hat{\sigma} = \frac{1}{N} \sum_{j=1}^{N} \sigma_i. \] (2.2)

As the B-S formula is able to price some options more precisely than others, this suggestion is contra intuitive and dominated by other models in the succeeding literature. Chiras and Manaster (1978) suggested to weight the implied volatility by volatility elasticities,

\[ \hat{\sigma} = \frac{\sum_{i=1}^{N} \delta \times C_i \delta \times \sigma_i}{\sum_{i=1}^{N} \delta \times C_i \delta \times \sigma_i}, \] (2.3)

\(^{2}\) ISD equals implied volatility, the difference being only a matter of notation.
while Beckers (1981) and Whaley (1982) suggested yet an approach, where the following loss function should be minimized at any single observation day:

\[ f(ISD) = \frac{\sum_{i=1}^{I} w_i [C_i - BS_i(ISD)]^2}{\sum_{i=1}^{I} w_i}. \quad (2.4) \]

Here, \( C_i \) is the market price of option \( i \) and \( BS_i \) the Black and Scholes option price. The weights may be chosen in many ways, e.g., B-S vegas or equal weights as discussed earlier. Beckers did not disagree with the weighting scheme of Latané and Rendleman, though he argued that even less weight should be put on outliers from deep in and out of the money options. Becker concludes that most of the relevant information from implied volatilities is reflected in near-the-money options. Consistent with this, Whaley (1982) and Gemmill (1986), compared the different weighting schemes and found that most information can be found in nearest-the-money options, followed by the minimum-squared-pricing-error measure.

In addition to the paper of Latané and Rendleman (1976), Schmalensee and Trippi (1978), Chiras and Manaster (1978) and Beckers (1981) all found that implied volatility is a better volatility forecast than historical volatility. In the succeeding literature, critique has been directed towards these first research papers due to spurious data sets, focus on static cross-sectional tests, in addition to not, in some papers, have taken dividends into account (e.g., Jeff Fleming (1998)).

The ever-increasing popularity of the use of derivatives instruments and technological leapfrogs during the last thirty years, have partly motivated the direction of research within the field of volatility forecasting in general. One financial innovation as such is the index option, introduced by the Chicago Board Option Exchange (CBOE) in 1983. Several studies (E.g.: Day and Lewis, 1992; Canina and Figlewski, 1993; Christensen and Prabhala, 1998) has been carried out by analyzing the S&P 100 index option, which is listed at the CBOE and use the Standard & Poor 100 (S&P 100) as
underlying. In addition to the opportunity of using index option in their studies, the subsequent scholars were able to analyze data more thoroughly with the means of time series analysis. This also created problems, as the analysis suffered from datasets with “built-in” errors and econometric problems when doing the regression analysis. No unified answer has yet been found to these problems, as researchers are trying to cope with them in different ways.

Harvey and Whaley (1991) examine the problem of errors-in-variables (EIV) when studying S&P 100 index option implied volatility. Four sources of error - estimation procedures, nonsimultaneous price problem, bid/ask price effect and infrequent trading of index stocks - are identified and scrutinized with regards to impact on implied volatility calculations. With a estimation procedure taking into account the special features of American options they find that nonsimultaneous prices and bid/ask price effect creates a significant problem when calculating implied volatility, while infrequent trading does not appear to affect implied market volatility in a meaningful way.

Day and Lewis (1992) study S&P 100 index options with expiries from 1985 to 1989, while Lamoureux and Lastrapes (1993) study options on ten stocks with expiries from 1982 to 1984. Both conclude that implied volatility is biased and inefficient; moreover they find that historical volatility subsumes the information existing within implied volatility. Christensen and Prabhala (1998) point out that both the study by Day and Lewis (1992) and by Lamoureux and Lastrapes (1993) use overlapping samples and are characterized by a “maturity mismatch” problem. Day and Lewis (1992) use options with up to 36 trading days in examining a one-week-ahead prediction power and Lamoureux and Lastrapes (1993) use options with up to 129 trading days in examining the one-day-ahead power of implied volatility. This apparent mismatch makes the results difficult to interpret.

Canina and Figlewski (1993) findings are even more negative then the two above-mentioned studies concerning the conclusion, finding that implied volatility has virtually no correlation with future volatility, and that implied volatility does not incorporate the information contained in lagged realized volatility. Their data sample constitute of
closing prices for all call options on the OEX index\(^3\) in the period from March 1983 to March 1987. Options with more than 127 or fewer than 7 days to maturity and those more than 20 points in- or out-of-the-money were eliminated. To take into account the value of early exercise, they used a binomial model with 500 time steps, accounting for dividends. After dividing the data into four “time-to-maturity” groups and eight “intrinsic value” groups, they calculated the realized volatility over the remaining life of the option for each group and regressed those estimates on the implied volatility for each group. Canina and Figlewski (1993) explain the unexpected negative results with limits to arbitrage, because arbitrage between the OEX index option and the underlying is difficult, in addition to liquidity constraints and investor tastes for special payoff patterns. Geske and Kim (1994) and Christensen and Prabhala (1998) have argued against Canina and Figlewski’s (1993) explanations. They point out that inefficiency is unlikely, taken into account the liquidity, depth and trading activity in the OEX options market. Further they argue that the Black-Scholes is valid as an index option formula even though continuous trading in the cash index markets is not possible. They refer to Brennan (1979) and Rubinstein (1976) and their work on one-period equilibrium representative agent models, which also lead to the Black-Scholes formula. Also, Constantinides (1994) shows that transaction costs do not have first-order influence on option prices. Hence, the arbitrage argument is turning rather weak in the eyes of Christensen and Prabhala (1998). Finally, they find it odd that no significant relationship between implied and realized volatility can be established, as “…all option pricing theory of which we are aware of implies that option prices should be positively correlated with the underlying asset’s volatility…”.

Jorion (1995) finds that both moving average and GARCH forecasts has lower explanatory power than implied volatility, and he describes the results as being in sharp contrast to the work of Canina and Figlewski. However, the forecast is still biased, even after accounting for measurement errors and statistical problems. Jorion (1995) focus on options on currency futures traded on the Chicago Mercantile Exchange (CME). Here, both the option and the underlying are traded side-by-side, and close at the same time. Problems with nonsimultaneous quotes are also minimized as transaction costs between the future and option market is low. The data are gathered from three different currencies,

\(^3\) OEX is S&P 100 index option’s ticker

Another approach for evaluating implied volatility is taken upon by Engle et.al. (1994). They assess the performance of a GARCH forecast and a implied volatility forecast in the time period April 1986 to December 1991 for S&P 500 index options. An economic analysis is applied, by devising trading rules using the two forecast methods to trade at-the-money straddles. They find that a GARCH forecast returns a greater profit than the rule based on an implied volatility regression model. Thus, they conclude: “…These results suggest that volatilities incorporated in option prices do not fully utilize historical information, and that GARCH volatility forecasts can therefore add value.”

Fleming (1998) found that implied volatility dominates historical volatility in terms of *ex ante* forecasting power. Like former studies, which find a statistical relationship between implied and realized volatility - the results indicate that implied volatility is a biased forecast. More precisely, Fleming (1998) finds that implied volatility is an upward biased forecast of future volatility. The data set constitutes of S&P 100 transaction price history in the period from October 1985 through April 1992, eliminating all observations that overlap the October 1987 stock market crash.

Christensen and Prabhala (1998) employ a different research design than earlier studies, using a monthly sampling frequency leading to nonoverlapping data that spans a longer period of time, in addition to apply instrumental regression to account for errors in variables. They find stronger evidence for the use of implied volatility than Jorion (1995) and Fleming (1998) and results that directly contradicts those of Canina and Figlewski (1993): “…Our conclusions are significantly different from those of previous literature, and the difference is robust to variations in econometric approach. We find that implied volatility does predict future realized volatility in isolation as well as in conjunction with the history of past volatility in some of our specifications…”. They find that implied volatility is an unbiased and efficient forecast of subsequent realized volatility, when instrumental regression is applied to adjust for measurement errors in variables. Their empirical work analyse S&P 100 index options with one-month expiration cycles and a data time span from November 1983 to May 1995. Critique has been given for evaluating
efficiency with respect to a rather limited information set, namely lagged realized
volatility (Becker et al., 2006). Christensen and Hansen (2002) study the relationship
between S&P 100 index option implied volatility realized volatility, using monthly
sampling to avoid “telescoping” data samples. Their sample is more recent than that of
Christensen and Prabhala (1998), constituting April 1993 to February 1997. In addition
to ordinary least squares and two stage least squares, they also employed a three-
equation-system to shed light over the problem at hand. A trade weighted average of all
near term call and put OEX options traded the last five days is used as the implied
volatility estimate. Their results confirm those of Christensen and Prabhala (1998),
namely that implied volatility is an unbiased and efficient forecast of future volatility,
and subsumes the information contained in lagged realized volatility. Hansen (1999)
studies the relationship between implied volatility and realized volatility in the Danish
option and equity markets. In particular, the author wants to investigate whether KFX
infrequent trading and low volume may lead to bias and inefficiency in the implied
volatility forecasts. An implied volatility forecast that is equally weighted between the
call and put closest to at-the-money manages to subsume the information in lagged
realized volatility, and the forecast bias is insignificant. The study is in large inspired of
that of Christensen and Prabhala (1998) and apply monthly sampling. Hansen (1999) is a
natural study to compare our results from the Norwegian market with, taken into account
the similar market size and sampling procedure.

Szakmary et al. (2003) find results that are qualitatively similar to those of
Christensen and Prabhala (1998) and Christensen and Hansen (2002). They examine 35
future markets and find that implied volatility, even though not a completely unbiased
predictor, performs well in a relative sense. For what they describe as an overwhelming
majority of the 35 commodities studied, implied volatility outperforms historical
volatility and a GARCH model. Due to the fact that the futures and options are traded on
the same floor, some problems with errors in variables are avoided, similar to Jorion

Mayhew and Stivers (2003) add new knowledge to the implied volatility
literature by suggesting a link between which model perform the best, implied volatility
or a GARCH-type forecast , and the liquidity of the option. By examining 50 individual
stocks-options with the highest trading volume out of the stock-options listed on the CBOE in the period 1988 to 1995, they find evidence that in highly liquid option markets, implied volatility subsumes nearly all information about future firm volatility from other forecasts. However, time-series models are comparable or even outperform implied volatility if the option trading volume of the stock is low in a relative sense.

Becker et al. (2006) test the efficiency of CBOE’s implied volatility index (VIX), which is based upon the S&P 500. They are testing whether implied volatility is encompassing a range of different volatility forecasts, among others GARCH (1,1) GJR-GARCH, ARMA and ARFIMA. The forecast-encompassing test is building upon the framework put forward by Fleming (1998). Data from January 1990 to October 2003 is used, finding evidence against the hypothesis of VIX being an efficient volatility forecast. This evidence is strongest when information is sampled daily, and considerably weaker when information is sampled monthly, which leads to the conclusion that implied volatility forecasts are efficient. In addition to the VIX index, Corrado and Miller (2004) also examine the VXN index, which is the implied volatility index for the Nasdaq 100, and the VXO index, which is the implied volatility index for the S&P 100. With data that spans the period January 1988 to December 2003, they find that the VIX and VXO yield an upwardly biased volatility forecast, while VXN yields a nearly unbiased volatility forecast. For all indices, implied volatility provides a more efficient forecast than realized volatility, measured by mean squared forecast errors. To account for econometric problems in the regression, instrumental regression is applied. However, compared to ordinary least square regression, no enhanced support for the forecast quality of implied information could be found, except in the period before 1995.

Fairly recent developments in time-series models (e.g. Andersen and Bollerslev 1998; Andersen et al. 2001), which allow the models to incorporate high-frequency time-series, have once again opened the debate of which volatility forecast method is the best. A comprehensive overview of competing volatility forecasting models, and how they perform, is given by Poon and Granger (2005). Comparing 93 studies divided in four different groups - namely, implied volatility, historical volatility, models in the autoregressive conditional heteroscedasticity (ARCH) class, and stochastic volatility...
models- they find implied volatility being the superior volatility forecast. Martens and Zein (2004) re-examined this evidence by comparing a GARCH and an ARFIMA model with implied volatility. Data was gathered from three asset classes, utilizing options on futures with the following assets as underlying: S&P 500, YEN/USD, and Light Sweet Crude Oil. Support is found in favour of earlier results (e.g. Christensen and Prabhala (1998); Jorion (1995)) when daily data is used. Nevertheless, the outcome is altered when estimating a fractionally integrated autoregressive model for realized volatilities computed from squared high-frequency returns. It is shown that both implied volatility and the long memory volatility forecast have information that the other does not contain. Further, they find that long memory volatility forecasts can compete, and in some instances even outperform implied volatility. Pong et al. (2004) also compare implied volatilities with AR(FI)MA models. In short, their conclusion states a relationship between forecasting horizon and the optimal volatility-forecasting model. ARFIMA, ARMA and GARCH forecasts have incremental information over implied volatility for the short horizons, which are one day and one week, while implied volatility is found to incorporate most information when the horizons is either one or three months.

Another branch in the volatility forecasting literature is that of stochastic volatility. In a Black and Scholes framework, the underlying asset price process is assumed to be a lognormal Brownian motion with constant mean and variance. This is an invalid assumption, something which is displayed graphically in later chapters. In this branch of the literature, the focus is on stochastic volatility models which assume that volatility of the stock price process is not constant, but stochastic itself. Well known papers using this approach are Hull (1987), Heston (1993) or Stein and Stein (1991). The survey of Poon and Granger (2005) can not conclude clearly whether stochastic volatility forecasts are better or worse than GARCH and implied volatility forecast due to limited research. However, it seems clear that the mathematical complexity increases when estimating stochastic volatility models, thus diminishing the accessibility to this tool for practitioners.
3 The Norwegian stock and option markets

3.1 Institutions and regulations

A standardized option market consists of a market place, commonly a stock exchange, in addition to a clearinghouse. Together, these two institutions allow investors to take advantage of opportunities in the market, and constitute the fundamental institutions when trading with options.

Oslo Børs, also referred to as Oslo stock exchange, holds the position as the market place in the Norwegian market. Their main tasks are to receive buy and sell orders from relevant parties and rank them according to price and time. Orders can be directly submitted through Oslo Børs’ electronical trading platform, or submitted by communication with Oslo Børs’ own brokerage service. Oslo Børs close trades between overlapping buy and sell interests and distribute price and trade information to the market. In addition to this, Oslo Børs is responsible for the listing of new option series and the surveillance of daily trades. The opening hours for continuous trading in equities and bonds are 09.00 to 16.20, while the official opening hours for derivative follow the underlying instruments, and are therefore equal, 09.00 to 16.20.

VPS Clearing holds the position as clearing house in the Norwegian market. The most crucial task of VPS Clearing is to act as a counterpart in trades. In other words, they step into the trade and guarantee that the trade will be successfully executed. In this way counterparty risk is avoided when a trade is recognized.

The investor is the most important player in the option market, and is able to sell and buy options through an authorized broker. The options bought by the investor are registered in an option account at VPS. If an investor wish to write options, a certain nominal amount in options or cash is held as security for the trade.

As a framework for the market place, clearinghouse and brokerage houses, there are several laws they have to obey. These laws are enforced by the financial supervisory authority of Norway (Kredittilsynet). All the institutions mentioned, Oslo Børs, VPS Clearing and the brokerage houses need approval from the financial supervisory authority of Norway to carry out their business.
3.2 The OBX index

The OBX index consists of the 25 most traded shares traded at Oslo Børs. The index was introduced in 1987 at a starting value of 200 and is a capitalization weighted index, sometimes also referred to as a market weighted index. The constituents are selected on the basis of the latest six months turnover rating and are semi-annually revised. The composition change takes place on the third Friday in December and the third Friday in June. New weights are given to the constituents every day before the stock exchange opens, on the basis of current market value of the firm constituting the OBX index at the current time. In the period between the composition review dates, the number of shares for each constituent is fixed with exception of continuous adjustments for corporate actions with priority for existing shareholders. The OBX index is an adjusted total return index, thus are the dividends adjusted for. It is a tradable index, with exchange traded futures and options available.

Since the introduction in 1987, one major structural break has taken place. The 21. April 2006 a four to one (4:1) split took place. In addition to this, it was from this date onwards the OBX index started to adjust for dividends. Thus, in the period 1987 to April 2006, the OBX index did not take dividends into account. Opposite to the S&P 500, the underlying stocks do not pay out dividend uniformly throughout the year. Norwegian companies usually pay out dividends in the spring, with a peak around May. This issue needs to be properly addressed when calculating implied volatility.

We have access to both an unadjusted OBX index series, in addition to an adjusted OBX index series and will utilize both in the thesis. The series which is not adjusted for dividends will simply be referred to as the unadjusted OBX index series, while the series that is adjusted for dividends will be referred to as the adjusted OBX index series.
3.3 **OBX index options**

Oslo Børs introduced stock options and the OBX index options in 1990. The OBX index options are European with expiring date the third Thursday every month. As the name suggest, the underlying of the OBX index options, is the OBX index itself. To be able to trade options on an index, the index has to be translated into a nominal value. This is done by multiplying the OBX index by 100, hence an index level of 500 has a nominal value of 50,000 NOK when investing in options.

New series of OBX index options are introduced monthly at the third Thursday in the month. The new options’ strike prices are determined by the OBX index level the previous trading day. At least five call options and five put options are listed, where one call and one put have a strike price as close as possible to the previous day’s closing index level. Subsequently, two puts and two calls are listed with the closest strike prices higher the initial listed option, and two puts and two calls are listed with the closest strike prices lower than the initial listed option. If the level of the underlying OBX index is lower (higher) than the second lowest (highest) strike price, OBX will issue minimum one new option with lower (higher) strike price than already existing in the market.

The interval between the strike prices are decided upon the level of the OBX index. Under 150, the interval is 3, between 150 and 500 the interval is 5, between 500 and 1000, the interval is 10 and finally, above 1000, the interval is 20. The new options listed have a cycle of three months and are issued each month, thus it always exists OBX index options in the market with three months, two months and one month to expiration.

3.4 **Descriptive statistics**

The black line in figure 1 represent the OBX adjusted index series in the period from 1 February 1997 to 5 February 2007. The level series have a minimum in this period of 79.96 and a maximum of 404.24. We observe rather small fluctuations in the range between 80 and 200 in the first part of the period, and a bull market from 2003 and onwards, only interrupted by minor corrections by the market.
Figure 1: Daily OBX level and the log-return series

The grey lines show the logarithmic return series for the OBX adjusted index in the same period. Observing figure 1 carefully, we can see some indications of what is known from the literature as volatility clustering or volatility pooling.\footnote{John Hull (2005)} Volatility clustering is a description of the return behavior in financial time series, where small (high) absolute returns tend to be followed by small (high) absolute returns. As an example, we can see that volatility soared in the end of 1998, and the first large absolute returns were followed by several subsequent large absolute returns. This period also reveal another stylized fact about the financial markets. In the period with large volatility, we can observe that the index dropped significantly in value, while in the period after, the index moved slightly upwards. When the index moves slightly upwards, the absolute

\footnote{John Hull (2005)}
returns are notably smaller, thus indicating asymmetric volatility; the tendency that financial data exhibits higher volatility when, e.g. the OBX adjusted index, is decreasing in value compared to when the index is increasing in value.

![Series: OBX_RETURN_SERIES](figure2)

**Figure 2: Daily OBX adjusted log-return series - histogram and descriptive statistics**

The mean of the OBX adjusted log-return series is 0.053% at a daily basis, which equals 13.28% on an annual basis when annualizing using 252 trading days. The Norwegian inter-bank interest rate (NIBOR) had an average of 4.84% in the same period, revealing an average risk premium of 8.44%. This is high compared to the risk premium of S&P 500, and indicates that the OBX adjusted index is more risky than the S&P 500. Due to the higher risk premium and the relationship between risk and return, we would expect that the OBX adjusted index will exhibit higher average volatility compared to the average volatility found in papers examining the S&P 500. The maximum daily return was 6.91%, while the largest drop in value experienced by the OBX adjusted index in the sampled period was -7.24%.

Another financial market stylized fact can be observed by looking at the histogram in figure 2. We can see that the tails of the distribution are spread in a wider fashion than what the normal distribution entails. The high frequency of these small probability/large impact observations create what is known in the literature as fat tails.
More observations around the mean than what is implied by the normal distribution, is also a common feature of financial data. Both the *fat tails* and that the number of observations is too high around the mean, compared to normality, can be checked by calculating the *kurtosis*. When a distribution conform to normality, it should have a kurtosis close to 3, while the OBX adjusted return series exhibits a kurtosis of twice this value, namely 6.10. This indicates that the distribution is *leptokurtic*, with both fatter tails and a higher “peak” than a normal distribution.

If the distribution should conform to normality, it should also be symmetric around its mean and a skewness taking the value of 0. The OBX adjusted return series has a skewness of -0.44, which implies that the distribution is clustering slightly to the right in figure 2. However, the skewness is not predominant.

![Figure 3: QQ-plot of daily OBX adjusted return series](image)

Finally, we can comment on the normality of the distribution by looking at figure 3 and evaluate the Jarque-Bera statistics. In figure 3, the points should follow the line to conform to normality. The S form that appears can be interpreted as the distribution having fatter tails than in a normal distribution. Further, we can reject normality on the basis of the Jarque-Bera test at all significance levels.
3.5 Statistical properties

We use an Augmented Dickey-Fuller (ADF, 1979) procedure in Eviews to test if the levels series or the OBX adjusted return series are non-stationary (unit root) or stationary. In an ADF test, we test the null hypothesis; “the time series is non-stationary” against the alternative hypothesis; “the time-series is stationary”.

As expected, the daily level series is non-stationary at all significance levels, with a 0.99% probability of being wrong if we rejected the null hypothesis. When calculating the logarithmic returns, we expect the series to become stationary. Indeed, the test reveals a 0.01% probability of being wrong if the alternative hypothesis is chosen. Thus, we can not reject that the daily OBX adjusted return series is stationary. No autocorrelation in the error terms is one of the assumptions in regression analysis. We can test this both by a visual check of the autocorrelation function (ACF), in addition to more formal test, like the Ljung-Box Q statistics. The ACF is the normalized autocovariances, taking a value between minus one and one, where lag \( t \) is autocorrelated if the value exceed the critical value of \( \pm 1.96^*(1/\sqrt{T}) \), where \( T \) is the number of observations in total. With the Ljung-Box Q statistics, we can test the hypothesis of whether it exist autocorrelation up to lag \( t \), the null hypothesis being that all ACF’s up to and including lag \( t \) are jointly equal to zero.

Table 1: ACF and Q-statistics

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<th>3</th>
<th>4</th>
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<td>0.028</td>
<td>0.015</td>
<td>-0.024</td>
<td>0.054</td>
<td>-0.026</td>
<td>0.013</td>
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<td>-0.009</td>
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<td>PAC</td>
<td>0.028</td>
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<td>0.266</td>
<td>0.247</td>
<td>0.019</td>
<td>0.019</td>
<td>0.030</td>
<td>0.049</td>
<td>0.075</td>
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<tbody>
<tr>
<td>ACF</td>
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<td>-0.027</td>
<td>-0.009</td>
<td>-0.020</td>
<td>0.008</td>
<td>0.008</td>
<td>0.020</td>
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<tr>
<td>PAC</td>
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<td>Q-Stat</td>
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<td>45.458</td>
<td>58.414</td>
<td>81.139</td>
<td>91.837</td>
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<td>111.97</td>
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</tr>
<tr>
<td>Prob</td>
<td>0.029</td>
<td>0.035</td>
<td>0.030</td>
<td>0.004</td>
<td>0.005</td>
<td>0.004</td>
<td>0.011</td>
<td>0.016</td>
<td>0.024</td>
</tr>
</tbody>
</table>

Estimated on the basis of daily OBX return series.
Table 1 shows that the ACF have a higher value than the critical value at lag four, thus implies that the series have an autocorrelation of order four. The Q-statistics also show that the time series exhibit autocorrelation up to lag $t$, at a 5% significance level, when 20 or more lags are included. The Q-statistic also suggests the presents of autocorrelation at a 5% significance level when 4 to 7 lags are included. We expect the problem of autocorrelation to be less pronounced when sampling our data-set at a monthly frequency. The level series do, as expected, exhibit strong autocorrelation at all lags.

Homoscedasticity, which means that the error terms have a constant variance, is another basic assumption when working with regression analysis. We can see indications of this assumption being violated when observing figure 1 and see that the volatility has a tendency to cluster. This is a larger problem in higher-frequency financial data then with data of lower frequency, which we will use in our analysis. Results from heteroscedasticity tests will be displayed when working with the regression analysis.
4 Data

4.1 Data-set sampling and calculations

4.1.1 Recording the OBX index level, option prices and strike prices

Our empirical analysis focuses on implied volatility from OBX index options that have around one month left until expiration. As options with one-month expiration cycles do not exist in the Norwegian market, the analysis looks at options with three-month expiration cycles. The data available enables us to sample data-series that start in February 1997 and ends in February 2007. Thus, the data constitutes of 121 observations in total, which also is the number of months the data spans over.

By large, the sampling procedure follows Christensen and Prabhala (1998). They sampled data at a low (monthly) frequency, creating data-series with beneficial properties when used in econometric analysis. The non-overlapping nature of the sampled data-series makes it possible to avoid serious problems with serial correlation in the error terms.

The OBX options expire on the third Thursday of every month, and if this is not a business day, on the day before. In order to avoid overlapping data, we move to the following Tuesday and record the OBX index level, $S_t$. On the same date, we look up the two closest put option prices and the two closest call option prices closest to being at-the-money. We record their closing prices, $P_{i,t}$, as well as the corresponding strike price, $K_{i,t}$. Here, $t$ denotes the month at hand, and $i$ denotes which option we are referring to, $1$ being the call option closest to at-the-money, $2$, the call option second closest to at-the-money,$3$ and $4$, the put options closest and second closest to being at-the-money, respectively. These options expire on the third Thursday in the following month $t + 1$, and the next set of call and put options is sampled at the Tuesday that immediately follows. This sampling procedure is repeated throughout the lifetime of the data we have available.
The extensive data set sampled is an extension in comparison to Christensen and Prabhala’s (1998) study, which limits their analysis to at-the-money call options. A sample of both put and call options enables us to undertake a more nuanced analysis, and create a weighted implied volatility estimate, which combine the information content of both the call and the put options.

In contrast to Christensen and Prabhala (1998), who studies one of the world’s most liquid option markets, we study OBX index options that are much thinner traded. This, in addition to the possibility of public holiday interfering with our sampling date, calls for a strategy in months the option is not traded at the original sampling date or the stock exchange is closed due to public holiday. With inspiration from Hansen (1999), we apply the following sampling procedure for such months:

1. Record the closing prices, \( P_{it} \), and their corresponding strike prices, \( K_{it} \), for the two call and two put options closest to being at-the-money the original sampling day, i.e., Tuesday following the third Thursday.
2. Record the closing prices, \( P_{it} \), and their corresponding strike prices, \( K_{it} \), for the two call and two put options closest to being at-the-money the previous day, i.e., Monday following the third Thursday.
3. If no option prices can be recorded on Monday, move two days ahead and record the closing prices, \( P_{it} \), and their corresponding strike prices, \( K_{it} \), for the two call and two put options closest to being at-the-money, i.e., Wednesday following the third Thursday.
4. If no option prices can be recorded at Wednesday, move to the Friday before the weekend and record the closing prices, \( P_{it} \), and their corresponding strike prices, \( K_{it} \), for the two call and two put options closest to being at-the-money the previous day, i.e., the day after the third Thursday.
5. If no option prices can be recorded at Friday, record the closing prices, \( P_{it} \), and their corresponding strike prices, \( K_{it} \), for the two call and two put options closest to being at-the-money two days after the original sampling day, i.e., Thursday following the third Thursday.
6. If it is still not possible to record any option prices, move one trading day ahead until successful sampling of the closing prices, \( P_{it} \), and their corresponding strike prices, \( K_{it} \), for the two call and two put options closest to being at-the-money is accomplished. This is to ensure the non-overlapping feature of our data.

The resulting data-set constitutes of two series of near at-the-money call closing prices and two series of near at-the-money put closing prices with a duration from February 1997 to February 2007. Out of these observations, we have recorded the data at the original recording day in 116 out of the total 121 time periods, being Tuesday after the third Thursday. Six observation were recorded at the Wednesday after the third Thursday, one observation was recorded the following Monday and one observation was recorded Thursday, one week after the Third Thursday in the month. In March 1997 it was not possible to obtain any closing prices for put options, thus, when analysis are done on the basis of put options, the time series include a missing value for March 1997.

It is not possible to illustrate the price series in a comparable useful way, as the prices depends upon the strike price, index value, time to maturity and interest rate at the given time. The data are rather displayed visually when the implied volatilities are calculated.

### 4.1.2 Recording dividends

No dividend series were available for the OBX index. However, as previously mentioned, we both have access to an adjusted, as well as an unadjusted version of the OBX index time series. Since the only difference between the series is whether dividend is adjusted for or not, it is possible to estimate the dividend yield on the basis of these two series.\(^5\) We denote the unadjusted series by \( P_i \) and the adjusted series \( P_i + d_i \).

\(^5\) This is confirmed by Oslo Børs
where \( d_t \) is the dividend at time \( t \). Further, we calculate the logarithmic return series for the adjusted series, denoted by \( A \) and the unadjusted series, denoted by \( B \):

\[
A = \ln(P_t + d_t) - \ln(P_{t-1})
\]  
\[
B = \ln(P_t) - \ln(P_{t-1})
\]  

We want to calculate the difference between series \( A \) and \( B \), which we denote as the return difference:

\[
\Delta \text{Return} = A - B = \ln(P_t + d_t) - \ln(P_t)
\]  

To find the dividend yield at time \( t \), we solve the equation with regards to \( d_t / P_t \):

\[
\text{div.yield} = \exp(\Delta \text{Return}) - 1
\]  

As we want to find the dividend yield over the remaining lifetime of an option, we add up the daily dividend yields:

\[
\text{Sum of div.yields} = \sum_t^{\tau} \frac{d_t}{P_t}
\]  

Further, to use dividend yields in the Black and Scholes option pricing formula, we need to annualize the summed dividend yields. This is done by dividing by the number of days in remaining lifetime of the option and multiplying by the number of trading days in one year. As convention we use 252 trading days, as suggested by Hull (2005):

\[
\text{Annualized sum of div.yields} = \frac{\sum_t^{\tau} \frac{d_t}{P_t}}{T - t} * 252
\]
In figure 4, we can see the annualized dividend yields displayed as a function of time.

![Annualized dividend yield graph]

Figure 4: Annualized dividend yields on the OBX index, calculated from equation (4.6)

This reveals a distinct pattern, which correspond to what we expected. The highest annualized dividend yields, displayed by all the highest bars in figure 4, take place in the period that starts late April and ends late May every year. As the dividend payouts are exceptionally large during these periods, we need to explicitly address the issue of dividends, in order to get correct results when calculating the implied volatility.

### 4.1.3 Calculate implied volatility

We have now sampled four monthly time-series of option closing prices and their corresponding strike prices. In addition to this, we have constructed a series of OBX index prices, which were used to extract the option prices. These are labeled $S_t$ in the following. To make an even clearer distinction between put and calls, we distinguish between put and call prices not only by the $i$ denotation, but also by separating $P_{t,t}$ into $C$
when referring to calls and $P_{i,t}$ when referring to puts. Further, we use the Norwegian inter-bank interest rate\(^6\) (NIBOR) as a proxy for the risk free interest rate faced by option traders when making the decision whether or not to buy an option at a specific price. The NIBOR time-series is gathered from Datastream, denoted by $r_t$, and sampled in the same way as the OBX index level.

To calculate implied volatility we apply the Black and Scholes option pricing formula taking dividends into account. We assume that the amount and timing of the dividends can be predicted with certainty. For short-life options, like those we are dealing with, this is a reasonable assumption. Then, by inserting the closing price of the option, we can invert the formula and solve for the volatility variable, applying numerical iteration procedures.\(^7\) Equation (4.7) is solved to calculate the call implied volatilities, while equation (4.8) is solved to calculate the put implied volatilities. The formulas are as follows;

\[
C_{i,t} = S_t e^{-rT} N(d_1^x) - e^{-rT} K_{i,t} N(d_2^x) \tag{4.7}
\]

and

\[
P_{i,t} = K_{i,t} e^{-rT} N(-d_2^x) - S_t e^{-rT} N(-d_1^x) \tag{4.8}
\]

where

\[
d_1^x = \frac{\ln(\frac{S_t}{K_{i,t}}) + (r_t - q + \sigma_{x,i,t}^2 / 2)T}{\sigma_{x,i,t} \sqrt{T}}, \quad x = c, p
\]

and

\[
d_2^x = d_1^x - \sigma_{x,i,t} \sqrt{T}, \quad x = c, p \tag{4.10}
\]

\(^6\) This corresponds to the rate used by Christensen and Prabhala (1998), which used LIBOR when studying implied volatility on S&P 100 index options.

\(^7\) Excel and the command “Goal seek” was used to solve for the implied volatility.
N(\bullet) denotes the standard normal probability distribution. Further, \( \sigma_{c,i,t} \) denotes the call implied volatility, where \( i \) indicates which of the call series the volatility belongs to and \( t \) denotes which month the option is sampled in. Likewise, \( \sigma_{p,i,t} \) denotes the put implied volatility, where \( i \) indicates which of the put series the volatility belongs to and \( t \) denotes which month the option is sampled. \( T \) is referring to time to maturity, given in years. As with calculating the dividends, we use 252 trading days. The \( q \) denotes the annualized dividend yield previously calculated. We never experience option prices to not obey the arbitrage boundaries. This would have led to negative implied volatility variables, which we could not have used in further analysis. This contradicts the findings of Hansen (1999), who studies the implied volatility extracted from KFX index options, where the arbitrage boundaries were broken 11 times in a much smaller data set.

To exploit all the information in our time-series, we construct a new implied volatility measure \( \sigma_{a,t} \), which use the information from the put series and the call series. The new implied volatility measure is given by

\[
\sigma_{a,t} = \frac{\sigma_{c,1,t} + \sigma_{c,2,t} + \sigma_{p,1,t} + \sigma_{p,2,t}}{4},
\]

where \( a \) is a descriptive denotation used as an abbreviation for average. In February 1997, when we do not have values for the put options, we let the implied average volatility be determined by an average of the two call implied volatilities.

This inherently unsophisticated weighting scheme was first applied, as mentioned in the literature review, by Trippi (1977) and Schmalensee and Trippi (1978). This proposal was criticized, as the Black and Scholes formula price some options more precisely than others. Close to being at-the-money options gives the most precise implied volatilities, while deep-in-and-out-of-the-money options gives less precise implied volatilities. A plot of the implied volatility of an option as a function of its strike price is

---

8 This precise notation of the Black and Scholes formula can be found in Hull(2005), page 314
9 Call price boundaries are \((S - Ke^{-rT}) \leq c \leq S\) and \((Ke^{-rT} - S) \leq p \leq S\) for put options.
known in the literature as a *volatility smile*, describing how the implied volatility tends to vary as a function of the strike price.

![Figure 5: Implied volatility for the two put and two call options closest to at-the-money on the OBX index. K denotes the strike price, while S denotes the index value.](image)

Figure 5 shows that we do not have an especially distinct volatility smile pattern, with exception for a possible gathering of higher than average implied volatility from $K/S$ equals 0.95 till the shift between 0.98/0.99. The indistinct form of the volatility smile is taken as an indication of the Black & Scholes implied volatilities not being especially biased, due to the fact that they are all sufficiently close to being at-the-money. A similar pattern can be found with respect to time to maturity, where long time or very short time to maturity gives less precise implied volatility estimates compared to realized volatility. However, the four option series we base our implied volatility analysis on are all close to being at-the-money and have an advantageous time to experience, thus the Black and Scholes pricing formula precision should be approximately similar. This indicates that there is no need for a more sophisticated weighting scheme than what proposed in equation (4.11).
It has also been shown in the literature by, e.g., Beckers (1981), Feinstein (1989) and Gemmill’s (1986), that a naive single option forecast is superior to more advanced weighting schemes. Ederington and Guan (2002) cast nuance to these findings by arguing that the averaging procedure is in general of secondary importance, but could matter much more in less liquid markets. The Norwegian market could be an example of such a “less liquid market”. It could be interesting to calculate a trade weighted implied volatility average; however, we do not have access to data concerning numbers of contracts traded on a daily basis.

It is not possible to extract a broad set of implied volatilities at each sampling date from the OBX index options; nevertheless we take the advice from Ederington and Guan (2002) into account, and extract a data-set, which is as broad as possible. Due to options’ approximately similar distance from at-the-money and that the time to maturities are in the same range, we expect an approximately equal behavior when inverting the Black and Scholes formula. We apply the simple weighting scheme, since the more advanced weighting schemes aim on eliminating noise, which is not particularly pronounced in our implied volatility estimates. The implied volatility time series will be thoroughly scrutinized and graphically displayed in chapter 4.3, descriptive statistics.

### 4.1.4 Calculate realized volatility

We have now defined how to calculate the different measures of implied volatility; however, we have not yet established a formal way of deriving our realized volatility time-series. The realized volatility time-series is in an informal sense our “answer-key” and the independent variable in the regression analysis introduced later. The realized volatility for each period is calculated as the standard deviation of the daily index returns during the remaining life of the options. First, we calculate the daily index returns,

\[ R_k = \ln\left(\frac{S_k}{S_{k-1}}\right), \]  

(3.11)
where $S_k$ denotes the closing index level on day $k$. Let $R_{a,t}$ denote the average daily index
returns over the remaining life of the option. The annualized realized volatility of the
index returns for month $t$ is then given by,

$$
\sigma_{r,t} = \sqrt{\frac{252}{(T_t - 1)} \sum_{k=1}^{T_t} (R_{t,k} - \bar{R}_t)^2}, \quad (3.12)
$$

where $k$ runs from the day after the actual sampling day and stops at the maturity date of
the option.\textsuperscript{10} Repeating this 121 times, we create one value for realized volatility for each
month in the data-sample, which together constitute a non-overlapping time-series for
realized volatility. $T_t$ denotes the number of trading days to maturity for option with
expiry in $t+1$.

It should be noted that the literature also suggest to use more sophisticated
estimates of the unobservable volatility process than standard deviation of daily return.
Andersen and Bollerslev (1998) assert that squared daily innovations are unbiased but
noisy estimates of the variance process. Sampling the return process at increasing
frequency, estimates of the underlying volatility becomes evermore efficient, something
which allows volatility to be treated as observed data at very high frequencies.
Poteshman (2001) investigated options on the S&P 500 and found that high-frequency
measures eliminated half the bias in prediction from S&P 500 options. Unfortunately, we
do not have access to such an extensive data set, thus, implied volatility and our
GARCH(1,1) forecast will only be compared with standard deviation of daily returns.

4.1.5 GARCH forecast

To analyze implied volatility in a broader perspective, we choose to calculate a
General Autoregressive Conditional Heteroscedasiticity forecast, which we can compare

\textsuperscript{10} This notation is inspired by Hansen (1999), while a qualitatively similar formula is
with implied volatility. In contrast to option implied volatility, which is inherently forward looking, statistical volatility forecast strictly utilizes historical information to predict volatility. The GARCH(p,q) model was introduced by Bollerslev in 1986 and is a more flexible version of the precursor - the ARCH(q) model introduced by Engle in 1982. With a GARCH(p,q) model, it is possible to incorporate the observed volatility clustering and asymmetric volatility into our model. On a general form, the variance equation can be expressed as:

\[ \sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i u_{t-i}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2, \]

where \( \alpha_0, \ldots, \alpha_q \geq 0, \beta_1, \ldots, \beta_p \geq 0 \) (3.13)

When forecasting the variance, different ARMA(p,q) specifications were fitted to the mean equation, however, the AR(1) model gave the best results with regards to the Ljung-Box Q-statistics.\(^{11}\) Next, we overfitted a GARCH(p,q) model, and found that \( p \) and \( q \) lags with higher order than one were insignificant, thus a GARCH(1,1) model will be used. We also tried to fit a TGARCH model, with threshold order one to take the observed asymmetric volatility into account. However, this term was not significantly different from zero, and therefore left out of the model. We estimate 121 rolling forecasts, each with a individually estimated GARCH(1,1) model. The models are estimated upon daily data from 2.january 1993 up until the day before the forecast takes place. Next, the length of the forecast is matched with the number of days until maturity, \( T \), for the given options in month \( t \), and the variance forecasts are summed:

\[ GARCH(1,1)_{Forecast} = \sum_{i=1}^{T} \sigma_t^2. \] (3.14)

To make it possible to analyze and compare the forecast, the variances are first annualized by dividing by \( T \) days and multiplying with 252, second, the annualized

\(^{11}\) More information about ARMA(p,q) models can be found in chapter 4.4
variances are transformed to volatilities by taking the square root. The GARCH(1,1) forecast time-series is displayed visually in chapter 4.3, descriptive statistics.

**4.2 Sources of error**

Across different asset classes and sample periods, scholars have found that implied volatility is a significantly biased predictor of realized volatility. Frequently, the \( \alpha_0 \) in equation (1.1) has been reported taking a positive value, while \( \alpha_1 \) has taken a value significantly less than unity (E.g. Canina and Figlewski (1993), Lamoureux and Lastrapes (1993), Jorion (1995), Fleming (1998) and Szakmary, Ors, Kim and Davidson (2003)). Other studies, like Christensen and Prabhala (1998), Hansen (1999) and Christensen and Hansen (2002), have found the implied volatility to be an unbiased forecast.

Explanations for the apparent bias and inefficiency, advocated by parts of the literature, fall into two major groups: (1) failure of the efficient market hypothesis and (2) failure in the testing procedures. Understandably, researchers have been reluctant to pursue the first explanation, as it seems unlikely that investors would leave money on the table and offer a “free lunch” to their competitors, thus the main focus has been on trying to improve the testing procedure.

The literature regarding different testing procedures is vast in numbers and an in-depth discussion of all explanations is out of the scope of this paper. It is also out of the scope of the current paper to quantify the errors. In addition to this, we also take note of that it is not impossible that certain of the errors neutralize each other, or neutralize themselves, due to negative daily correlation. To give the reader a certain overview of the explanations, we mention a set of explanations proposed by Neely (2004) and Figlewski (2004) in section 4.2.1. While we focus more narrowly on the error in variable problems that Christiensen and Prabhala (1998) address, which are in large inspired by Harvey and Whaley (1991), in section 4.2.2.
4.2.1 Various explanations for bias and inefficiency offered by Neely (2004) and Figlewski (2004)

In the following, four explanations will be briefly discussed, being 1) peso problems; 2) use of overlapping samples and; 3) a non-zero price of risk volatility 4) the role of the option trader.

4.2.1.1 Peso problems

Poteshman (2001) discusses a problem, referred to as peso problems, which might lead to estimates that appear to be biased in-sample due to unusual sampling variation. More specific, investors may price option rationally and take into account low probability/large impact events. However, it could be that these events do not take place if the data spans over a relatively short time. The opposite could of course also be true, being that the low probability/large impact events take place more frequent than you would expect. Neely (2004) concludes that the implied volatility estimates would appear biased, producing overly volatile predictors. However, the key to solve this problem is by utilizing data that spans over a longer time. With our data-set, sampled from a period over ten years, the peso problems are a very unlikely explanation for bias and inefficiency.

4.2.1.2 Overlapping sample

In the literature, regression is usually conducted on data with overlapping forecasts. Christensen and Prabhala (1998) point out that the overlapping forecasts might produce very poor small-sample estimates. To solve the problem, Neely (2004) suggests two ways to investigate the consequences of such overlapping forecasts. First, he suggests to simulate the distribution of the test statistics under the null hypothesis of unbiased forecasts. Here, you will obtain greater power by pooling the horizons together. Second, he suggests to independently estimate the regression analysis in equation (1.1)
for each forecast horizon. This is the approach adapted by Christensen and Prabhala (1998) and applied in the current paper. Thus, due to the methodology chosen in the thesis, we will not have any problems with “telescoping” data.

4.2.1.3 Non-zero price of risk volatility

The justification for using a constant-volatility model to predict what is, de facto, stochastic volatility, is provided by Hull and White (1987). To show this, they assumed that volatility evolves independently of the underlying price and that no priced risk is associated with the option. In Neeley (2004) they explain the volatility risk problem as follows: “If the investor only hedges with the underlying asset – not using another option too – then the return to the investor’s portfolio is not certain. It depends on changes in volatility. If such volatility fluctuations represent a systematic risk investors must be compensated for the exposure. In this case, the Hull-White result does not apply and the implied volatility from the Black Scholes formula will not be the correct volatility estimate.” Pricing of volatility risk has been discussed by Hull and White (1987), in addition to Heston (1993) and Lamoureux and Lastrapes (1993). A more recent branch of research represented by, e.g. Poteshman (2001), Bates (2003), Chernov (2002), Pan (2002) has paid great attention to the role of volatility risk in options and equity markets. In the S&P 500 option market, Benzoni (2002) finds evidence that volatility risk is priced. However, Neeley (2004) does not find that permitting a price-of-volatility risk eliminates the forecasting bias in implied volatility. It is beyond the scope of this thesis to investigate whether pricing of volatility risk could explain a bias in forecasting ability of implied volatility. However, we take note of the potential measurement error that stems from not taking the price of volatility risk into account.

4.2.1.4 From an option trader’s perspective

Academics strongly believe that markets are efficient and all accessible information is taken into account when valuing security prices. When addressing options, this
information-set includes all information that is relevant when predicting the future volatility of the underlying asset’s return. This would typically include straightforward calculation of historical sample volatility, in addition to more complex behavior which can be revealed by the use of GARCH or other statistical techniques. One would also have access to all relevant news from the past, in addition to the current news picture and what is going to take place in the future, e.g., political elections, macroeconomic news on international financial conditions etc. Thus, it is frequently argued in academia that since the volatility forecast implied in current option prices incorporate all relevant information, it will make additional or other forecasts redundant. However, this requires that the one making the forecast must calculate implied volatility from the same model as the market use pricing options. Figlewski (2004) argues that option traders agree upon the importance of implied volatility, however for other reasons: “For them, the great value of a pricing model is that it allows them to estimate what the option price will do when the underlying asset price moves to a different level.” Thus, he argues that the importance of an option pricing formula for an option trader is not to predict the volatility over the remaining lifetime of the option, but rather that the model gives an indication of how the market is currently pricing options relative to the underlying assets. With this knowledge of current implied volatility, traders are able to consistently price different option in relation to each other, in addition to hedge individual options against the underlying asset.

Further, Figlewski (2004) argues that traders will not go into a “riskless” arbitrage trades when the market miss price options, due to the fact that it is very difficult for them to be sure about their volatility prediction. The following example is given: “If the market is currently pricing options such that implied volatility is about 15%, the market maker may quote bids based on a volatility of 14.8% and offers on 15.2%. Even if she were certain that the true volatility over the option’s lifetime would turn out to be 20%, the market maker would very likely continue to make markets based on 15% volatility, because that would produce more frequent turnover from which she would earn the bid-ask spread. If, on the other hand, she were determined to take positions based on the true 20% volatility, she would immediately buy as many contracts as she could afford to carry, hedge the
position, and the have no further role as a market maker. There would be no revenue earned from the bid-ask spread until the contracts expired or investors eventually changed their views about volatility.”

If the scenario Figlewski (2004) is drawing here frequently takes place, this is an argument against the validity of using the Black and Scholes implied volatility as the “market’s” volatility forecast. It is also an interesting discrepancy between theory and practice and how academia and practitioners both find value in a model, though for difference purposes. Finally, if this is predominant behavior in the market, it may be an explanation why different studies have found that implied volatility not necessarily is efficient and subsumes all information in other statistical forecast alternatives.

4.2.2 Errors in variables initially proposed by Harvey and Whaley (1991)

Our estimates of implied volatility are, as mentioned, affected by sources that impose measurement errors to our variables. Harvey and Whaley (1991) wrote a paper purely dedicated to this topic. They essentially pointed out four main areas that impact the measurement accuracy; estimation procedure, non simultaneous price problem, bid/ask price effect and problems due to infrequent trading. These are also the measurement errors discussed by Christensen and Prabhala (1998) in their paper. In the next sections we will shed light over why relevant sources of measurement errors will or will not impact the implied volatility calculations.

4.2.2.1 Non-simultaneous trading

This problem takes place when the option price and the price of the underlying are not traded at the same time. This leads to an implied volatility, which is incorrect calculated. Mainly, this is a problem when the exchange that trades the underlying and the exchange that trades the options have different opening hours. E.g. the S&P 100 index option market closes at 3:15 P.M., while the stock market closes at 3:00 P.M. During the last 15 minutes, information will continue to enter the option market, which
will lead to an unobservable change in option traders’ opinion about the value of the underlying. When the implied volatility is calculated using closing prices, this will lead to wrong results, where the implied volatility from call options is higher (lower) than it should be when the market news was positive (negative). The opposite is true for put implied volatility, namely that the calculated implied volatility is higher (lower) than it should be when news is bad (good). Christensen and Prabhala (1998) also mention that the non-simultaneous problem also can take place if some of the closing prices of the stocks that constitute the underlying index are stale. Stale prices can take place, and would probably be a larger issue in the Norwegian market compared to the OEX index, due to the fact that the Norwegian market is less liquid. However, measurement errors due to different opening times in the option and index market is non existent, as the Norwegian option and index markets have equal opening hours.

4.2.2.2 Black and Scholes model

A version of the Black and Scholes model, which takes dividend into account, was introduced in chapter 4.1.3. The formula holds for European options, and this corresponds to the OBX index options, which are European, thus can not be exercised before the maturity date. To derive at this formula, a set of assumptions is applied and according to Hull (2005) they are stated approximately as follows:

1. The underlying price of the financial asset follows a log-normal geometric Brownian motion diffusion process.
2. The short selling of securities with full use of proceeds is permitted.
3. There are no transaction costs or taxes. All securities are perfectly divisible.
4. There are no dividends during the life of the derivative.
5. There are no riskless arbitrage opportunities.
6. Security trading is continuous.
7. The risk-free rate of interest, r, is constant and the same for all maturities.
In our previously presented version of the Black and Scholes formula, dividends were taken into account, thus point number four is relaxed, nevertheless we still need to know the future dividend yield at the time we price the option. This is not necessarily fulfilled, as uncertainty is often attached to these payout ratios.

It is possible to give criticism to several of the assumptions underpinning the Black and Scholes formula. To start at the end, the risk-free rate is not necessarily constant, not even at short maturities, and is often argued to follow a mean-reverting process. Further, option trading in the Norwegian market is absolutely not continuous, with Oslo Børs closing both over night and over the weekends. Assumption five is often regarded as valid, as investors would not “give away” money to competing investors, thus making the market efficient. Torricelli and Brunetti (2005) study the efficiency of the Italian option market, using high-frequency data from the start of September to the end of December 2002. They found that the market was efficient, as the frequency of (ex-post and ex-ante) arbitrage possibilities are low for arbitrageurs and much lower for occasional retails. They find the results similar to studies presented in other European countries. This gives us an indication, in addition to the intuitive appeal of the argument itself, that no arbitrage argument holds for the Norwegian market. Assumption three is violated as tax and transaction costs are of course present in the Norwegian market and not all securities are perfectly dividable. Short sale is restricted in the Norwegian market, e.g., with regards to the trading by institutional security funds. Finally, and one of the sharpest critiques against the Black and Scholes formula, is that the underlying asset do in reality not follow a generic geometric Brownian motion. The geometric Brownian motion diffusion process is given by:

\[
dS(t) / S(t) = r * dt + \sigma * dZ(t),
\]

where \( r \) is the risk-free rate of return and \( Z \) is a Wiener process. The increments \( dZ \) are independent and normally distributed with mean zero and variance \( dt \). With the assumption that the prices follow a Brownian motion process, this implies that log-return are independently and normally distributed with a constant volatility. We can observe

\[\text{Market Models (2001), Chapter 2, page 21-23}\]
from the OBX adjusted daily return series in figure 1 that the volatility most likely is not constant. We found indications of volatility clustering as well as leverage effects. It is possible to relax the assumption of a constant volatility and rather treat the volatility of the underlying asset as a *stochastic process*. In the literature, it mainly exist two approaches when you want to model a stochastic process. In the first approach, the volatility depends on the time as well as the value of the underlying, giving rise to a *local volatility* model. Second, it is possible to consider that the volatility has a stochastic component of its own. This is conceptually more challenging, and the number of factors in total will increase by the number of stochastic factors entering the volatility modeling.\(^{13}\)

The underlying asset in a Black and Scholes framework follows a *log-normal* diffusion process, which is not necessarily a correct assumption. For the option prices to exhibit a log-normal diffusion process, the continuously compounded returns need to be normally distributed. Looking at the daily OBX adjusted log-return series in figure 2, we observed a leptokurtic distribution, with too many observations centered around the mean and fatter tails than the normal distribution entails. However, this can alter when observing the monthly return series.

As mentioned earlier, Hull and White (1987) shows that a constant volatility can be used if the volatility evolves independently of the underlying price and that no priced risk is associated with the option. In addition, both inverting the Black and Scholes and the log-normal diffusion process have intuitive appeal, at least for the sake of understandability and simplicity.

### 4.2.2.3 Infrequent trading

Infrequent trading can be categorized into two different groups. First and previously discussed, is non-simultaneous trading. This occurs when the stock or option is actually traded in a certain interval, e.g. daily, but not necessarily at the close of each interval. Second, we have non-trading, when an asset is not traded at all during a specific

\(^{13}\) Can, e.g., be found in Busca and Berestycki (2004)
interval. From the definitions, we can see that the main difference between these two infrequent trading forms stems from how we define the time interval. If we define the time interval as a month, most stock indices will have been traded at least once, in contrast to if we define the time interval over a few seconds or minutes. We observe that as the time interval shrinks non-simultaneous trading becomes non-trading.

Mayhew and Stivers (2003) finds results from the American stock option market, which indicates that option trading volume do influence the ability of implied volatility to subsume all relevant information about conditional variance. They study the fifty most actively traded firms with options listed on the CBOE, with regard to option volume. Further, they compare how well the implied volatility subsumes other forecast in the top quintile and bottom quintile of firms, where the firms are sorted according to total option trading volume. From their results, we can see that the implied volatility for stocks with a very active option market subsumes almost all information about future firm volatility. However, for firms with a relatively lower option trading volume, they find that information from implied volatility is comparable, or even inferior, to approaches that rely on past time-series, like GJR-GARCH.

Table 2: Top ten and bottom ten firms by total trading volume

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<tr>
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<tbody>
<tr>
<td>IBM</td>
<td>37,008,649</td>
<td>4,626,081</td>
<td>385,507</td>
</tr>
<tr>
<td>Merck</td>
<td>8,278,922</td>
<td>1,034,865</td>
<td>86,239</td>
</tr>
<tr>
<td>General Motors</td>
<td>8,206,743</td>
<td>1,025,843</td>
<td>85,487</td>
</tr>
<tr>
<td>Chrysler</td>
<td>6,975,108</td>
<td>871,889</td>
<td>72,657</td>
</tr>
<tr>
<td>General Electric</td>
<td>6,534,711</td>
<td>816,839</td>
<td>68,070</td>
</tr>
<tr>
<td>AT&amp;T</td>
<td>5,575,220</td>
<td>696,903</td>
<td>58,075</td>
</tr>
<tr>
<td>Ford</td>
<td>5,233,991</td>
<td>654,249</td>
<td>54,521</td>
</tr>
<tr>
<td>Eastman Kodak</td>
<td>5,000,864</td>
<td>625,108</td>
<td>52,092</td>
</tr>
<tr>
<td>Wal-Mart</td>
<td>4,758,688</td>
<td>594,836</td>
<td>49,570</td>
</tr>
<tr>
<td>Hewlett-Packard</td>
<td>4,596,612</td>
<td>574,577</td>
<td>47,881</td>
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<tbody>
<tr>
<td>Alcoa</td>
<td>944,056</td>
<td>118,007</td>
<td>9,834</td>
</tr>
<tr>
<td>Honeywell</td>
<td>913,414</td>
<td>114,177</td>
<td>9,515</td>
</tr>
<tr>
<td>Amoco</td>
<td>896,638</td>
<td>112,080</td>
<td>9,340</td>
</tr>
<tr>
<td>ITT</td>
<td>847,406</td>
<td>105,926</td>
<td>8,827</td>
</tr>
<tr>
<td>First Interstate</td>
<td>838,402</td>
<td>104,800</td>
<td>8,733</td>
</tr>
<tr>
<td>Fluor</td>
<td>707,369</td>
<td>88,421</td>
<td>7,368</td>
</tr>
<tr>
<td>Halliburton</td>
<td>697,708</td>
<td>87,214</td>
<td>7,268</td>
</tr>
<tr>
<td>Black &amp; Decker</td>
<td>652,372</td>
<td>81,547</td>
<td>6,796</td>
</tr>
<tr>
<td>Mead</td>
<td>639,748</td>
<td>79,969</td>
<td>6,664</td>
</tr>
<tr>
<td>Bell Atlantic</td>
<td>634,737</td>
<td>79,342</td>
<td>6,612</td>
</tr>
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From Mayhew and Stivers (2003).
The average monthly trading volume numbers for the top ten and bottom ten firms, from the fifty firms with the highest stock option trading volume in the period from 1988 to 1995, is shown in table 2. We observe a monthly trading volume in the range from 47.881 contracts to 385.507 contracts among the top ten firms, and notably lower among the bottom ten firms, where the average monthly volume of contract varies between 6.612 and 9.834. The study did not take into account any nominal value of the contracts, so it is unfortunately not possible to shed light over that aspect. This could be interesting when comparing to the volume for the Norwegian OBX index option.

In figure 6, we find the monthly OBX index option volume and monthly nominal value of the same trades, both with value denoted on the left axis. The data were only available from May 2001 and onwards, and data for July 2001 were missing. We can see

![Figure 6: Monthly index option volume and nominal value for the OBX index option](image)

Figure 6: Monthly index option volume and nominal value for the OBX index option

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14 Data are publicly available and found at www.osloboers.no
a fairly stable volume with some peaks, and a tendency of being somewhat higher towards the end of the period. The fluctuations are centered approximately around a volume of 50,000 contracts. In table 3 we get the visual observation confirmed, stating an average of 62,626 contracts, including some occasional peaks and a increasing tendency towards the end of the period.

Table 3: Descriptive statistics, monthly OBX index option volume

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<table>
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<tbody>
<tr>
<td><strong>Average</strong></td>
<td>62.626</td>
</tr>
<tr>
<td><strong>Maximum</strong></td>
<td>286.661</td>
</tr>
<tr>
<td><strong>Minimum</strong></td>
<td>21.839</td>
</tr>
<tr>
<td><strong>Standard deviation</strong></td>
<td>46.585</td>
</tr>
</tbody>
</table>

When comparing the traded OBX index option volume with the study of Mayhew and Stivers (2003), we can see that the average OBX index option volume would appear in the middle amongst the top ten firms in table 2. According to the findings in Mayhew and Stivers (2003), implied volatility from OBX index options should subsume the information of alternative statistical forecasting techniques based on historical time-series. However, the OBX volume statistics is based on all options and not only the two closest calls and two closest puts, who are utilized in the current thesis. As nothing else is mentioned, we assume that the volume for the firms in the Mayhew and Stivers (2003) study also include all options. This allows us to compare the figures with regards to number of contracts traded.

Table 4: Monthly average trading volume for OEX and SPX options

<table>
<thead>
<tr>
<th></th>
<th>OEX (S&amp;P 100)</th>
<th></th>
<th>SPX (S&amp;P 500)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Annual Volume</td>
<td>Monthly Average</td>
<td>Annual Volume</td>
<td>Monthly Average</td>
</tr>
<tr>
<td>2001</td>
<td>10,339,217</td>
<td>861,601</td>
<td>24,271,222</td>
<td>2,022,602</td>
</tr>
<tr>
<td>2002</td>
<td>13,530,159</td>
<td>1,127,513</td>
<td>29,939,598</td>
<td>2,494,967</td>
</tr>
<tr>
<td>2003</td>
<td>14,343,992</td>
<td>1,195,333</td>
<td>36,754,720</td>
<td>3,062,893</td>
</tr>
<tr>
<td>2004</td>
<td>16,426,392</td>
<td>1,368,866</td>
<td>49,472,117</td>
<td>4,122,676</td>
</tr>
<tr>
<td>2005</td>
<td>18,672,964</td>
<td>1,556,080</td>
<td>71,802,558</td>
<td>5,983,547</td>
</tr>
</tbody>
</table>

15 From CBOE 2005 Market Statistics, can be found at www.cboe.com
Christensen and Prabhala (1998) study the relationship between implied and realized volatility in the American market, focusing on the implied volatility from the call option being closest to at-the-money. They used OEX options, where the S&P 100 is the underlying index. In table 4 we can see the monthly average volume for the OEX and SPX options. We can clearly see, as expected, that OBX index options are considerably less traded than both OEX and SPX option. According to the study of Mayhew and Stivers (2003) it can seem like a higher volume leads to a higher information content in the implied volatility estimates. In this sense, implied volatilities inverted from OEX options should contain more information than implied volatilities inverted from OBX options. However, intuitively, it could be argued that the marginal increase in information content diminishes as the volume reaches a level that is fairly high. A discussion of how good the implied volatility forecast from inverted OBX options is compared to that of OEX options can be found in the result section.

We can conclude that, on a daily basis, non-trading in the underlying OBX index is as good as non-existing. However, some of the stocks constituting the index may be stale, as discussed under non-simultaneous trading. The OBX index options are traded each day, however, not necessarily in the “moneyness” we prefer. In case of a non-trading problem, we outlined a sampling procedure when the preferred option was not traded. Stale and non-simultaneous option prices will be a problem, as it could happen that an option is only traded in the middle of the day, and the index closing prices are used. Overall, the comparison with the trading volumes from Mayhew and Stivers (2003) and OEX options lead us to the conclusion that, \textit{ceteris paribus}, it should be possible to invert implied volatilities from the OBX index options with a strong information content.

4.2.2.4 \textit{Bid-ask price effect}

The fact that any transaction, and therefore also the closing prices, must take place at a bid or ask level creates the bid-ask price effect. Hentschel (2003) argues that this effect is especially strong when options are deep-in-or-out-of-the-money and options that are near maturity. Further, Roll (1984) argues that the random movement between the bid and ask price level in successive transactions will produce a significant negative first-order serial
covariance in price change series. To some extent, we are able to minimize these errors by utilizing options, which not are close to the money, neither too close to maturity, albeit some measurement errors will remain.

Jorion (1995) is estimating the error in implied volatility, when subject to the bid-ask effect, stale prices and infrequent trading. He uses an error of 0.25 percent in the measurement of the underlying spot price, which he finds conservative. Using this number, he calculates an annual error in implied volatility of 1.2 percent. He argues that this error is substantial, as it is approximately the same as a daily average variation in actual volatility. Even though we eliminate some of the sources of error in our analysis, due to similar opening hours in the option and stock market, it is likely that errors will be larger due to less frequent trading in the Norwegian market, compared to the study of Christensen and Prabhala (1998). However, we are aware of the measurement errors and choose two stage least squares as an analysis tool in addition to conventional regression analysis. The two stage least square regression might lead to better results than conventional regression analysis when measurement errors are present.

4.3 Descriptive statistics

Table 5 presents descriptive statistics for the implied volatility time series and the realized volatility time series, while table 6 present descriptive statistics of the same series in logarithmic form. We observe that the means are of lower value for the log and level implied volatility series, compared to log and level realized volatility. This is similar to what is found in Christensen and Prabhala (1998), (CP), Christensen and Hansen (2002), (CH) and Hansen (1999). Further, we can see that the Norwegian OBX index is more volatile than both the S&P 100 index, studied by CP and CH, and the Danish KFX index, studied by Hansen (1999). The realized volatility in CP is 0.13 in the period 1987 to 1995, while CH reports a realized volatility of 0.12 in the period 1993 to 1997 and Hansen (1999) report a KFX index realized volatility of 0.15 in the period 1995 to 1998. This confirms our descriptive statistics finding in the daily return series, where the OBX index is reported to most probably have a higher volatility than the S&P 100.
We see that put implied volatility have the highest mean, followed by average implied volatility and call implied volatility. CH also find the average put implied volatility being the highest, and explain that buying put options is a convenient and relative inexpensive way to implement portfolio insurance. They argue that the excess demand for puts will lead to higher prices on put options, thus a higher implied volatility compared to call implied volatility. The GARCH(1,1) forecast mean is closer to that of realized volatility, when comparing with the means of the implied volatility series. One possible explanation for this, is that the bias present in the implied volatility calculations is not present in the GARCH(1,1) forecast.

Table 5: Descriptive statistics for level series

<table>
<thead>
<tr>
<th>Level series</th>
<th>Average</th>
<th>Call</th>
<th>Put</th>
<th>Realized Volatility</th>
<th>GARCH(1,1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.21</td>
<td>0.20</td>
<td>0.22</td>
<td>0.19</td>
<td>0.19</td>
</tr>
<tr>
<td>Variance*100</td>
<td>0.33</td>
<td>0.33</td>
<td>0.37</td>
<td>0.72</td>
<td>0.39</td>
</tr>
<tr>
<td>Skewness</td>
<td>1.44</td>
<td>0.96</td>
<td>1.04</td>
<td>1.60</td>
<td>2.43</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>5.29</td>
<td>4.22</td>
<td>4.05</td>
<td>6.49</td>
<td>9.91</td>
</tr>
<tr>
<td>Jarque-Bera</td>
<td>68.14</td>
<td>26.28</td>
<td>27.53</td>
<td>113.36</td>
<td>359.95</td>
</tr>
<tr>
<td>P-value</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Descriptive statistics for monthly time series of level implied volatility from the call series and put series calculated from options closest to at-the-money, average implied volatility level series, realized volatility series and GARCH(1,1) forecast series. The individual implied volatility series are computed each month by using the Black and Scholes formula. The average implied volatility is extracted by calculating the average for the two call and two put implied volatility series calculated for each month. Realized volatility is the annualized ex-post daily return volatility over the life time of the option. The descriptive statistics are calculated on the basis of 121 observations in the time span February 1997 to February 2007.

Table 6: Descriptive statistics for logarithmic series

<table>
<thead>
<tr>
<th>Log series</th>
<th>LN Average All</th>
<th>LN Call</th>
<th>LN Put</th>
<th>LN Realized Volatility</th>
<th>LN GARCH(1,1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>-1.58</td>
<td>-1.63</td>
<td>-1.56</td>
<td>-1.73</td>
<td>-1.68</td>
</tr>
<tr>
<td>Variance*100</td>
<td>6.10</td>
<td>7.39</td>
<td>7.00</td>
<td>15.85</td>
<td>6.89</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.69</td>
<td>0.09</td>
<td>0.36</td>
<td>0.33</td>
<td>1.35</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>3.36</td>
<td>3.19</td>
<td>2.77</td>
<td>2.98</td>
<td>5.45</td>
</tr>
<tr>
<td>Jarque-Bera</td>
<td>10.26</td>
<td>0.33</td>
<td>2.91</td>
<td>2.17</td>
<td>67.19</td>
</tr>
<tr>
<td>P-value</td>
<td>0.01</td>
<td>0.85</td>
<td>0.23</td>
<td>0.33</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Descriptive statistics for monthly time series of log implied volatility from the call series and put series calculated from options closest to at-the-money, log-average implied volatility level series, and log-realized volatility series. The individual implied volatility series are computed each month by using the Black and Scholes formula. The log-average implied volatility is extracted by calculating the average for the two call and two put implied volatility series calculated for each month. Log-realized volatility is the annualized ex-post daily return volatility over the life time of the option. In addition to this, descriptive statistics for the logarithmic
transformed GARCH(1,1) forecast series are presented. The descriptive statistics are calculated on the basis of 121 observations in the time span February 1997 to February 2007.

Further, we can see that the variances for the implied volatility series are lower than the variance for the realized volatility series, something which is also observed in comparable studies. The variance for realized volatility is around twice as high as for the implied volatility measures. The variance of both implied volatility and realized volatility is also higher than what is found by Hansen (1999) for the KFX and by CH for the S&P 100. The only variances that are comparable, are those of CP for the full period 1983 to 1995, however, if you study the subperiod 1987 to 1995, the variance of CP’s call implied volatility estimate is significantly lower. Thus, we find that the Norwegian OBX index is not only volatile compared to the KFX and S&P 100, but the volatility itself is also more volatile.

![Figure 7: Quantile to quantile plots. Log series at the bottom and levels series at the top, going from left to right – average implied volatility, call implied volatility, put implied volatility, realized volatility and our GARCH(1,1) forecast.](image)

The Jarque-Bera normality test reveals, as expected, that all the logarithmic series conform closer to normality than the level series. We can see that normality is rejected for all the level series with the Jarque-Bera statistics. In addition, it is possible to visually observe that the series deviate from normality in figure 7, where we in the upper part of
the figure can see that the observations are more “bended” compared to the straight line. When we transform the series, we find that the Jarque-Bera test statistic conforms much better to reality, except for the \( \ln \text{average} \) series and \( \text{GARCH}(1,1) \) forecast series. This also confirms that the natural logarithm of the monthly realized volatility series conforms closer to normality than daily log-return series, as predicted earlier in chapter 3.4. Figure 7 shows that the observations follow the straight line more closely, thus they are in line with normality. We also observe that the \( \ln \text{average} \) and the \( \text{GARCH}(1,1) \) forecast series still has a somewhat non-normal shape. With exception of certain graphs, which are presented in their levels form to facilitate interpretation, the logarithmic transformations will be used in the following analysis, due to their beneficial properties with regards to normality and to allow comparison with results from Christensen and Prabhala (1998), Hansen (1999) and Christensen and Hansen (2002).

Wherever applicable, we will use the denotation \( h_t \) for the natural logarithmic realized volatility series, \( i_{x,t} \) for the natural logarithmic implied volatility series and \( j_t \) for the logarithmic transformed \( \text{GARCH}(1,1) \) series. The \( t \) tells which period we are analyzing, and the \( x \) will tell which implied volatility series we analyze. The \( x \) will be replaced with either \( c \) for the call implied volatility series, \( p \) for the put implied volatility series, or \( a \) for the average implied volatility series.

To enhance the visual interpretation we choose to display the outcome of the data gathering and calculation process in two graphs. In figure 8, we have plotted call and put implied volatility, in addition to realized volatility and the OBX adjusted index level against time. In figure 9, we have plotted the average implied volatility series, realized volatility, the unadjusted OBX index, in addition to our \( \text{GARCH}(1,1) \) forecast. From figure 8 and figure 9, we can observe the same \textit{stylized facts} as commented upon in part 3.2.1, descriptive statistics on the daily OBX log-return series. First, we can observe that the volatility not is constant; it rather looks like periods have different levels of volatility, where the volatility cluster and tend to be followed by volatility in the same range. This is obviously evidence against a constant volatility framework, as in the Black and Scholes formula, when options have a long time to maturity. Second, both realized, implied volatility and the \( \text{GARCH}(1,1) \) forecast tends to be higher when OBX index displays negative returns.
Figure 8: The level series of call and put implied volatility, in addition to realized volatility and the OBX adjusted index level. For illustrative purposes, the OBX index level 4:1 split on 21.04.2006 is not taken into account. The data comprise 121 observations, from February 1997 to February 2007.
Figure 9: The level series of average implied volatility, realized volatility and the GARCH(1,1) forecast with values denoted at the left axis. Values for the unadjusted OBX index are shown on the right axis, and the stock split of 21.04.2006 is not taken into account to facilitate interpretation. The data comprise 121 observations, from February 1997 to February 2007.
Further, we see that the realized volatility series is, as earlier stated, more volatile than both the implied volatility time series and the GARCH(1,1) forecast. Taking a closer look at the put and call implied volatility series in figure 8, we can see that they follow each other closely, and it is visually difficult to observe that the put implied volatility series has a somewhat higher average than the call implied volatility series. From the graph, it looks like both the implied volatility series and the GARCH(1,1) manage to capture the signal in the realized volatility series. Visually, it is very difficult to tell which implied volatility is superior as a volatility forecast and if the GARCH(1,1) forecast is better or worse than an implied volatility forecast, nevertheless, this will, of course, be statistically scrutinized in the result section.

4.4 Time-series properties

With univariate time series models, we model and predict financial variables using information contained in the time series own past values, as well as current and past values of the error term. In contrast, structural models are multivariate in nature and try to explain changes in a variable by modeling the movements in current or past values of other explanatory variables. However, it may not always be convenient to apply a structural model, as the variables thought to drive a certain process not are observable or measurable. In the following we are going to analyze the call, put, average implied volatility series, as well as the realized volatility and GARCH(1,1) forecast time-series in a univariate framework.

The Autoregressive Moving Average (ARMA) model class is often associated with Box and Jenkins (1976), because they were the first to create a methodology behind how to estimate an ARMA model. Their method is described as practical and pragmatic, and involves three steps:

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16 General information about the topic was found in ”Introductory Econometrics for Finance” by Chris Brooks (2002)
1. **Identification** – graphical procedures are used to identify and determine the order of the model required to capture the dynamic feature in the data.

2. **Estimation** – this involves estimation of the parameters of the model specified at step 1.

3. **Diagnostic checking** – determine whether the model identified and estimated is adequate. This is done in two ways, either by overfitting the model or by residual diagnostics where the autocorrelation function, partial autocorrelation function, or Ljung-Box Q-statistics could be used.

In the following we will estimate ARMA(p,q) models on the basis of our different time-series, and comment upon their properties.

### 4.4.1 ARMA(p,q)

An ARMA(p,q) model is a combination of an autoregressive, AR(p), and a moving average, MA(q), model. An ARMA(p,q) states that the current value of some series $y$ depend linearly on it’s own previous values, in addition to previous and current values of the error term. An intuitive denotation, though not the most compact one, could be written

$$
y_t = \mu + \phi_1 y_{t-1} + \phi_2 y_{t-2} + ... + \phi_p y_{t-p} + \theta_1 u_{t-1} + \theta_2 u_{t-2} + ... + \theta_q u_{t-q} + u_t, \quad (3.13)
$$

with $E(u_t) = 0; \ E(u_t^2) = \sigma^2; \ E(u_t, u_s) = 0, t \neq s$.

In table 7 we can observe values for estimated on the basis of equation (3.13) for the logarithmic transformation of our five time series. It is our goal to form a parsimonious model, which can describe the dynamics in the data using as few parameters as possible. We fitted larger models than required and reduced the number of terms, until non are insignificant. We present one AR(p) model, one MA(q) model and one ARMA(p,q) model for each time-series, in addition to this we present an AR(1) model if the AR(q) model is of higher order than one. We reject the presence of unit-root for all five time-
series at all significance levels, thus, the estimation of an ARIMA(p,d,q) model is redundant.

Starting at the top in table 7, we can see that the dynamic in the realized volatility series is best described by past values of itself, with the means of a first or second order autoregressive model. The Schwarz information criteria punish extra terms and less degrees of freedom harder than the Akaike information criteria. This is the explanation behind why the Schwarz criterion is recommending an AR(1) model, while the Aikaike criterion recommend an AR(2) model. Both models have joint insignificant autocorrelation up to lag twelve, but the AR(2) model show the lowest value and is preferable. The second order autoregressive parameter, which only is significant at a 10% level, is an argument against the AR(2) model. In sum, it could be argued that the principle of parsimonious modeling suggest that an AR(1) process best describes our realized volatility time-series. Further, the call implied volatility is best described by a third order moving average model. All the parameters are significant at a 1% level, the autocorrelation is low compared to the other model alternatives, and both information criteria suggest this model. This corresponds to the results of French et.al. (1986), who also utilized the MA(3) model to describe monthly volatility series. For the put implied volatility time-series, the information criterions suggest either an AR(2) or an ARMA(1,2) model. However, the the Ljung-Box Q-statistics is lower for the ARMA(1,2) model, and it becomes a trade-off between lower overall autocorrelation, or a simpler model with fewer terms. The same inconclusive situation is found with regards to the average implied volatility time-series. Here, the Aikaike information criterion recommends a MA(3) model and the Schwarz criterion recommends an AR(1) model. The MA(3) model have a lower Q-statistics than the AR(1) model, on the other hand, it could be argued that the AR(1) model is preferable due to its simplicity. The information criteria suggest that the GARCH(1,1) forecast time-series is best described by an AR(1) process. However, the autocorrelation left when fitting a MA(2) series is somewhat less. It should be noted that the percentage explained by the model, R-squared, is much smaller when fitting the GARCH(1,1) forecast to an ARMA(p,q) model then when fitting implied volatility. Thus, implied volatility has a better ability to predict its own future values.
Table 7: ARMA(p,q) models for all data-series

<table>
<thead>
<tr>
<th>Panel A: Realized volatility</th>
<th>μ</th>
<th>Φ₁</th>
<th>Φ₂</th>
<th>θ₁</th>
<th>θ₂</th>
<th>θ₃</th>
<th>Q-stat (12)</th>
<th>Degrees of freedom</th>
<th>Adjusted R-squared</th>
<th>Akaike info criterion</th>
<th>Schwarz criterion</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR(1)</td>
<td>-1.72 a)</td>
<td>0.42 a)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>6.96</td>
<td>11</td>
<td>17 %</td>
<td>0.8343</td>
<td>0.8807</td>
</tr>
<tr>
<td></td>
<td>(-30.05)</td>
<td>(5.04)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AR(2)</td>
<td>-1.73 a)</td>
<td>0.35 a)</td>
<td>0.16 c)</td>
<td></td>
<td></td>
<td></td>
<td>3.80</td>
<td>10</td>
<td>19 %</td>
<td>0.8305</td>
<td>0.9005</td>
</tr>
<tr>
<td></td>
<td>(-25.31)</td>
<td>(3.86)</td>
<td>(1.75)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ARMA(1,1)</td>
<td>-1.72 a)</td>
<td>0.64 a)</td>
<td>-0.27</td>
<td>(-1.37)</td>
<td></td>
<td></td>
<td>4.81</td>
<td>10</td>
<td>18 %</td>
<td>0.8310</td>
<td>0.9007</td>
</tr>
<tr>
<td>MA(3)</td>
<td>-1.73 a)</td>
<td>0.33 a)</td>
<td>0.28 a)</td>
<td>0.16 c)</td>
<td></td>
<td></td>
<td>4.66</td>
<td>9</td>
<td>18 %</td>
<td>0.8347</td>
<td>0.9272</td>
</tr>
<tr>
<td></td>
<td>(-25.32)</td>
<td>(4.13)</td>
<td>(3.04)</td>
<td>(1.78)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Panel B: Implied volatility call</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AR(1)</td>
<td>-1.63 a)</td>
<td>0.58 a)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>15.80</td>
<td>11</td>
<td>33 %</td>
<td>-0.1399</td>
<td>-0.0835</td>
</tr>
<tr>
<td></td>
<td>(-33.64)</td>
<td>(7.68)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AR(2)</td>
<td>-1.62 a)</td>
<td>0.49 a)</td>
<td>0.16 c)</td>
<td></td>
<td></td>
<td></td>
<td>11.70</td>
<td>10</td>
<td>34 %</td>
<td>-0.1417</td>
<td>-0.0717</td>
</tr>
<tr>
<td></td>
<td>(-28.27)</td>
<td>(5.32)</td>
<td>(1.71)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ARMA(1,1)</td>
<td>-1.62 a)</td>
<td>0.74 a)</td>
<td>-0.25 c)</td>
<td>(-1.68)</td>
<td></td>
<td></td>
<td>12.18</td>
<td>10</td>
<td>34 %</td>
<td>-0.1475</td>
<td>-0.0779</td>
</tr>
<tr>
<td>MA(3)</td>
<td>-1.63 a)</td>
<td>0.46 a)</td>
<td>0.38 a)</td>
<td>0.40 a)</td>
<td></td>
<td></td>
<td>7.45</td>
<td>9</td>
<td>38 %</td>
<td>-0.1828</td>
<td>-0.0903</td>
</tr>
<tr>
<td></td>
<td>(-36.76)</td>
<td>(5.50)</td>
<td>(4.39)</td>
<td>(4.82)</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Panel C: Implied volatility put</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AR(1)</td>
<td>-1.56 a)</td>
<td>0.56 a)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>25.85 a)</td>
<td>11</td>
<td>31 %</td>
<td>-0.1653</td>
<td>-0.1188</td>
</tr>
<tr>
<td></td>
<td>(-34.23)</td>
<td>(7.30)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AR(2)</td>
<td>-1.55 a)</td>
<td>0.38 a)</td>
<td>0.31 a)</td>
<td></td>
<td></td>
<td></td>
<td>10.88</td>
<td>10</td>
<td>37 %</td>
<td>-0.2429</td>
<td>-0.1728</td>
</tr>
<tr>
<td></td>
<td>(-24.57)</td>
<td>(4.32)</td>
<td>(3.54)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ARMA(1,2)</td>
<td>-1.55 a)</td>
<td>0.75 a)</td>
<td>-0.42 a)</td>
<td>(-3.40)</td>
<td>0.24 b)</td>
<td>(2.39)</td>
<td>7.05</td>
<td>9</td>
<td>38 %</td>
<td>-0.2601</td>
<td>-0.1672</td>
</tr>
<tr>
<td>MA(3)</td>
<td>-1.56 a)</td>
<td>0.41 a)</td>
<td>0.33 a)</td>
<td>0.44 a)</td>
<td></td>
<td></td>
<td>13.00</td>
<td>9</td>
<td>37 %</td>
<td>-0.2571</td>
<td>-0.1647</td>
</tr>
<tr>
<td></td>
<td>(-37.85)</td>
<td>(4.96)</td>
<td>(3.86)</td>
<td>(5.27)</td>
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<td>Panel D: Implied volatility average</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>AR(1)</td>
<td>-1.58 a)</td>
<td>0.69 a)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>13.79</td>
<td>11</td>
<td>48 %</td>
<td>-0.5763</td>
<td>-0.5293</td>
</tr>
<tr>
<td></td>
<td>(-29.18)</td>
<td>(10.37)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ARMA(1,1)</td>
<td>-1.57 a)</td>
<td>0.80 a)</td>
<td>0.13 c)</td>
<td>(-1.68)</td>
<td></td>
<td></td>
<td>10.22</td>
<td>10</td>
<td>48 %</td>
<td>-0.5817</td>
<td>-0.5113</td>
</tr>
<tr>
<td>MA(3)</td>
<td>-1.58 a)</td>
<td>0.56 a)</td>
<td>0.42 a)</td>
<td>0.44 a)</td>
<td></td>
<td></td>
<td>11.54</td>
<td>9</td>
<td>49 %</td>
<td>-0.5984</td>
<td>-0.5059</td>
</tr>
<tr>
<td></td>
<td>(-40.92)</td>
<td>(6.79)</td>
<td>(4.67)</td>
<td>(5.37)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Panel E: GARCH(1,1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AR(1)</td>
<td>-1.67 a)</td>
<td>0.47 a)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>8.69</td>
<td>11</td>
<td>22 %</td>
<td>-0.0586</td>
<td>-0.0121</td>
</tr>
<tr>
<td></td>
<td>(-41.59)</td>
<td>(5.82)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>ARMA(1,1)</td>
<td>-1.67 a)</td>
<td>0.59 a)</td>
<td>-0.16</td>
<td>(-0.85)</td>
<td></td>
<td></td>
<td>7.05</td>
<td>10</td>
<td>23 %</td>
<td>-0.0499</td>
<td>0.0197</td>
</tr>
<tr>
<td>MA(2)</td>
<td>-1.67 a)</td>
<td>0.44 a)</td>
<td>0.25 a)</td>
<td></td>
<td></td>
<td></td>
<td>6.33</td>
<td>10</td>
<td>23 %</td>
<td>-0.0551</td>
<td>0.0141</td>
</tr>
<tr>
<td></td>
<td>(-46.91)</td>
<td>(4.98)</td>
<td>(2.86)</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

a) Significant at a 1% level  b) Significant at a 5% level  c) Significant at a 10% level

Estimates of ARMA(p,q) models fitted onto the logarithmic transformed implied volatility call, put and average series, in addition to the log-realized volatility series and log-GARCH(1,1) forecast, on the form of equation (3.13). The method of ordinary least squares is used to solve equation (3.13) and estimated with the help of EViews. The data consists of 121 non-overlapping observations in the time interval February 1997 to February 2007. Asymptotic t-statistics in parenthesis and the numbers in bold present the best value for the Q-stat(12), Akaike information criterion and Schwarz criterion.
However, the GARCH(1,1) forecast’s R-square is still higher than the R-square of realized volatility.
5 Models

In the following, models are presented, which enables us to analyze and statistical compare implied volatility, realized volatility and our GARCH(1,1) forecast.

5.1 The information content

We apply a regression model to analyze the information content of implied volatility in the Norwegian stock market,

\[ h_t = \alpha_0 + \alpha_1 i_{x,t} + e_t, \]

where \( h_t \) denotes the log-realized volatility for period \( t \) and \( i_{x,t} \) denotes the \textit{ex ante} log-implied volatility forecast for implied volatility series \( x \), in period \( t \). In the literature it is usual to assess at least three hypotheses on the basis of this model:

1. If \( \alpha_1 \) is non-zero and significant, this means that implied volatility contains \textit{some} information about future realized volatility.

2. It can be said that the forecast is \textit{unbiased} if it cannot be significantly rejected that \( \alpha_0 = 0 \) and \( \alpha_1 = 1 \).

3. Finally, implied volatility is \textit{efficient} if the residuals \( e_t \) are white noise and uncorrelated with any variable in the market information set.

Two aspects considering the data and our analysis framework need to be commented upon. First, as discussed earlier, e.g. bid-ask spreads, infrequent trading and the invalidity of the Black and Scholes formula, will most probably lead to measurement errors in our variables. The presents of measurement errors will impose a downward bias in our slope.
and intercept coefficient. This, in turn, will lead to that the hypothesis of unbiasedness might be rejected too often. Second, Christensen and Prabhala (1998) points out that the logarithmic transformation of our data-series makes it hard to interpret the intercept term in the regression analysis. From table 5 and 6, we can see that $\bar{h}_t < \bar{i}_{x,t} < 0$, and in addition to this, we will see in later results that the slope coefficients tend to be less than one. This leads to a negative intercept $\alpha_0$, since the equation $\alpha_0 = \bar{h}_t - \alpha_1 \bar{i}_{x,t}$ must be satisfied. As an example, we use values from table 5 and 6 for call and realized volatility, with $\alpha_1 = 0.8$. This leads to $\alpha_0 = 0.35$ for the level series and $\alpha_0 = -0.43$ for the logarithmic series. As we can see, the intercept for the logarithmic series is further away from zero than the intercept for the levels series, leading to too frequent rejection of the hypothesis of unbiased forecasts. Hansen (1999) and Christensen and Hansen (2002) argues that due to the above mentioned reasons, a forecast is unbiased if only the restriction $\alpha_1 = 1$ cannot be statistically rejected. Since we will analyze the logarithmic series, this “weaker” test of unbiased forecast will be used alongside the traditional test. However, it will be carefully stated when we use which hypothesis.

### 5.2 Encompassing regression

To analyze if implied volatility subsumes the information in lagged realized volatility, we estimate an encompassing regression on the form,

$$ h_t = \alpha_0 + \alpha_1 i_{x,t} + \alpha_2 h_{t-1} + e_t \tag{5.2} $$

where we introduce the lagged realized volatility, $h_{t-1}$, as an independent variable. If implied volatility subsumes all the information in lagged realized volatility, $\alpha_2$ should be insignificantly different from zero. In the same way, it is possible to check if some of the implied volatility forecast subsumes the information in another implied volatility forecast by estimating an encompassing regression on the form,
\[ h_t = \alpha_0 + \alpha_1 i_{x,t} + \alpha_2 i_{y,t} + e_t, \]  

(5.3)

where

\[ x = c, p, a, y = c, p, a \text{ and } x \neq y. \]

When estimating this regression, we can also see if a combined forecast is preferable, constituting, e.g., the put and call series. If we find that both the put and call slope coefficient are significantly different from zero, the slope coefficients can be seen upon as weights to use in a weighted average forecast. Further, we can analyze if our implied volatility forecast subsumes the information in our GARCH(1,1) forecast, by estimating a similar equation to (5.1) and (5.2),

\[ h_t = \alpha_0 + \alpha_1 i_{x,t} + \alpha_2 j_t + \alpha_3 h_{t-1} + e_t. \]  

(5.4)

Here, \( j_t \) is the logarithmic transformed GARCH(1,1) forecast series. We will use equation (5.4) in mainly three ways, (1) to compare implied volatility and our GARCH(1,1) forecast, (2) to compare our GARCH(1,1) forecast with lagged realized volatility, and (3) analyze the information content in our GARCH(1,1) forecast itself. We can also see if a combined implied volatility/GARCH(1,1) forecast is preferable. The information content of the GARCH(1,1) forecast and the encompassing regressions with the GARCH(1,1) forecast and the implied volatility series will be presented in chapter 6.3, comparison with a GARCH(1,1) forecast.

5.3 Two stage ordinary least squares\(^{17}\)

One of the fundamental assumptions behind regression analysis is that the independent variables are uncorrelated with the error term. If you estimate ordinary least squares

\(^{17}\) General information about the topic is gathered from *Introductory Econometrics* by J. Wooldridge and from the Eviews’ user manual.
square, the result will be biased and inconsistent if this assumption is violated. Two classical situations where this violation takes place are:

1. When there are endogenously determined variables on the right-hand side of the equation.
2. When independent variables are measured with error.

However, it is possible to obtain consistent estimators when conducting a two stage least square procedure. The idea behind least square procedure with two or more stages is to find a set of instrumental variables that are both correlated with the independent variable in the equation, and uncorrelated with the error term. These instruments are used to eliminate the correlation between right-hand side variables and the error term.

In chapter 4.2, we argue convincingly that there exist measurement errors in the implied volatility estimates. Hence, we also estimate the equations presented under the two last sub-chapters, with exception of equation (5.3), by using the two stage least squares method. However, we will only use two stage least squares when comparing implied volatility with our GARCH(1,1) forecast if we cannot reject the presents of measurement errors in the former analysis.

5.4 Root mean square deviation

In addition to our regression analysis, we also apply root mean square deviation (RMSE) to measure forecast accuracy. This is the square root of the mean of the squared prediction errors, and is frequently used in the literature:

\[ RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^{n} u_i^2} / n , \]  

(5.5)

where

\[ u_i = \sigma^2_{(Realized)} - \sigma^2_{(Predicted)} . \]  

(5.6)
The RMSE measure will be used to compare our three implied volatility forecast with our GARCH(1,1) forecast. This prediction-test will be complementary to our encompassing regression, which due to multicollinearity problems have difficulties comparing more than two forecasts.
6 Results and discussion

6.1 Conventional analysis

6.1.1 The information content

In Table 8, we find the results from estimating equation (5.1) with the logarithmic transformed average, call and put implied volatility (IV) series as independent variables, and log-realized volatility as the dependent variable. In addition, we present the result from autocorrelation, normality and heteroskedasticity residual tests, and show the results from our two Wald coefficient tests for each estimated equation, to shed light over whether the forecasts are biased or not. We see that all three implied volatility estimates have a slope coefficients that are different from zero at a 1% significance level. This shows that our implied volatility series, as expected, contains some information about future realized volatility. Average IV has the largest slope coefficient, suggesting that average IV tells most about future volatility, followed by put IV and call IV, respectively.

<table>
<thead>
<tr>
<th>OLS Estimates</th>
<th>Residual tests, p-values: L-B Q(12) J-B White test</th>
<th>Wald coefficient tests, p-values: a0=0 and a1=1 a1=1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>Average IV Call IV Put IV Adj. R²</td>
<td>L-B Q(12) J-B White test</td>
</tr>
<tr>
<td>-0.39 c)</td>
<td>0.84 a)</td>
<td>27 %</td>
</tr>
<tr>
<td>(-1.96)</td>
<td>(6.64)</td>
<td></td>
</tr>
<tr>
<td>-0.64 a)</td>
<td>0.67 a)</td>
<td>20 %</td>
</tr>
<tr>
<td>(-3.24)</td>
<td>(5.64)</td>
<td></td>
</tr>
<tr>
<td>-0.62 a)</td>
<td>0.71 a)</td>
<td>22 %</td>
</tr>
<tr>
<td>(-3.23)</td>
<td>(5.81)</td>
<td></td>
</tr>
</tbody>
</table>

a) Significant at a 1% level  b) Significant at a 5% level  c) Significant at a 10% level

OLS estimates on the form of equation (5.1). Average implied volatility, call implied volatility and put implied volatility in their logarithmic form are substituted as independent variables, and log-realized volatility is the dependent variable. Adjusted R-squared are displayed, in addition to Wald coefficient test for unibiasness and residual tests: Ljung-Box Q-statistic with 12 lags to test for autocorrelation, Jarque-Bera test for normality and White test to test for heteroskedasticity. All test are presented by their p-values, and the null-hypothesis are, respectively, no autocorrelation, normality and homoskedasticity. The White test follows a Chi-Squared distribution, the same as the Wald coefficient test. The Wald coefficient tests have null hypothesis of, first $\alpha_0 = 0$ and $\alpha_1 = 1$, second $\alpha_1 = 1$. 
We also observe that average IV is able to explain more of the variation in realized volatility, measured by adjusted R-square. It can seem like the bias in call and put volatility is, to a certain extent, reduced when taking the average of the two call and two put implied volatilities closest to at-the-money. This is confirmed by the coefficient tests, where it cannot be rejected that the slope coefficient for average IV is different from unity. The slope coefficient for call IV and put IV can be rejected to be equal to unity at a 5% significance level, however, not at a 1% significance level. The joint hypothesis, where the intercept equals zero and the slope coefficient equals one, is rejected for all three implied volatility series. Further, we can see that all residuals are normally distributed, homoskedastic and do not show any sign of autocorrelation. These results are far better than what is found in the Danish market by Hansen (1999), where the log-call implied volatility slope coefficient is estimated to be 0.38, and only significant at a 5% level. The adjusted R-square is also lower, 16.6%, compared to 20% for OBX call implied volatility. This is rather surprising, since the two countries are very similar, and you would intuitively think that the two markets are relatively similar. However, call implied volatility from OBX index options gives estimates that contains almost twice as much information about future realized volatility, as the reported call IV slope coefficient is 0.67 at a 1% significance level. Christensen and Prabhala (1998) reports the slope coefficient for OEX call implied volatility to be 0.76 and significant at a 1% level, while Christensen and Hansen (2002) reports the slope coefficient for OEX call implied volatility for a more recent period to be 0.83 at a 1% significance level. These results from the OEX implied volatilities contains, as expected, somewhat more information about future realized volatility. In addition to this, the R-square is considerably higher – 39% in Christensen and Prabhala’s study and 26.7% in the study by Christensen and Hansen. Mayhew and Stivers (2003) argues that there exists a relationship between option market liquidity and the forecasting ability of implied volatility, something that may explain the difference in the information content. Though, if you take the enormous liquidity difference into account, it is impressing how close the information content from the OBX call implied volatility is to the information content from the S&P 100 call implied volatility, where the options are substantially more liquid.
6.1.2 Encompassing regression

In table 9, OLS estimates of equation (5.2) are reported. We can see that lagged realized volatility explains future realized volatility, however, with a lower information content and adjusted R-square than all the implied volatility series. Further, we observe

Table 9: Encompassing regression: OLS estimates

<table>
<thead>
<tr>
<th>OLS Estimates</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th>Residual tests, p-values:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>Average IV</td>
<td>Call IV</td>
<td>Put IV</td>
<td>RV_{t-1}</td>
<td>Adj.R²</td>
</tr>
<tr>
<td>-0.41 b)</td>
<td>0.76 a)</td>
<td></td>
<td></td>
<td>0.07</td>
<td>26 %</td>
</tr>
<tr>
<td>(-1.98)</td>
<td>(3.93)</td>
<td></td>
<td></td>
<td>(0.56)</td>
<td></td>
</tr>
<tr>
<td>-0.61 a)</td>
<td></td>
<td></td>
<td></td>
<td>0.20 c)</td>
<td>22 %</td>
</tr>
<tr>
<td>(-3.12)</td>
<td></td>
<td></td>
<td></td>
<td>(1.83)</td>
<td></td>
</tr>
<tr>
<td>-0.60 a)</td>
<td></td>
<td></td>
<td>0.52 a)</td>
<td></td>
<td>23 %</td>
</tr>
<tr>
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<td></td>
<td></td>
<td>(3.10)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-0.99 a)</td>
<td>0.18 c)</td>
<td></td>
<td></td>
<td>0.42 a)</td>
<td>17 %</td>
</tr>
<tr>
<td>(-6.73)</td>
<td></td>
<td>(1.66)</td>
<td></td>
<td>(5.04)</td>
<td></td>
</tr>
</tbody>
</table>

Table 9: Encompassing regression: OLS estimates

<table>
<thead>
<tr>
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<td>17 %</td>
</tr>
<tr>
<td>(-6.73)</td>
<td></td>
<td>(1.66)</td>
<td></td>
<td>(5.04)</td>
<td></td>
</tr>
</tbody>
</table>

OLS estimates on the form of equation (5.2). An encompassing regression, used to compare average, call and put volatilities with lagged realized volatility. Realized volatility as dependent variable, and all data are estimated in their logarithmic form. Adjusted R-square is displayed, in addition to residual tests’ p-values.

that average IV subsume all information in lagged realized volatility. This suggests that average IV is an efficient forecast of realized volatility in accordance to the test put forward by Christensen and Prabhala (1998), Hansen (1999) and Christensen and Hansen (2002). However, the market’s information set doesn’t solely consist of lagged realized volatility – the market utilizes more sophisticated time series models. Therefore, we compare our implied volatility forecasts with a GARCH(1,1) forecast in chapter 6.3. This is an extension compared to the efficiency tests in the three previously mentioned papers, and will shed a clearer light on the question of whether the implied volatility forecasts really are efficient or not. Call and put implied volatility do also subsume most of the information in lagged realized volatility. We can observe that the slope coefficient values for lagged realized volatility are halved when entering into an encompassing regression alongside call and put implied volatility. The significance level also drops from 1% to a 10% level, being close to insignificant. The residual tests are not showing any signs of
Results and discussion

non-normality, autocorrelation or heteroskedasticity. These results are actually better than those of Christensen and Prabhala (1998) and those of Hansen (1999), implying a strong information content in implied volatility relative to lagged realized volatility in the Norwegian market. However, in the more recent study on OEX by Christensen and Hansen (2002), realized volatility enters insignificantly into the encompassing regression, both when compared with put and call implied volatility.

Table 10: Encompassing regression: OLS estimates

<table>
<thead>
<tr>
<th>OLS Estimates</th>
<th>Residual tests, p-values:</th>
<th>Wald coefficient tests, p-values:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dependent variable: Realized Volatility</td>
<td>L-B Q(12)</td>
<td>J-B</td>
</tr>
<tr>
<td>Intercept</td>
<td>Average IV</td>
<td>Call IV</td>
</tr>
<tr>
<td>-0.40 c)</td>
<td>0.93 a)</td>
<td>-0.08</td>
</tr>
<tr>
<td>(-1.97)</td>
<td>(3.19)</td>
<td>(-0.31)</td>
</tr>
<tr>
<td>-0.40 c)</td>
<td>0.87 a)</td>
<td>-0.02</td>
</tr>
<tr>
<td>(-1.95)</td>
<td>(2.76)</td>
<td>(-0.08)</td>
</tr>
<tr>
<td>-0.44 b)</td>
<td>0.37 b)</td>
<td>0.44 b)</td>
</tr>
<tr>
<td>(-2.13)</td>
<td>(2.26)</td>
<td>(2.60)</td>
</tr>
</tbody>
</table>

a) Significant at a 1% level  b) Significant at a 5% level  c) Significant at a 10% level

OLS estimates on the form of equation (5.3). An encompassing regression used to compare average, call and put volatilities, in addition to see if a combined forecast is preferable. The dependent variable is realized volatility. Residuals tests are displayed, in addition to coefficient test, to test if the forecast is biased or not. A natural restriction test would be to see if the two slope coefficient in each regression sums up to unity. This test is displayed under the heading a1+a2=1. The second coefficient test also restricts the intercept term to be equal to zero. Realized volatility as dependent variable, and all data are estimated in their logarithmic form.

In table 10, OLS estimated of equation (5.3) are reported. We can observe that average implied volatility subsumes the information in both call and put implied volatility.

Further, none of the estimated equations exhibit non-normality, autocorrelation or heteroskedasticity. The coefficient tests are slightly altered to make sense in a context where we can choose to combine two forecasts. We introduce the restriction \( \alpha_1 + \alpha_2 = 1 \), which replace the old restriction, \( \alpha_1 = 1 \). Here, \( \alpha_1 \) represents the first implied volatility slope coefficient, and \( \alpha_2 \) represent the second implied volatility slope coefficient. The result from the coefficient tests in the first and second line in table 10, equals those of table 8, where the information content of implied volatility is investigated. This is as expected, since put and call implied volatility enters insignificantly into the equation (3.3) when compared with average implied volatility. Thus, average implied volatility
subsumes all the information in both the call and put implied volatility series. In line three in table 10, we can see that put and call implied volatility both enters significantly into equation (3.3) at a 5% level. The least restrictive slope coefficient test also fails to reject that the sum of the put and call implied volatility slope coefficient sums up to unity, with a p-value of 0.14. This suggests that a combined forecast with call and put implied volatility slope coefficient’s as weights is unbiased, and it could be interesting to test as a volatility forecast. We notice this result, however, to study different weighting schemes is out of the scope of the current paper.

6.2 Two Stage Least Squares

6.2.1 The information content

Consistent estimation when errors-in-variable problems are present can be achieved by using an instrumental variable method. Thus, instruments are called for. Lagged values of implied volatility itself and realized volatility are natural candidates used in the previous literature. Hansen (1999) suggests testing down the most parsimonious model using OLS. The resulting model takes the following form:

\[ i_{x,t} = \alpha_0 + \alpha_1 i_{x,t-1} + \alpha_2 i_{x,t-2} + \alpha_3 h_{t-1} + e_t \]  \hspace{1cm} (6.1)

Estimates on the basis of this equation, in addition to the two stage least squares (TSLS) estimates on the form of equation (5.1), are presented in table 11.

We can see from table 11 that the average IV estimates are not much altered, compared to those from the OLS regression. The only difference being a stronger p-value with regards to the coefficient test, confirming even stronger that the slope coefficient not is significantly different from unity. The call IV and put IV slope coefficient have higher information content then when estimated with OLS regression. The call IV slope coefficient takes a value of 0.80 and the put IV slope coefficient takes a value of 0.88, compared to OLS estimates of 0.67 and 0.71, respectively.
Table 11: Information content: TSLS estimates

<table>
<thead>
<tr>
<th>Average IV</th>
<th>Dependent variable: Average IV</th>
<th>Residual tests, p-values:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>Average IV&lt;sub&gt;t-1&lt;/sub&gt;, Average IV&lt;sub&gt;t-2&lt;/sub&gt;, RV&lt;sub&gt;t-1&lt;/sub&gt;</td>
<td>Adj.R&lt;sup&gt;2&lt;/sup&gt;</td>
</tr>
<tr>
<td>-0.27 a)</td>
<td>0.30 a)</td>
<td>0.70</td>
</tr>
<tr>
<td>(-2.98)</td>
<td>(3.78)</td>
<td>(2.41)</td>
</tr>
</tbody>
</table>

Dependent variable: Realized Volatility

Instruments for Average IV: Ave. IV<sub>t-1</sub>, Ave. IV<sub>t-2</sub> and RV<sub>t-1</sub>

<table>
<thead>
<tr>
<th>Intercept</th>
<th>Average IV</th>
<th>Adj.R&lt;sup&gt;2&lt;/sup&gt;</th>
<th>L-B Q(12)</th>
<th>J-B</th>
<th>White test</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.41 c)</td>
<td>0.84 a)</td>
<td>0.27</td>
<td>0.83</td>
<td>0.87</td>
<td>0.67</td>
</tr>
<tr>
<td>(-1.67)</td>
<td>(5.45)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Call IV

<table>
<thead>
<tr>
<th>Intercept</th>
<th>Call IV&lt;sub&gt;t-1&lt;/sub&gt;, Call IV&lt;sub&gt;t-2&lt;/sub&gt;, RV&lt;sub&gt;t-1&lt;/sub&gt;</th>
<th>Adj.R&lt;sup&gt;2&lt;/sup&gt;</th>
<th>L-B Q(12)</th>
<th>J-B</th>
<th>White test</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.36 a)</td>
<td>0.25 a)</td>
<td>0.56</td>
<td>0.07</td>
<td>0.01</td>
<td>0.93</td>
</tr>
<tr>
<td>(-2.95)</td>
<td>(3.02)</td>
<td>(2.11)</td>
<td>(7.64)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Dependent variable: Realized Volatility

Instruments for Call IV: Call IV<sub>t-1</sub>, Call IV<sub>t-2</sub> and RV<sub>t-1</sub>

<table>
<thead>
<tr>
<th>Intercept</th>
<th>Call IV</th>
<th>Adj.R&lt;sup&gt;2&lt;/sup&gt;</th>
<th>L-B Q(12)</th>
<th>J-B</th>
<th>White test</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.43 c)</td>
<td>0.80 a)</td>
<td>0.20</td>
<td>0.80</td>
<td>0.67</td>
<td>0.95</td>
</tr>
<tr>
<td>(-1.66)</td>
<td>(5.00)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Put IV

<table>
<thead>
<tr>
<th>Intercept</th>
<th>Put IV&lt;sub&gt;t-1&lt;/sub&gt;, Put IV&lt;sub&gt;t-2&lt;/sub&gt;, RV&lt;sub&gt;t-1&lt;/sub&gt;</th>
<th>Adj.R&lt;sup&gt;2&lt;/sup&gt;</th>
<th>L-B Q(12)</th>
<th>J-B</th>
<th>White test</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.30 a)</td>
<td>0.17 b)</td>
<td>0.58</td>
<td>0.49</td>
<td>0.53</td>
<td>0.30</td>
</tr>
<tr>
<td>(-2.74)</td>
<td>(2.17)</td>
<td>(3.51)</td>
<td>(7.66)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Dependent variable: Realized Volatility

Instruments for Put IV: Put IV<sub>t-1</sub> and RV<sub>t-1</sub>

<table>
<thead>
<tr>
<th>Intercept</th>
<th>Put IV</th>
<th>Adj.R&lt;sup&gt;2&lt;/sup&gt;</th>
<th>L-B Q(12)</th>
<th>J-B</th>
<th>White test</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.36 a)</td>
<td>0.88 a)</td>
<td>0.21</td>
<td>0.39</td>
<td>0.74</td>
<td>0.95</td>
</tr>
<tr>
<td>(-1.43)</td>
<td>(5.44)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a) Significant at a 1% level  b) Significant at a 5% level  c) Significant at a 10% level

Two stage least squares estimates of equation (6.1). The most parsimonious model is tested down by using OLS, and instrument variable coefficients, test statistic, Wald coefficient test p-values and residual test p-values output is presented. The numbers in parenthesis are asymptotic t-statistics. All data are estimated in their logarithmic form.

This is an indication that errors-in-variables are present. To formally test this indication, we introduce a Hausman test, defined by\(^\text{18}\),

\[
H = (b_{IV} - b_{OLS})\left[[SE^2(b_{IV}) - SE^2(b_{OLS})]^{-1} \right] (b_{IV} - b_{OLS}),
\]

\(^{18}\) Yale lecture note; http://research.yale.edu/vote/Lecture%209.doc
Results and discussion

where $b$ is the coefficient estimate found by either instrumental variable regression or ordinary least squares. The null hypothesis of no measurement errors present is tested against the alternative hypothesis of measurement errors present. The test statistics follows a chi-square distribution with degrees of freedom equal to the rows of each coefficient vector, thus we have one degree of freedom.

Table 12: Hausman p-values

<table>
<thead>
<tr>
<th></th>
<th>Hausman test-statistics</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average IV</td>
<td>0.008</td>
<td>0.93</td>
</tr>
<tr>
<td>Call IV</td>
<td>1.492</td>
<td>0.22</td>
</tr>
<tr>
<td>Put IV</td>
<td>2.575</td>
<td>0.11</td>
</tr>
</tbody>
</table>

Hausman test-statistics follow a chi-squared distribution with 1 degree of freedom.

The alternative hypothesis of measurement error present is not significant at any reasonable significant level for either of the implied volatility estimates. However, our findings indicate that the probability of measurement error is highest in put implied volatility, followed by call implied volatility - average implied volatility shows no sign of measurement errors. These findings contradict those of Christensen and Prabhala (1998) and Hansen (1999), which find the Hausman test to be significant at a 5% level. However, the literature is not unanimous, as Christensen and Hansen (2002) neither can reject the null hypothesis of no measurement error. Further, the slope coefficients for average, call and put IVs cannot be rejected to be equal to unity at any reasonable confidence level.

These are positive results for the use of implied volatility as a volatility forecast in the Norwegian market, and suggest that average, call and put volatility forecasts are efficient. The results are, as expected, not as good as those found in Christensen and Prabhala (1998) and Christensen and Hansen (2002). However, the results are better than those of Hansen (1999), who studies a much more comparable market. Using a comparable implied volatility estimate to our “average volatility”, Hansen (1999) reports the slope coefficient to be 0.61 and not significantly different from 1. The slope coefficients from the Norwegian market are noticeably higher, revealing a stronger information content in the implied volatility forecasts in the Norwegian market.
6.2.2 Encompassing regression

Two stage least square estimates on the form of equation (5.2) can be found in table 13 - the instrumental variable estimates are not presented here, but can be found in table 11. From table 13, we can see that implied volatility, overall, subsumes the information in lagged realized volatility. However, the slope coefficient estimated

<table>
<thead>
<tr>
<th>Average IV</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Dependent variable: Realized Volatility</td>
<td>Instruments for Average IV: Average IV_{t-1}, Average IV_{t-2} and RV_{t-1}</td>
<td>Residual tests, p-values:</td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>Average IV</td>
<td>Realized Volatility _1</td>
<td>Adj.R²</td>
</tr>
<tr>
<td>-0.60 c)</td>
<td>0.51</td>
<td>0.19</td>
<td>25 %</td>
</tr>
<tr>
<td>(-1.98)</td>
<td>(1.47)</td>
<td>(1.08)</td>
<td></td>
</tr>
<tr>
<td>Call IV</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dependent variable: Realized Volatility</td>
<td>Instruments for Call IV: Call IV_t-1, Call IV_t-2 and RV_t-1</td>
<td>Residual tests, p-values:</td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>Call IV</td>
<td>Realized Volatility _1</td>
<td>Adj.R²</td>
</tr>
<tr>
<td>-0.68 c)</td>
<td>0.39</td>
<td>0.24</td>
<td>22 %</td>
</tr>
<tr>
<td>(-1.91)</td>
<td>(0.97)</td>
<td>(1.19)</td>
<td></td>
</tr>
<tr>
<td>Put IV</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dependent variable: Realized Volatility</td>
<td>Instruments for Put IV: Put IV_t-1, Put IV_t-2 and RV_t-1</td>
<td>Residual tests, p-values:</td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>Put IV</td>
<td>Realized Volatility _1</td>
<td>Adj.R²</td>
</tr>
<tr>
<td>-0.47</td>
<td>0.68 c)</td>
<td>0.11</td>
<td>22 %</td>
</tr>
<tr>
<td>(1.51)</td>
<td>(1.89)</td>
<td>(0.59)</td>
<td></td>
</tr>
</tbody>
</table>

Table 13: Encompassing regression: TSLS estimates

Two stage least square estimates on the form of equation (5.2). Instrument estimation is shown in table 11. Residual test p-values are shown – asymptotic t-statistic in parenthesis. Realized volatility as dependent variable, and all data are estimated in their logarithmic form.

for the put implied volatility series is the only one that is significant, though only at a 10% level. All the implied volatility slope coefficients are larger than the corresponding lagged realized volatility slope coefficients, and with exception of call implied volatility, the t-statistics are larger as well. This shows that implied volatility subsumes most of the information in lagged realized volatility. However, the use of instrumental variables to eliminate errors-in-variables does not improve the information content in average and call implied volatility. On the contrary, the call and average implied volatility slope
coefficient actually diminish and are now insignificant. The put IV slope coefficient is larger, compared to the OLS estimates, and stays significant at the same time as lagged realized volatility becomes insignificant. Though, with a Hausman test taking on a value of 0.27, we can reject the alternative hypothesis of “measurement error present” at all significance levels. Even though we have argued for the presents of measurement errors in implied volatility calculations, the Hausman test and the poor performance of instrumental variable regression do not support this view. It is possible that the different measurement errors, to a large extent, neutralize each other. These findings do, as mentioned earlier, contradict those of Christensen and Prabhala (1998) and Hansen (1999), which cannot reject the presents of measurement errors.

6.3 Comparison with a GARCH(1,1) forecast

As the previous analysis have rejected the presents of measurement errors, the argument in favor of a two stage least squares estimation method is significantly weaker. Thus, the following analysis is estimated with OLS, in addition to the calculation of

<table>
<thead>
<tr>
<th>Intercept</th>
<th>GARCH(1,1)</th>
<th>(RV_{t-1})</th>
<th>Adj. R²</th>
<th>Residual tests, p-values:</th>
<th>Wald coefficient tests, p-values:</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.29 (-1.47)</td>
<td>0.86 a) (7.51)</td>
<td>32 %</td>
<td>0.25 0.49 0.65</td>
<td>0.01 0.22</td>
<td></td>
</tr>
<tr>
<td>-0.28 (-1.40)</td>
<td>0.90 a) (5.01)</td>
<td>-0.04 (-0.30)</td>
<td>31 %</td>
<td>0.22 0.44 0.17</td>
<td>0.11 0.26</td>
</tr>
</tbody>
</table>

a) Significant at a 1% level  b) Significant at a 5% level  c) Significant at a 10% level

OLS estimates on the form of equation (5.4). Asymptotic t-statistics are denoted in parenthesis. Residual test p-values are presented, in addition to coefficient restriction test p-values. The data sample constitute 121 observation in the time span from February 1997 to February 2007.

root mean square deviation to check the robustness of our findings. In table 14, OLS estimates on the form of equation (5.4) are reported. The slope coefficient is 0.84 for our GARCH(1,1) forecast, with a adjusted R-square of 32%. These results are clearly better than those of call and put implied volatility, and somewhat better than the results of
average implied volatility. The slope coefficient of average implied volatility is 0.84 and the adjusted R-square is 27%, compared to the GARCH(1,1) forecast results, estimated to be 0.6 and 32%, respectively. Opposite to our implied volatility forecasts, the GARCH(1,1) forecast cannot be rejected to have an intercept that equals zero and a slope coefficient that equals unity. The GARCH(1,1) forecast also clearly subsumes all the information in lagged realized volatility. These are all strong results, both independently and when compared with the results from our implied volatility series.

In table 15, results are presented from an encompassing regression on the form of equation (5.4). Here, our GARCH(1,1) forecast is compared to the implied volatility series. We can see that when the GARCH(1,1) forecast is added as an explanatory variable in addition to one of the implied volatility forecasts, the implied volatility slope coefficients drop noticeably. The GARCH(1,1) slope coefficient drops less and continues to be significant at a 1% level. This suggests a stronger information content in the GARCH(1,1) forecast, compared to our implied volatility forecasts. However, the implied volatility OLS estimates still are significant, which suggests that a combined forecast is preferable. Further, the results suggest that an implied volatility forecast is inefficient in the Norwegian market. The poor performance of the implied volatility forecast in the Norwegian market, compared to our GARCH(1,1) forecast might be attributed to the liquidity of the Norwegian option market, even though the study of

<table>
<thead>
<tr>
<th>Table 15: Encompassing regression: TSLS estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>OLS Estimates</strong></td>
</tr>
<tr>
<td>Depend: Realized Volatility</td>
</tr>
<tr>
<td>Intercept</td>
</tr>
<tr>
<td>-0.08</td>
</tr>
<tr>
<td>(-0.39)</td>
</tr>
<tr>
<td>-0.10</td>
</tr>
<tr>
<td>(-0.49)</td>
</tr>
<tr>
<td>-0.17</td>
</tr>
<tr>
<td>(-0.81)</td>
</tr>
</tbody>
</table>

a) Significant at a 1% level  b) Significant at a 5% level  c) Significant at a 10% level

OLS estimates on the form of equation (5.4). Asymptotic t-statistics are denoted in parenthesis. Residual test p-values are presented, in addition to coefficient restriction test p-values. The data sample constitutes 121 observations in the time span from February 1997 to February 2007.
Mayhew and Stivers (2003) suggest otherwise. Our findings contradict the common opinion in the literature, and intuition in general, which argues that an implied volatility forecast should be superior to a statistical forecast, due to its forward looking nature.\footnote{See Poon and Granger (2005) for an extensive survey}

<table>
<thead>
<tr>
<th>Table 16: RMSE results</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Garch(1,1)</td>
</tr>
<tr>
<td>2 Average IV</td>
</tr>
<tr>
<td>3 Call IV</td>
</tr>
<tr>
<td>4 Put IV</td>
</tr>
</tbody>
</table>

RMSE, calculated as shown in equation (3.5) and (3.6). The forecast with the best accuracy is shown at first place.

The coefficient restriction tests cannot reject that the slope coefficients are equal to unity, though the stricter test can be rejected at a 1% level for all three specifications. The root mean square errors are calculating as in equation (5.5) and (5.6). The results are presented in table 16, and shows that the GARCH(1,1) has the highest forecast accuracy, followed by average, call and put implied volatility. This result underpins our previous findings, and favor a GARCH(1,1) forecast as oppose to an implied volatility forecast in the Norwegian market.
7 Conclusion

The fundamental question addressed in this thesis is whether the implied volatility from OBX index options do predict realized volatility in the Norwegian market. Several studies have shown that implied volatility is in fact an unbiased, efficient forecast of realized volatility (e.g., Christensen and Prabhala (1998), Christensen and Hansen (2002), Szakmary et. Al. (2003), Corrado and Miller (2004)), however, almost all articles study implied volatilities calculated from highly liquid option markets. The only truly comparable study found, is that of Hansen (1999), who studies option implied volatility in the Danish option and equity markets.

We adapt the research methodology of Christensen and Prabhala (1998), who employ a lower (monthly) sampling frequency. As an extension to the study of Christensen and Prabhala (1998), which only studies call implied volatility, we also study put implied volatility and an average implied volatility estimate, being the average of the two closest to at-the-money put implied volatilities and two closest to at-the-money call implied volatilities. Our results reveal the slope coefficient from the OLS estimation to be 0.67, 0.71 and 0.84, for call, put and average implied volatility, respectively. These results clearly show that implied volatility contains information about ex-post realized volatility. However, only average implied volatility can be argued to be an unbiased predictor of realized volatility. These results are strong compared to the findings in the Danish market, which reveals a call implied volatility slope coefficient of 0.38, and, they are surprisingly strong compared to the much more liquid US market, where Christensen and Prabhala (1998) find the call implied volatility slope coefficient to be 0.67.

While average implied volatility clearly subsumes all the information in lagged realized volatility, put and call implied volatility manage to subsume a considerable share of the information in lagged realized volatility. These results are also noteworthy stronger than what is found in the Danish option and equity markets. Christensen and Prabhala (1998), Christensen and Hansen (2002) and Hansen (1999) consider the market information set to be restricted to lagged realized volatility, and labels implied volatility efficient if implied volatility subsumes the information in lagged realized volatility. We
extend the market information set in a natural way, and also include a GARCH(1,1) forecast.

Several sources of error are identified in our implied volatility calculation process, e.g. bid-ask spread, invalidity of Black and Scholes assumptions, non-simultaneous trading and infrequent trading. To account for measurement error, the regression parameters are also estimated using an instrumental variable approach. However, a Hausman test rejects the presents of measurement error for all three implied volatility series, at all reasonable significance levels.

We calculate a GARCH(1,1) forecast, and compare it with our three implied volatility forecasts. We find that GARCH(1,1) dominates our implied volatility forecasts, is an unbiased predictor of future realized volatility and have a larger slope coefficient than implied volatility in an encompassing regression. The RMSE results confirm that the GARCH(1,1) forecast is more accurate than the implied volatility forecasts. However, both implied volatility are our GARCH(1,1) forecast are significant in the encompassing regression, thus the optimal forecast in the Norwegian market is a combined forecast between implied volatility and our GARCH(1,1) forecast. This is surprising given Mayhew and Stivers’ (2003) results, which argue that implied volatility should subsume the information of a GARCH forecast when the options are as frequently traded as the OBX index options are. The study of Mayhew and Stivers (2003) is an indication that there are other reasons than just the level of liquidity that hinders the OBX index implied volatility to be efficient and subsume the information in our GARCH(1,1) forecast.

As a final summarization we can state that implied volatility in the Norwegian market have a strong information content in comparison to the Danish market and the much more liquid US market, in addition to subsume all the information in lagged realized volatility. Our average implied volatility estimate is the best implied volatility forecast and subsumes the information in our call and put implied volatility series. Further, we can argue that implied volatility is unbiased, however, due to our stronger efficiency criteria, implied volatility can not be said to be efficient.
8 Limitations and suggestions for future research

During the thesis, we have pointed out some limitations to our analysis. The presents of these restrictions emerge, e.g., due to our limited data-set. To make our analysis even stronger, data with a longer time-span is beneficial, especially when taking into consideration our monthly sampling procedure. However, when comparing to similar studies, a data-set spanning 10 years is acceptable. A more extensive data-set would allow us to study the relationship between implied volatility and high-frequency return data. With high frequency return data we could get a deeper understanding of the inherently unobservable volatility in the Norwegian market. A data-set where the “number of contracts traded” is stated would also be preferable. With this information it would be possible to study at least to things: (1) Calculate a trade-weighted implied volatility estimate, and (2) it would be possible to sort our implied volatility data-series with regards to option-liquidity. Thus, it would be possible to carry out a similar study to that of Mayhew and Stivers (2003), and compare, e.g., the top quintile and bottom quintile implied volatilities. This would enable us to truly analyze if there are large discrepancies in implied volatility’s ability to predict future realized volatility or subsume the information in another volatility forecast, due to Norwegian option-market liquidity.

There are several additional issues that would be interesting to study in the Norwegian market, though they are not included in the thesis due to the thesis’ inherently narrow scope. When focusing on implied volatility, it could be interesting to test out and compare different weighting schemes. It would also be interesting to analyze the combined put/call forecast and average implied volatility/GARCH(1,1) forecast found in the current thesis.

We present several sources of error when calculating implied volatility, however, an analysis of the presents of the different measurement errors in the Norwegian market would shed light over an interesting topic and might lead to ways to improve our implied volatility calculations.
Looking at volatility forecasting in general, a broader comparison between different volatility forecasts could obviously have been done in the Norwegian market. In addition to study forecast accuracy, it would be interesting to investigate which forecast that performs best at different time-horizons. With regards to implied volatility and a broader volatility forecast comparison, it would be out of interest not only to compare the forecast on a statistical basis, but also include an economic analysis. This could take place by, e.g., device a trading rule for at-the-money straddles, as done by Engle et. al. (1994), and see which volatility forecast is producing the largest profit.

Finally, we know that when studying our volatility forecasts, we need to assume that the option market is efficient. To strengthen (or weaken) the argument in favor of the use of implied volatility as a forecast in the Norwegian market, it would be interesting to study the efficiency of the Norwegian option market.
REFERENCES


